ON THE THEORY OF FERROMAGNETISM IN METALS

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The methods of quantum field theory are used to investigate the properties of a ferromagnetic metal. The interaction between Fermi excitations and spin waves is examined and their contributions to the temperature dependence of the thermodynamic quantities are derived. The condition for occurrence of ferromagnetism in a metal is also determined.

1. In an earlier paper [1] we developed the theory of an isotropic ferromagnetic Fermi liquid. Here we extend the results to the case of a ferromagnetic metal.

Our fundamental assumption is the one which lies at the root of Landau's theory of a Fermi liquid, [2] and has received support from a microscopic analysis [3]: that the switching on of the interaction between the fermions does not alter the classification of the energy levels of the system. For the electrons in a crystal this statement reduces to the assumption that the Fermi excitations of the system, like the levels of the corresponding one-electron problem, form energy bands; in the case of a metal some of these will be incompletely filled. The transition metals, with which we shall be concerned in this work, are characterized by the presence not only of bands of conduction-electron (s-electron) excitations, which originate in the atomic valence electrons and are distinguished by a high mobility, but also of d-bands; the excitations of the latter originate in the incompletely filled inner shells of atomic electrons, and have only a low mobility. As a result the d-bands are considerably narrower than the s-bands. We shall assume that the metal considered is ferromagnetic, i.e., that the degeneracy in the energy of the excitations corresponding to spin orientation is lifted and that there is a finite mean magnetic moment. As a result, the Fermi surfaces for excitations of different spin orientation do not coincide even when they belong to the same band.

2. As shown by Abrikosov [4], the one-particle Green function of the electrons in a metal can be written in the form:

$$G_{\alpha\beta}(x, x') = -i \langle T(\tilde{\psi}_\alpha(x)\tilde{\psi}_\beta^+(x'))\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')}$$

$$\times G_{\alpha\beta}(p, \mathbf{r}, \mathbf{r}') = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \sum_n G_n(p) u_n(p, \mathbf{r}, \alpha)$$

$$\times u_n^*(p, \mathbf{r}', \beta).$$

Here and subsequently $p = (\epsilon, \mathbf{p})$ is the energy and pseudo-momentum, $px = \mathbf{p} \cdot \mathbf{r} - \epsilon t$, $d^4p = d\epsilon\, d\mathbf{p}$, with the $\mathbf{p}$-integration taken over an elementary cell of the reciprocal lattice of the crystal; n includes the two indices ν, which numbers the irreducible representation of the little group of the vector $\mathbf{p}$, realized by the functions $u_n(p, \mathbf{r}, \alpha)$, and s, which numbers the functions of a given representation.

We notice that the functions $u_n(p, \mathbf{r}, \alpha)$ depend on both the quasi-momentum $\mathbf{p}$ and the energy ϵ, and so cannot be eigenfunctions of any Hamiltonian. Their fundamental property is that in the representation defined by them the Green function is diagonal. It follows from the results of [4] that if $\mathbf{p}_0$ lies on the Fermi surface of the n-th band and there is no exact degeneracy, then in the limit $\mathbf{p} \to \mathbf{p}_0$, $\epsilon \to 0$ the Green function has the form

$$G_{\alpha\beta}(p, r, r') = \frac{a_n(\mathbf{p}_0) u_n(p, \mathbf{r}, \alpha) u_n^*(p, \mathbf{r}', \beta)}{\varepsilon + \mu - \varepsilon_n(\mathbf{p}) + i\delta\, \text{sign}\, \varepsilon} \quad (2)$$

where μ is the chemical potential.

Since the functions $u_n(p, \mathbf{r}, \alpha)\, e^{ipx}$ form a complete set of mutually orthogonal functions, not only the Green functions but all other quantities of interest in quantum field theory can be decomposed in terms of them. These quantities now depend not only on the "momenta" (as in the isotropic case) but also on the band indices n; however, provided we work with the complete Green functions (to which the above remarks refer) the only change necessary in the diagram technique consists in carrying out, in addition to an integration over the "momenta" of the internal lines, a

summation over the corresponding band indices.

