# SOME PROCESSES INVOLVING HIGH-ENERGY POLARIZED ELECTRONS AND POSITRONS

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The formalism of two-component spinors is used to calculate the cross sections for some processes involving high-energy longitudinally polarized electrons and positrons, on the assumption that the charges and magnetic moments of the particles involved have extended distributions.

As has been pointed out in a number of papers,[1,2] experiments with polarized particles can provide additional relations for determining form factors. When one takes form factors into account and averages over the polarizations in the initial state and sums over those in the final state, the usual cross sections for processes involving electrons and positrons give rather complicated functions of the form factors, and there is considerable difficulty in determining these quantities on the basis of experimental data.

In this respect the cross sections for processes involving polarized particles (for example, the cross sections for scattering of polarized electrons by polarized electrons and positrons) have a simpler dependence on the form factors, and the determination of the form factors by comparison with experiment is much simpler.

1. Let us consider some processes which involve polarized electrons and positrons at large energies $\epsilon \gg m$ ($m$ is the rest mass). We assume that these particles have a certain distribution of charge and magnetic moment.

As has been shown by Sannikov,[3] it is convenient to calculate processes involving fast electrons by means of the formalism of two-component spinors. When the mass term is dropped the Dirac equation breaks up into two Weyl equations for two-component spinors, one describing electrons polarized along the momentum (R-electrons) and the other electrons polarized opposite to the momentum (L-electrons). The presence of a possible structure of the electron has the result that besides diagrams at whose vertices electron lines of the same helicity come together one must also consider diagrams at which electron lines of different helicities come together.

We shall start from the vertex operation for the interaction of electron and electromagnetic field in the most general four-component form[2]:

$$\Gamma_\mu^{(e)}(k) = \gamma_\mu a_e(k^2) + \frac{i}{2}(\gamma_\mu \hat{k} - \hat{k}\gamma_\mu)\frac{b_e(k^2)}{|k^2|^{1/2}}, \qquad (1)$$

where $k$ is the difference of the four-momenta of the electron in the final and initial states, and $a_e(k^2)$ and $b_e(k^2)$ are the electromagnetic form-factors of the electron, which are real in the region of spacelike four-momentum transfers $k^2 > 0$ and complex in the region of timelike transfers $k^2 < 0$. Using a representation in which the Dirac matrices are

$$\boldsymbol{\alpha} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad (2)$$

and writing the bispinor $u(\mathbf{p})$ in the form

$$u = \begin{pmatrix} u_R \\ u_L \end{pmatrix}, \qquad (3)$$

where $u_R$ and $u_L$ are the respective two-component spinor amplitudes of the R-electron and L-electron, we can put the expression $\bar{u}(\mathbf{q})\Gamma_\mu^{(e)}(\mathbf{k})u(\mathbf{p})$ in two-component form, and it follows that to a vertex at which R-electron lines meet we must assign the matrix $\Gamma_{R\mu}^{R} = -\sigma_\mu^+ a_e$; to a vertex at which L-electron lines meet, the matrix $\Gamma_{L\mu}^{L} = \sigma_\mu^- a_e$; to a vertex where an L-electron line enters and an R-electron line emerges, the matrix $\Gamma_{L\mu}^{R} = \frac{1}{2}(\sigma_\mu^+ k^- - k^+\sigma_\mu^-)\,|k^2|^{-1/2} b_e$; and finally, to a vertex where an R-electron line enters and an L-electron line emerges, the matrix $\Gamma_{R\mu}^{L} = \frac{1}{2}(\sigma_\mu^- k^+ - k^-\sigma_\mu^+)\,|k^2|^{-1/2} b_e$. Here we are using the notation of[3], namely:

$$\sigma_\mu^+ = (\boldsymbol{\sigma}, i), \quad \sigma_\mu^- = (\boldsymbol{\sigma}, -i), \quad k^+ = k_\mu\sigma_\mu^+,$$
$$k^- = k_\mu\sigma_\mu^-,$$

where $\sigma$ are the Pauli matrices, and the fourth component is a two-rowed unit matrix multiplied by

±i. It is not hard to see that the vertex operators are the same for positrons. The other rules of correspondence given in [3] are not altered.

