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A METHOD FOR MEASURING THE FIELD DISTRIBUTION IN AN OPEN RESONATOR

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A method for investigating the amplitude distribution of fields in open resonators is proposed which is based on the introduction of a small absorbing body into the resonance volume. Results of measurements for a resonator with concave circular mirrors are presented. The experimental results are compared with the theory. It is noted that in real resonators with circular mirrors the higher-order even-symmetry modes may split into components possessing an azimuthal dependence.

IN investigating the distribution of electromagnetic fields in open resonators one uses either the method of small perturbations or the method of a scattered modulated field. The method of small perturbations is based on determination of the cavity detuning caused by the introduction of a small perturbing nonuniformity[1]. The method employing scattering and modulation is as follows. An audio-frequency modulating voltage is applied to a miniature diode scatterer placed in the field under investigation. The scattered field is modulated by the variation in the diode resistance. The reradiated signal is received and detected synchronously.

This method can not be used close to the reflectors, since the direct and the reflected beams interfere and cause significant distortion of the true field distribution. A strong distortion of the field is also caused by the wires carrying the modulating voltage to the diode scatterer, although this drawback can be avoided by using a photoresistor in place of the diode and by illuminating it with a modulated light beam[2].

The above methods share a general drawback; namely, the necessity of using highly stable microwave signal sources. Thus, if the quality factor of the cavity is Q, one must have a frequency stability which is not worse than Q^{-1}. And since open resonators have extremely high Q's, of the order of 10^4—10^6, while microwave signal sources are, as a rule, rather poor in stability, in the region of the spectrum where open resonators are used, application of the above methods is difficult at best.

The method proposed here is a modification of the method of small perturbations, allows one to measure rather easily the amplitude distribution of the field for any mode in the cavity, and does not require great stability for the microwave source.

The open resonator studied was excited through a decoupling attenuator by a frequency modulated microwave source. After amplification the signal from the detector was fed into a wideband oscilloscope with a linear amplitude characteristic. The voltages producing the frequency modulation of the source and the time sweep of the oscilloscope were in synchronism. As a result the oscilloscope displayed a series of resonance peaks corresponding to the cavity modes and separated in frequency by an amount determined by the mirror separation and diameter. The amplitudes of the resonance peaks were determined by the Q of the cavity for each mode and by the coupling between the detector and the source at constant signal level.

The present method for measuring the field distribution is based on introducing into the resonant volume a small body that absorbs microwave energy strongly, e.g., a small piece of black rubber or polyethylene containing a large amount of lampblack. For sufficiently small dimensions of the absorber relative to the resonator volume, a decrease in the Q and in the corresponding amplitude of the resonance peak of the fundamental or of any higher mode occurs without any noticeable change in the resonance frequencies. Keeping in mind the quadratic character of the detector and the smallness of the axial components of the fields in an open cavity, it is easy to show that the field at the point where the probe is located is related to the amplitude of the oscilloscope signal by

$$E(x, y, z) / E_{max} = H(x, y, z) / H_{max} = Cf(\xi),$$

where

$$f(\xi) = \xi^{-1/4}(1-\sqrt{\xi})^{1/2}, \qquad \xi = h(x, y, z) / h_0,$$

h_0 is the amplitude of the signal from the mode under investigation in the absence of the absorbing probe, h(x, y, z) is the amplitude when the probe is placed at the point (x, y, z), and C is a normalizing factor determined for each series of measurements from the requirement that $|E(x, y, z)/E_{max}|_{max} = 1$.

By determining the function h(x, y, z) we thus obtain the amplitude distribution of the field in the cavity. To simplify the data reduction it is useful to employ either a table or a graph of the function $f(\xi)$.