3. We shall be interested in the singularities of the vertex part $\Gamma(p_1, p_2; k)$ (defined as in [1]) as a function of the "momentum" transfer $k = (\omega, \mathbf{k})$; these correspond to spin waves. The relevant singularities occur in the transverse spin components of Γ. Without going through the derivation, which is completely analogous to the one used for an isotropic Fermi liquid [1], we write down the expression for the pole term of the transverse component of Γ, neglecting magnetic interactions:

$$G^{II}_{l_+ m_- n_- s_+}(p_1, p_2; k)$$

$$= G_l^+(p_1) G_n^-(p_1 + k) \Gamma_{l_+ m_- n_- s_+}(p_1, p_2; k)$$

$$\times G_m^-(p_2 + k) G_s^+(p_2)$$

$$= i\Delta_l^+(p_1, k) D(k) \Delta_m^-(p_2, k) \delta_{ln}\delta_{ms}; \qquad (3)$$

$$D(k) = (\omega - 2\mu_0 H - \alpha_{ij}^{lm}k_i k_j + i\delta)^{-1}; \qquad (4)$$

$$\Delta_l^\pm(p, 0) = \sqrt{\mu_0 / M_0}[G_l^+(p) - G_l^-(p)],$$

$$M_0 = -i\mu_0 \lim_{\tau \to 0} \sum_n \int \frac{d^4 p}{(2\pi)^4} e^{i\epsilon\tau}[G_n^+(p) - G_n^-(p)]. \qquad (5)$$

Here M_0 is the magnetic moment of the system at zero temperature, μ_0 is the Bohr magneton, and the signs $\pm$ in Γ and in the Green functions refer to the spin directions. The quantities $\Delta_l^\pm(p, k)$ are the amplitudes for emission (absorption) of a spin wave by the Fermi excitations.

An analysis similar to the one carried out in [1] shows that the expansion of the quantities of interest at the pole in powers of $\mathbf{k}$ preserves their analytic form at least up to terms of fourth order.

We shall show that the quantities α_{ij}^{lm} do not depend on the indices l and m. To do this we use the integral equation which connects the irreducible vertex part $\Gamma^{(1)}$ with Γ:

$$\Gamma_{lmns}(p_1; p_2; k) = \Gamma_{lmns}^{(1)}(p_1; p_2; k)$$

$$- i\sum_{m's'}\int \frac{d^4 q}{(2\pi)^4}\Gamma_{lm'ns'}^{(1)}(p_1, q; k) G_{s'}(q) G_{m'}(q + k)$$

$$\times \Gamma_{s'mm's}(q, p_2; k). \qquad (6)$$

We substitute in (6) our expression for the relevant component of the vertex part, which in the absence of a magnetic field has the form

$$\Gamma_{l_+ m_- l_- m_+}(p_1, p_2; k)$$

$$= \frac{\mu_0}{M_0}\frac{[\Sigma_l^+(p_1) - \Sigma_l^-(p_1)][\Sigma_m^+(p_2) - \Sigma_m^-(p_2)]}{\omega - \alpha_{ij}^{lm}k_i k_j + i\delta}.$$

Here $\Sigma_l^\pm(p)$ is the irreducible electron self-energy part; and we have set $\omega = 0$ and $\mathbf{k} = 0$ everywhere except in the denominator, which is respon-

sible for the pole. Now if we use the following relation (which is a consequence of the fact that $\Gamma^{(1)}$ is the functional derivative of Σ with respect to G)

$$\Sigma_l^+(p) - \Sigma_l^-(p)$$

$$= -i\sum_m \int \frac{d^4 q}{(2\pi)^4}\Gamma_{l_+ m_- l_- m_+}^{(1)}(p, q; 0)[G_m^+(q) - G_m^-(q)]$$

and neglect the first term on the right-hand side of (6) near the pole, we see immediately that $\alpha_{ij}^{lm} = \alpha_{ij}$. For the same reason α_{ij} must also be independent of p_1 and p_2.