We can now calculate the cross sections for processes involving longitudinally polarized electrons and positrons. We present the results of the calculations on the cross sections of several processes. The calculations were made for the case of one-photon exchange [4] in the center-of-mass system of the colliding particles.

2. We begin with the simplest case—the scattering of an electron by a charged spinless particle ($\pi^{\pm}$ meson, $\alpha$ particle). The differential cross section without change of helicity of the electron is (in units that make $\hbar = c = 1$, which we shall use always hereafter)

$$d\sigma_R{}^R = d\sigma_L{}^L = Z^2\alpha^2 \frac{\cos^2(\theta/2)}{4\varepsilon^2 \sin^4(\theta/2)} a_e{}^2(k^2) A^2(k^2)\, do, \qquad (4)$$

and that with change of helicity is

$$d\sigma_R{}^L = d\sigma_L{}^R = Z^2\alpha^2 \frac{[\varepsilon\cos^2(\theta/2)+\omega]^2}{4\varepsilon^2(\varepsilon+\omega)^2\sin^4(\theta/2)} b_e{}^2(k^2) A^2(k^2)\, do, \qquad (5)$$

where $\alpha = e^2/4\pi = 1/137$, $\epsilon$ and $\omega$ are the respective energies of the electron and the scalar particle, $\theta$ is the scattering angle, $k^2 = 4\epsilon^2 \sin^2(\theta/2)$, $A(k^2)$ is the charge form factor of the scalar particle, and the upper and lower indices on $d\sigma$ indicate the polarizations of the electron before and after the scattering. Half the sum of all the cross sections (4) and (5) is the cross section for scattering of an electron by a charged spinless particle, averaged over the polarizations of the electron in the initial state and summed over those in the final state. Setting $Z = 2$ and going over to the laboratory system, we get the cross section for scattering of an electron by an $\alpha$ particle which was given in a paper by Avakov and Ter-Martirosyan.[2]

It is an equally simple task to calculate the cross sections for the annihilation of a polarized electron-positron pair by conversion into a $\pi$-meson pair. These cross sections are

$$d\sigma_{RR} = d\sigma_{LL} = \alpha^2 \frac{q^3}{16\varepsilon^5} \sin^2\theta\, |a_e(k^2)|^2 |A_\pi(k^2)|^2\, do,$$

$$d\sigma_{RL} = d\sigma_{LR} = \alpha^2 \frac{q^3}{16\varepsilon^5} \cos^2\theta\, |b_e(k^2)|^2 |A_\pi(k^2)|^2\, do, \qquad (6)$$

where $\epsilon$ is the energy of the particles involved in the process, $q$ is the absolute value of the three-momentum of the $\pi$ meson, $A_\pi(k^2)$ is the charge form-factor of the $\pi$ meson, and $k^2 = -4\epsilon^2$. Then the cross section for annihilation of an unpolarized electron-positron pair with conversion into a $\pi$-meson pair is

$$d\sigma = \alpha^2 \frac{q^3}{32\varepsilon^5} (\sin^2\theta\, |a_e(k^2)|^2 + \cos^2\theta\, |b_e(k^2)|^2) |A_\pi(k^2)|^2\, do. \qquad (7)$$

For $b_e = 0$ this goes over into the expression obtained by Afrikyan and Garibyan.[5]