An additional experiment was carried out to determine the degree to which the probe perturbed the field distribution. Curves of the field distribution in one of the higher order modes were plotted. Then an additional probe of the same size as the original probe was placed in the reflector aperture; and the field distribution redetermined. The resultant field distributions were practically independent of the position of the additional probe in the aperture. Whereas the additional probe decreased the Q of the cavity by more than a factor of ten, the differences in the field distribution, due to distortion of the field by the additional probe, were of the order of 6—10%. The accuracy of the measurements increases with the Q of the resonator. In cavities with low mirror reflectivities and with strong coupling to the signal source it becomes necessary to utilize large probes; this increases the errors of measurement.

The present method was used to determine the fields at the mirrors of an open resonator with spherical reflectors. The mirrors had a diameter $2a = 85\ \lambda$ and radii of curvature $r_0 = 1250\ \lambda$ and their separation was $2l = 500\ \lambda$. The Q of the highest modes in this case was comparable with the Q of the fundamental mode; however the mode degeneracy characteristic of confocal geometry (where $2l = r_0$) was absent.

In Fig. 1 we show the radial dependence of the field of the fundamental TEM_{00} mode, calculated according to the theory of Vaĭnshteĭn[3], and also the experimentally determined points. Here and elsewhere we use the mode designation for cavities with circular mirrors proposed by Fox and Li[1]. As is clear from the figure, the experimental results are practically independent of the choice (within reasonable limits) of the size of the absorbing probe and are in good agreement with the theory.

If the resonator is carefully aligned and the aperture coupling it to the microwave source is in the center of the mirror, only even symmetric modes TEM_{0n}, where n = 0, 1, 2, . . ., are effectively excited. Field measurements showed that only the fundamental mode exhibited circular symmetry, which persisted for not too great a misalignment of the mirrors. The theoretically predicted higher-order even symmetry modes turned out to be unstable in the sense that in the real cavity they easily split into a series of modes

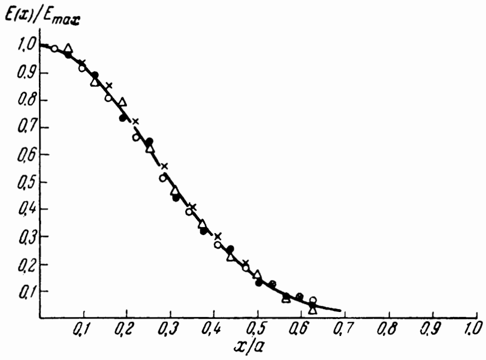

FIG. 1. The radial distribution of the field of the fundamental mode of an open resonator, close to one of the mirrors. The solid line is the calculated distribution. The experimental points are taken for absorbing cylindrical elements of length 0.8λ and of cross-section area: ○ – 0.25 λ^2, △ – 0.8λ^2, ● – 1.5λ^2, × – 3.0 λ^2.

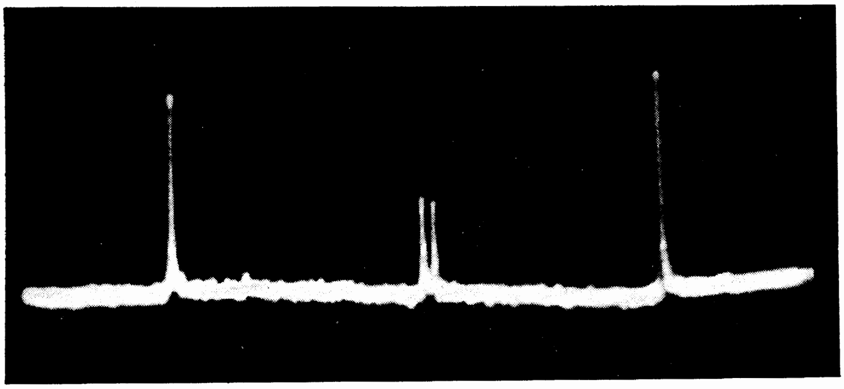

FIG. 2. Oscillogram of the resonance peaks for the modes TEM_{00q}, TEM'_{01q}, TEM''_{01q} and $TEM_{00(q+1)}$ (left to right).