4. We now consider the effect of magnetic anisotropy. Since the strength of the relativistic interactions is of order $10^{-3} - 10^{-5}$ times that of the exchange interactions, the former can be treated by lowest-order perturbation theory.

Owing to the nonconservation of spin in the presence of magnetic interactions, we have, besides $G^{II}_{l_+ m_- l_- m_+}(p_1, p_2; k)$ two other transverse components:

$$G^{II}_{l_+ m_+ l_- m_-}(p_1, p_2; k) = i\Delta_l^+(p_1, k) D_-(k) \Delta_m^+(p_2, k),$$

$$G^{II}_{l_- m_- l_+ m_+}(p_1, p_2; k) = i\Delta_l^-(p_1, k) D_+(k) \Delta_m^-(p_2, k).$$

We shall find it convenient to write the irreducible part of the anisotropic interaction (that is, the part irreducible with respect to spin waves) in the form $S^i_{\alpha\gamma}\Pi_{ij}S^j_{\beta\delta}$. Then if we define

$$\Pi(k) = s_{\gamma\alpha}^- s_{\alpha\gamma}^i \Pi_{ij} s_{\beta\delta}^j s_{\delta\beta}^+,$$

$$B_+(k) = s_{\gamma\alpha}^+ s_{\alpha\gamma}^i \Pi_{ij} s_{\beta\delta}^j s_{\delta\beta}^+, \qquad B_-(k) = s_{\gamma\alpha}^- s_{\alpha\gamma}^i \Pi_{ij} s_{\beta\delta}^j s_{\delta\beta}^+,$$

the quantities D, $D_\pm$ can be expressed in terms of Π and $B_\pm$ exactly as in the isotropic case [1].

Suppose we expand Π_{ij} in powers of the small vector $\mathbf{k}$. For $\mathbf{k} = 0$ the analytic part of Π_{ij} is a second-rank tensor with the symmetry of the crystal. For a uniaxial crystal, in which the minimum of the energy corresponds to orientation of the magnetic moment along the crystal symmetry axis (we are assuming that the external magnetic field is also oriented along this direction, which we take as the z-axis) Π_{ij} is diagonal, with $\Pi_{xx} = \Pi_{yy} = 4\beta\mu_0 M_0$. Hence the analytic parts of Π and $B_\pm$ are given for $\mathbf{k} = 0$ by

$$\Pi(0) = 2\beta\mu_0 M_0, \qquad B_\pm(0) = 0.$$

The nonanalytic terms in Π arise from the magnetic dipole interaction of the electron spins, since this contains the Green function of the magnetic field. The calculations are now similar to those carried out in [1], for the isotropic Fermi liquid, the only difference being that we must now

expand the Green function of the magnetic field $F_{ij}(x_1, x_2)$ not in a Fourier integral but in terms of the functions $u_n(p, r)e^{ipx}$:

$$F_{lmns}(p_1, p_2; k) = \int dr_1 dr_2 \frac{dq}{(2\pi)^3} u_l(p_1, r_1) u_n^*(p_1 + k, r_1)$$

$$\times F(\omega, q)\exp[i(q - k)(r_1 - r_2)]u_m(p_2 + k, r_2)u_s^*(p_2, r_2).$$

Since the product $u_l(p_1, r_1)u_n(p_1 + k, r_1)$ has the spatial periodicity of the crystal lattice, we can expand it in a Fourier series:

$$u_l(p_1, r_1)u_n^*(p_1 + k, r_1) = \sum_\varkappa B_\varkappa^{ln}(p_1, k)\exp(iK_\varkappa r_1),$$

where the summation is taken over all vectors $K_\varkappa$ of the reciprocal lattice and

$$B_\varkappa^{ln}(p_1, k) = \frac{1}{v_c} \int dr_1 \exp(-iK_\varkappa r_1)u_l(p_1, r_1)u_n^*(p_1 + k, r_1),$$