3. Let us now consider the scattering of an electron by a particle with spin ½ ($\mu$ meson, nucleon). The cross sections for scattering without change of helicity, averaged and summed over the polarizations of the $\mu$ meson, are

$$d\sigma_R{}^R = d\sigma_L{}^L = \frac{\alpha^2}{4\varepsilon^2 w^2 \sin^2(\theta/2)} \Big\{ \Big(2\varepsilon^2 \sin^2\frac{\theta}{2} + w^2 \cot^2\frac{\theta}{2}\Big) \times A_\mu{}^2(k^2)\, 4M\varepsilon \sin\frac{\theta}{2} A_\mu(k^2) B_\mu(k^2) + \Big(w^2 \cot^2\frac{\theta}{2} + 2M^2\Big) \times B_\mu{}^2(k^2) \Big\} a_e{}^2(k^2)\, do, \qquad (8)$$

and with change of helicity,

$$d\sigma_R{}^L = d\sigma_L{}^R = \frac{\alpha^2}{4\varepsilon^2 w^2 \sin^4(\theta/2)} \Big\{ w\Big(w - 2\varepsilon \sin^2\frac{\theta}{2}\Big) A_\mu{}^2(k^2) - 2M\varepsilon \sin^3\frac{\theta}{2} A_\mu(k^2) B_\mu(k^2) + \Big[\Big(w - \varepsilon\sin^2\frac{\theta}{2}\Big)^2 - M^2 \sin^2\frac{\theta}{2}\Big] B_\mu{}^2(k^2) \Big\} b_e{}^2(k^2)\, do, \qquad (9)$$

where $\epsilon$ is the energy of the electron, $w$ is the total energy of the colliding particles, $M$ is the mass of the $\mu$ meson, $A_\mu(k^2)$ and $B_\mu(k^2)$ are the electromagnetic form factors of the $\mu$ meson, introduced in accordance with (1), and $k^2 = 4\epsilon^2 \sin^2(\theta/2)$. Half of the sum of the cross sections (8) and (9) is the cross section for scattering of an electron by a $\mu$ meson, averaged over the polarizations in the initial state and summed over those in the final state. For the scattering of an electron by a nucleon we must take $M$ to be the mass of the nucleon, and insert instead of $A_\mu$ and $B_\mu$ the electromagnetic form-factors $A_N$ and $B_N$ of the nucleon.

We also give the expressions for the cross sections for annihilation of an electron-positron pair with conversion into a $\mu$-meson pair, summed over the polarizations of the $\mu$ mesons. These cross sections are

$$d\sigma_{RR} = d\sigma_{LL} = \alpha^2 \frac{q}{8\varepsilon^3} \Big\{ \Big(1 + \frac{M^2}{\varepsilon^2} + \frac{q^2}{\varepsilon^2}\cos^2\theta\Big) |A_\mu(k^2)|^2 + 4\frac{M}{\varepsilon} \mathrm{Re}\,[A_\mu(k^2) B_\mu{}^*(k^2)] + \Big(1 + \frac{M^2}{\varepsilon^2} - \frac{q^2}{\varepsilon^2}\cos^2\theta\Big) \times |B_\mu(k^2)|^2 \Big\} |a_e(k^2)|^2\, do,$$

$$d\sigma_{RL} = d\sigma_{LR} = \alpha^2 \frac{q}{8\varepsilon^3} \Big\{ \Big(1 - \frac{q^2}{\varepsilon^2}\cos^2\theta\Big) |A_\mu(k^2)|^2 \qquad (10)$$

$$+2\frac{M}{\varepsilon}\operatorname{Re}[A_\mu(k^2)B_\mu^*(k^2)]$$

$$+\left(\frac{M^2}{\varepsilon^2}+\frac{q^2}{\varepsilon^2}\cos^2\theta\right)|B_\mu(k^2)|^2\Big\}|b_e(k^2)|^2\,do.$$

Here $\epsilon$ is the energy of the particles involved in the process, q is the absolute value of the three-momentum of the $\mu$ meson, and $k^2 = -4\epsilon^2$. The expression

$$d\sigma = {}^1/_2(d\sigma_{RR}+d\sigma_{RL}), \tag{11}$$

where $d\sigma_{RR}$ and $d\sigma_{RL}$ are given by (10), is the cross section for the annihilation of an unpolarized electron-positron pair with conversion into a $\mu$-meson pair, which agrees with the analogous cross section in the paper of Guliev and Épshteĭn,[6] when in the latter formula we take the limit $m \to 0$. The expressions (10) and (11) are valid for the annihilation of an electron-positron pair with conversion into any fermion-antifermion pair with spin $1/2$ (in particular, nucleon-antinucleon), with the exception of an electron-positron pair.