(the integral here is to be taken over the elementary cell of the reciprocal lattice, the volume of which is denoted by v_c). After some simple manipulations we obtain the following expression for the Green function of the magnetic field:

$$F_{lmns}(p_1, p_2, k) = \sum_\varkappa B_\varkappa^{ln}(p_1, k)[B_\varkappa^{ms}(p_2, k)]^* F(\omega, k - K_\varkappa)$$

$$= F(\omega, k)\delta_{ln}\delta_{ms} + A_{lmns} + F(\omega, k)o(k) + o(k). \quad (7)$$

Bearing in mind that $F_{ij}(k) = 4\pi(k_i k_j/k^2 - \delta_{ij})$ we see that for small k we may neglect the last two terms in (7). The second term is a constant, which we saw above was equal to $4\beta\mu_0 M_0$. Hence we can finally write

$$\Pi(k) = 2\beta\mu_0 M_0 + 4\pi\mu_0 M_0 k_+ k_- / k^2,$$

$$B_\pm(k) = 4\pi\mu_0 M_0 k_\pm^2 / k^2; \quad (8)$$

$$D(k) = \frac{\omega + A(k)}{\omega^2 - \omega^2(k)}, \qquad D_\pm(k) = \frac{B_\pm(k)}{\omega^2 - \omega^2(k)},$$

$$\omega^2(k) = A^2(k) - B_+(k)B_-(k),$$

$$A(k) = 2\mu_0 H + a_{ij}k_i k_j + \Pi(k). \quad (9)$$

5. So far we have assumed that the system exhibits the phenomenon of ferromagnetism, without concerning ourselves with the criteria for its occurrence. We now write down the condition for ferromagnetism to occur (the derivation is the same as for the isotropic case) in the form of a relation between the parameters of the paramagnetic state at the moment of onset of ferromagnetism:

$$\Sigma_l^+(p) - \Sigma_l^-(p) = -2\pi \sum_m \int \frac{d^4 p'}{(2\pi)^4} a_m^2(p')\Gamma_{l_+ m_- l_- m_+}^\omega(p, p')$$

$$\times \delta(\varepsilon')\delta(\varepsilon_m(p') - \mu)[\Sigma_m^+(p') - \Sigma_m^-(p')],$$

$$\Gamma^\omega(p, p') = \lim_{\substack{|k| \to 0 \\ \omega \to 0}} \Gamma(p, p'; k). \quad (10)$$

The condition for the appearance of ferromag-

netism is the appearance of a nontrivial solution of the system of homogeneous integral equations (10) for $\Sigma_l^+(p) - \Sigma_l^-(p)$. It can be shown that when this condition is fulfilled the paramagnetic susceptibility of the system tends to infinity.

6. We now consider the electronic contribution to the magnetic moment and susceptibility of a ferromagnetic metal at zero temperature.

As shown by Dzyaloshinskiĭ [5], the magnetic moment of a system of electrons in a crystal, if we neglect the effects of magnetic anisotropy, is given at T = 0 by the expression

$$M_0 = \mu_0 \sum_n \int \frac{dp}{(2\pi)^3}[\theta(\mu - \varepsilon_n^+(p)) - \theta(\mu - \varepsilon_n^-(p))]$$

$$= (2\pi)^{-3}\mu_0[l\Omega + V_+^F - V_-^F];$$

$$\theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases} \quad (11)$$

where Ω is the volume of an elementary cell of the reciprocal lattice, $V_\pm^F$ is the sum of the volumes enclosed by all the Fermi surfaces corresponding to the spin direction in question, and l is some integer. A calculation similar to the one carried out in [1] gives the correction to Eq. (11) introduced by the relativistic magnetic interactions; it is

$$\delta M_0 = -\mu_0 \int \frac{dk}{(2\pi)^3} \frac{A(k) - \omega(k)}{\omega(k)} \quad (12)$$

with $A(k)$, $\omega(k)$ given by formulae (9).

If we neglect the effects of magnetic anisotropy, the longitudinal magnetic susceptibility χ is determined solely by the interaction of the electronic excitations on the Fermi surface. From (11),

$$\chi_\mu \equiv \left(\frac{\partial M}{\partial H}\right)_\mu \equiv \mu_0^2 \sum_n \left(\int dS_n^+ B_n^+ - \int dS_n^- B_n^-\right). \quad (13)$$

The subscript μ indicates differentiation at constant chemical potential, and we have defined

$$B_n^\pm = \frac{1}{\mu_0} \frac{\partial \varepsilon_n^\pm(p)}{\partial H},$$

$$\int dS_n^\pm B_n^\pm = \int \frac{dp}{(2\pi)^3}\delta(\mu - \varepsilon_n^\pm(p))B_n^\pm(p).$$

Since the number of electrons in the system is fixed, the actual magnetic susceptibility is given by:

$$\chi = \chi_\mu + \left(\frac{\partial M}{\partial \mu}\right)_H \left(\frac{\partial \mu}{\partial H}\right)_N = \chi_\mu - \left(\frac{\partial N}{\partial H}\right)_\mu^2 \Big/ \left(\frac{\partial N}{\partial \mu}\right)_H, \quad (14)$$

where $N = N(\mu, H)$ is the number of particles at T = 0, which, in agreement with [2], is given by

$$N = \sum_n \int \frac{dp}{(2\pi)^3}[\theta(\mu - \varepsilon_n^+(p)) + \theta(\mu - \varepsilon_n^-(p))].$$

Introducing the notation $C_n^{\pm} = 1 - \partial \epsilon_n^{\pm}(\mathbf{p})/\partial \mu$, we write our final expression for the susceptibility in the form[1]

$$\chi = \mu_0^2 \left\{ \sum_n \left(\int dS_n^+ B_n^+ - \int dS_n^- B_n^- \right) \right.$$
$$- \left[\sum_n \left(\int dS_n^+ B_n^+ + \int dS_n^- B_n^- \right) \right]^2$$
$$\left. \times \left[\sum_n \left(\int dS_n^+ C_n^+ + \int dS_n^- C_n^- \right) \right]^{-1} \right\}. \quad (15)$$

The quantities $B_n^{\pm}$, $C_n^{\pm}$ can be expressed in terms of Landau's function[2]

$$f_{\alpha\beta\gamma\delta}^{mn}(\mathbf{p}_1, \mathbf{p}_2) = (a_{m_\alpha} a_{n_\beta} a_{m_\gamma} a_{n_\delta})^{1/2} \Gamma_{m_\alpha \beta m_\gamma n_\delta}(\mathbf{p}_1, \mathbf{p}_2)|_{\epsilon_1 = \epsilon_2 = 0}$$

by means of the following two systems of integral equations:

$$B_m^+(\mathbf{p}) + \sum_n \left[\int dS_n^+ f_{1111}^{mn}(\mathbf{p}, \mathbf{p}') B_n^+(\mathbf{p}') \right.$$
$$\left. + \int dS_n^- f_{1212}^{mn}(\mathbf{p}, \mathbf{p}') B_n^-(\mathbf{p}') \right] = 1, \quad (16)$$

$$B_m^-(\mathbf{p}) + \sum_n \left[\int dS_n^+ f_{2121}^{mn}(\mathbf{p}, \mathbf{p}') B_n^+(\mathbf{p}') \right.$$
$$\left. + \int dS_n^- f_{2222}^{mn}(\mathbf{p}, \mathbf{p}') B_n^-(\mathbf{p}') \right] = -1;$$