4. Finally, let us turn to the scattering of electrons by electrons and by positrons. For the cross sections for scattering of longitudinally polarized electrons we get expressions of the form

$$d\sigma_{AB}^{CD} = \frac{\alpha^2}{\varepsilon^2}\frac{1}{\sin^4\theta}(M_{AB}^{CD})^2\,do, \tag{12}$$

where A, B, C, D denote helicities R or L, and the quantities $M_{AB}^{CD}$ are

$$M_{RR}^{RR}=M_{LL}^{LL}=(1+\cos\theta)a_e^2+(1-\cos\theta)a_e'^2,$$

$$M_{RR}^{RL}=M_{RR}^{LR}=M_{RL}^{RR}=M_{RL}^{LL}=M_{LR}^{RR}=M_{LR}^{LL}=M_{LL}^{RL}=M_{LL}^{LR}$$

$$=(1+\cos\theta)\cos\frac{\theta}{2}a_eb_e-(1-\cos\theta)\sin\frac{\theta}{2}a_e'b_e',$$

$$M_{RR}^{LL}=M_{LL}^{RR}={}^1/_4[(1+\cos\theta)(3+\cos\theta)b_e^2$$

$$+(1-\cos\theta)(3-\cos\theta)b_e'^2],$$

$$M_{RL}^{RL}=M_{LR}^{LR}={}^1/_4[2(1+\cos\theta)^2a_e^2$$

$$-(1-\cos\theta)(3-\cos\theta)b_e'^2],$$

$$M_{RL}^{LR}=M_{LR}^{RL}={}^1/_4[2(1-\cos\theta)^2a_e'^2$$

$$-(1+\cos\theta)(3+\cos\theta)b_e^2]. \tag{13}$$

The lower indices on $d\sigma$ indicate the polarizations of the electrons before the scattering, and the upper indices those after the scattering, $\epsilon$ is the energy of each of the electrons, and

$$a_e\equiv a_e(k_1^2),\quad b_e\equiv b_e(k_1^2),\quad a_e'\equiv a_e(k_2^2),\quad b_e'\equiv b_e(k_2^2), \tag{14}$$

where $k_1$ and $k_2$ are the momentum transfers in the direct and exchange diagrams,

$$k_1^2=4\varepsilon^2\sin^2\frac{\theta}{2},\qquad k_2^2=4\varepsilon^2\cos^2\frac{\theta}{2} \tag{15}$$

These expressions for the cross sections for scattering of longitudinally polarized electrons agree (apart from a factor $1/2$) with the cross sections for scattering of electrons with a fixed projection of the spin on the direction of motion of one of the particles, as derived in a paper by Bogush and Satsunkevich [7] on the basis of a covariant method for direct calculation of the matrix elements of polarized particles. The same paper discusses the possibility of determining the electromagnetic form factors of the electron in the region $k^2 > 0$ from known experimental cross sections.

The sum of all the cross sections (12), divided by 4, is the cross section for electron-electron scattering averaged over the polarizations in the initial state and summed over those in the final state; it agrees with the result of Baĭer,[8] if in the latter we take the limit $m \to 0$.