$$C_m^+(\mathbf{p}) + \sum_n \left[\int dS_n^+ f_{1111}^{mn}(\mathbf{p}, \mathbf{p}') C_n^+(\mathbf{p}') \right.$$
$$\left. + \int dS_n^- f_{1212}^{mn}(\mathbf{p}, \mathbf{p}') C_n^-(\mathbf{p}') \right] = 1,$$

$$C_m^-(\mathbf{p}) + \sum_n \left[\int dS_n^+ f_{2121}^{mn}(\mathbf{p}, \mathbf{p}') C_n^+(\mathbf{p}') \right.$$
$$\left. + \int dS_n^- f_{2222}^m(\mathbf{p}, \mathbf{p}') C_n^-(\mathbf{p}') \right] = 1. \quad (17)$$

It is obvious from (15)–(17) that the magnetic susceptibility of a ferromagnetic metal is anomalously large owing to the contribution of the d-band excitations, with their large density of states at the Fermi surface.

7. We can use the technique employed in [1] to establish the following relation for the magnetic moment per unit volume of a ferromagnetic metal at low temperatures:

$$M(T) = M(0) - \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\partial \omega(\mathbf{k})/\partial H}{\exp\{\omega(\mathbf{k})/T\} - 1}$$
$$+ \frac{\pi^2}{6} T^2 \left[\frac{\partial^2 Z}{\partial \epsilon \partial H} - \frac{\partial Z}{\partial H} \frac{\partial}{\partial \epsilon} \ln \left(\frac{\partial Z}{\partial \mu} \right) \right] \Big|_{\epsilon=0} \quad (18)$$

where

$$Z(\epsilon, \mu, H) = \sum_n \int \frac{d\mathbf{p}}{(2\pi)^3} [\theta(\epsilon + \mu - \epsilon_n^+(\mathbf{p}))$$
$$+ \theta(\epsilon + \mu - \epsilon_n^-(\mathbf{p}))]. \quad (19)$$

The second term in the square brackets in (18) arises because, if N is held constant, the chemical potential changes when we go to finite temperatures.[2]

The specific heat due to Fermi excitations and spin waves is given by

$$C = \frac{\pi^2}{3} T \frac{\partial Z}{\partial \epsilon} \Big|_{\epsilon=0} + \frac{\partial}{\partial T} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\omega(\mathbf{k})}{\exp\{\omega(\mathbf{k})/T\} - 1}. \quad (20)$$

Owing to the large density of states at the Fermi surface of the d-electron band excitations, these give the dominant contribution to the heat capacity and magnetic moment of a ferromagnetic metal.

We shall not write out here the expressions for the contributions of the spin waves to M(T) and C in different temperature regions (they are given in [6]); we merely compare them with the contributions of the Fermi excitations, which are of order

$$\delta M_F \propto \frac{\mu_0}{a^3} \left(\frac{T}{\epsilon_F} \right)^2, \qquad C_F \propto \frac{1}{a^3} \frac{T}{\epsilon_F},$$

where a is the lattice constant, and ϵ_F the chemical potential measured from the bottom of

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[1] In [1] the term $(\partial M/\partial\mu)_H (\partial\mu/\partial H)_N$ was omitted in the expression for χ. Hence the correct formula for the magnetic susceptibility of a ferromagnetic Fermi liquid, which should replace (A.21) of [1], is:

$$\chi = \frac{\mu_0^2}{\pi^2} \left[\left(1 + \frac{m_+ p_+}{2\pi^2} a_+^2 \overline{\Gamma_{1111}^\omega} \right) \left(1 + \frac{m_- p_-}{2\pi^2} a_-^2 \overline{\Gamma_{2222}^\omega} \right) \right.$$
$$- \frac{m_+ m_- p_+ p_-}{4\pi^4} (a_+ a_- \overline{\Gamma_{1212}^\omega})^2 \Big]^{-1} \cdot 2m_+ m_- p_+ p_- \left\{ 1 + \frac{m_+ p_+}{2\pi^2} a_-^2 \overline{\Gamma_{2222}^\omega} \right.$$
$$\left. + \frac{m_- p_-}{2\pi^2} a_+^2 \overline{\Gamma_{1111}^\omega} + \frac{m_+ m_- p_+ p_-}{4\pi^4} a_+^2 a_-^2 [\overline{\Gamma_{1111}^\omega} \overline{\Gamma_{2222}^\omega} - (\overline{\Gamma_{1212}^\omega})^2] \right\}$$
$$\times \left[m_+ p_+ + m_- p_- + \frac{m_+ m_- p_+ p_-}{2\pi^2} (a_+^2 \overline{\Gamma_{1111}^\omega} - 2a_+ a_- \overline{\Gamma_{1212}^\omega} + a_-^2 \overline{\Gamma_{2222}^\omega}) \right]^{-1}.$$

[2] This term was omitted in [1] in the formula for M(T). The correct expression for the magnetic moment in the isotropic case is:

$$M(T) = M(0) - \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\partial \omega(\mathbf{k})/\partial H}{\exp\{\omega(\mathbf{k})/T\} - 1} + \frac{T^2}{12} \left\{ \frac{\partial}{\partial H} (m_+ p_+ + m_- p_-) \right.$$
$$+ \mu_0 \left[m_+ p_- - m_- p_- + \frac{m_+ m_- p_+ p_-}{2\pi^2} (a_-^2 \overline{\Gamma_{2222}^\omega} - a_+^2 \overline{\Gamma_{1111}^\omega}) \right]$$
$$\times \left[m_+ p_+ + m_- p_- + \frac{m_+ m_- p_+ p_-}{2\pi^2} (a_+^2 \overline{\Gamma_{1111}^\omega} - 2a_+ a_- \overline{\Gamma_{1212}^\omega} + a_-^2 \overline{\Gamma_{2222}^\omega}) \right]^{-1}$$
$$\times \frac{\partial}{\partial \mu} (m_+ p_+ + m_- p_-) \right\}.$$

the narrowest band. In the temperature region $T \gg 4\pi\mu_0 M_0$, $2\mu_0(H + \beta M_0)$ we find

$$\delta M_s \sim \frac{\mu_0}{a^3}\left(\frac{T}{\Theta_c}\right)^{3/2}, \qquad C_s \sim \frac{1}{a^3}\left(\frac{T}{\Theta_c}\right)^{3/2}$$

where Θ_C is the Curie temperature; hence the contribution of the Fermi excitations to the specific heat is larger than that of the spin waves, while the reverse is true of the contribution to the magnetic moment.

For $2\mu_0(H + \beta M_0) \ll T \ll 4\pi\mu_0 M_0$,

$$\delta M_s \sim \frac{\mu_0}{a^3}\left(\frac{T}{4\pi\mu_0 M_0}\right)^{1/2}\left(\frac{T}{\Theta_c}\right)^{3/2},$$

$$C_s \sim \frac{1}{a^3}\frac{T}{4\pi\mu_0 M_0}\left(\frac{T}{\Theta_c}\right)^{3/2}.$$

Although both contributions to the magnetic moment are of order T^2, the spin wave contribution of the spin waves to the heat capacity is small compared to that of the Fermi excitations.

Finally, in the region

$$2\mu_0(H + \beta M_0) \gg T, 4\pi\mu_0 M_0,$$

$$T \ll 2\mu_0(H + \beta M_0), 4\pi\mu_0 M_0$$

the spin wave contribution is exponentially small, and the temperature dependence of all quantities is determined by the Fermi excitation spectrum.

An analysis similar to the one carried out for the isotropic case[1] can be given for a metal too, to rule out the possibility of the spin waves changing the Fermi spectrum in such a way as to render the above results invalid.

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