The corresponding cross sections for the scattering of longitudinally polarized electrons and positrons are of the form

$$d\sigma_{AB}^{CD}=\frac{\alpha^2}{16\varepsilon^2}|M_{AB}^{CD}|^2\,do, \tag{16}$$

where

$$M_{RR}^{RR}=M_{LL}^{LL}=\frac{1+\cos\theta}{1-\cos\theta}[2a_e^2-(1-\cos\theta)a_e'^2],$$

$$M_{RR}^{RL}=M_{RR}^{LR}=M_{RL}^{RR}=M_{RL}^{LL}=M_{LR}^{RR}=M_{LR}^{LL}=M_{LL}^{RL}=M_{LL}^{LR}$$

$$=\sin\theta\left[\frac{2}{(1-\cos\theta)\sin(\theta/2)}a_eb_e+a_e'b_e'\right],$$

$$M_{RR}^{LL}=M_{LL}^{RR}=(1-\cos\theta)a_e'^2-\frac{3+\cos\theta}{1-\cos\theta}b_e^2,$$

$$M_{RL}^{RL}=M_{LR}^{LR}=\frac{4}{1-\cos\theta}a_e^2-\cos\theta\, b_e'^2,$$

$$M_{RL}^{LR}=M_{LR}^{RL}=\frac{3+\cos\theta}{1-\cos\theta}b_e^2+\cos\theta\, b_e'^2. \tag{17}$$

Here the lower indices on $d\sigma$ indicate the polarizations of the electron and positron in the initial state, and the upper, those in the final state; the form factors are as defined in (14), with

$$k_1^2=4\varepsilon^2\sin^2\frac{\theta}{2},\qquad k_2^2=-4\varepsilon^2. \tag{18}$$

By means of the expressions (16) and (17) we can get the cross section for the scattering of an electron by a positron, averaged over the polarizations in the initial state and summed over those in the final state:

$$d\sigma = \frac{\alpha^2}{16\varepsilon^2}\left\{\frac{2[4+(1+\cos\theta)^2]}{(1-\cos\theta)^2} a_e^4 - 2\frac{(1+\cos\theta)^2}{1-\cos\theta} a_e^2 \operatorname{Re}(a_e'^2) + (1+\cos^2\theta)|a_e'|^4 + \frac{16\sin^2\theta}{(1-\cos\theta)^3} a_e^2 b_e^2 + \frac{8\sin^2\theta}{(1-\cos\theta)\sin(\theta/2)} \times a_e b_e \operatorname{Re}(a_e' b_e') + 2\sin^2\theta\,|a_e'|^2 |b_e'|^2 - (3+\cos\theta) b_e^2 \operatorname{Re}(a_e'^2) - \frac{4\cos\theta}{1-\cos\theta} a_e^2 \operatorname{Re}(b_e'^2) + \frac{(3+\cos\theta)^2}{(1-\cos\theta)^2} b_e^4 + \frac{(3+\cos\theta)\cos\theta}{1-\cos\theta} b_e^2 \operatorname{Re}(b_e'^2) + \cos^2\theta\,|b_c'|^4\right\} do. \quad (19)$$

The expressions (16) and (17) enable us to obtain information about the complex form factors $a_e(k_2^2)$ and $b_e(k_2^2)$ in the region of timelike momentum transfers $k_2^2 < 0$ from the known experimental cross sections. Let us consider, for example, the cross section $d\sigma_{RR}^{RR}$. Regarding the form-factor $a_e(k_1^2)$ in the region of spacelike momentum transfers $k_1^2 > 0$ as known (from the cross sections for scattering of polarized electrons), we can readily see that the expression for $d\sigma_{RR}^{RR}$ at fixed energy $\epsilon$ and two different angles $\theta$ gives two equations, from which one can easily determine $\alpha_1^2$ and $\alpha_2^2$, where $\alpha_1$ and $\alpha_2$ are the real and imaginary parts of $a_e(k_2^2)$. Analogous data on the form-factor $b_e(k_2^2)$ can be obtained, for example, from a consideration of $d\sigma_{RL}^{RL}$. Then, knowing the form factors $a_e(k_2^2)$ and $b_e(k^2{}_2)$ in the entire region of momentum transfers, one can get from the cross sections given here information about the electromagnetic form factors of the particles involved in the various reactions we have considered.

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209