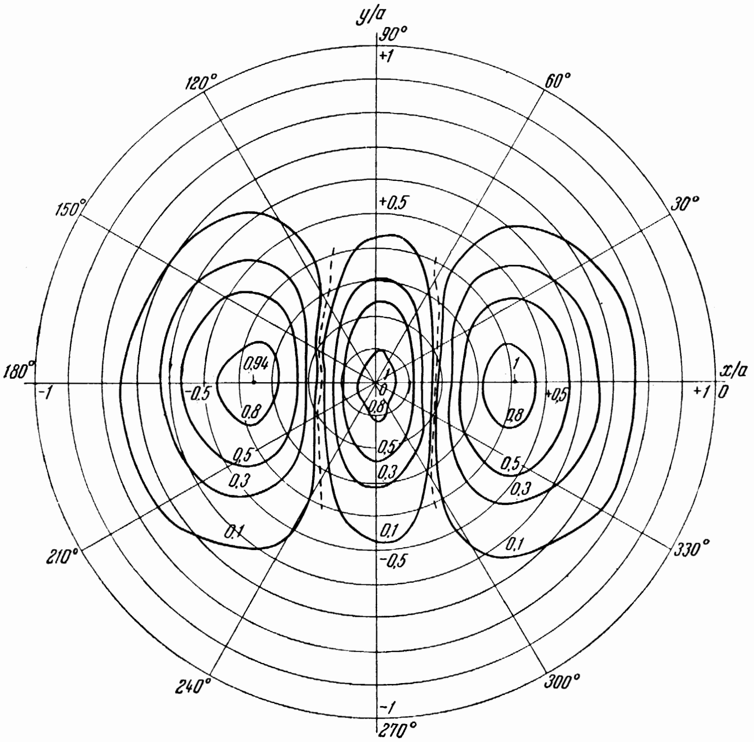

FIG. 3. Field distribution for the mode TEM'_{01}. The solid curves connect points of equal field strength on the mirrors

having more complicated field configurations, exhibiting an azimuthal dependence, and having nearly equal frequencies. This splitting is due probably to the non-ideal geometry of real cavities.

The TEM_{01} mode was observed to split into two modes with approximately the same resonance frequencies, which we will designate TEM'_{01} and TEM''_{01} (cf. Fig. 2). The field distribution of the TEM'_{01} mode is shown in Fig. 3. The field distribution of the TEM''_{01} mode differs only by a 90° rotation and is therefore not shown. If, according to [3], the TEM_{01} mode is described approximately by

$$\frac{E}{E_{max}} = \left(\frac{\tau^2}{2} - 1\right) e^{-\tau^2/4},$$

$$\tau = \left[\frac{4\pi (x^2 + y^2)}{r_0 \lambda} \sqrt{\frac{r_0}{l} - 1}\right]^{1/2}, \tag{1}$$

then the TEM'_{01} and the TEM''_{01} modes can be approximated, respectively, by the following empirical expressions.

$$\frac{E}{E_{max}} = \left(\frac{\tau^2}{2} + \frac{\tau^2}{2}\cos 2\varphi - 1\right) e^{-\tau^2/4},$$

$$\frac{E}{E_{max}} = \left(\frac{\tau^2}{2} - \frac{\tau^2}{2}\cos 2\varphi - 1\right) e^{-\tau^2/4}. \tag{2}$$

It is interesting to note that the field distributions of the above modes are very reminiscent of the types of modes characteristic of mirrors with square apertures. In cartesian coordinates they have the designations TEM_{02} and TEM_{20}. If one now multiplies and normalizes expressions (2) for the fields TEM'_{01} and TEM''_{01}, one obtains the above formula (1) for the TEM_{01} mode. Applying an analogous operation to the experimentally determined distributions of the same fields allows

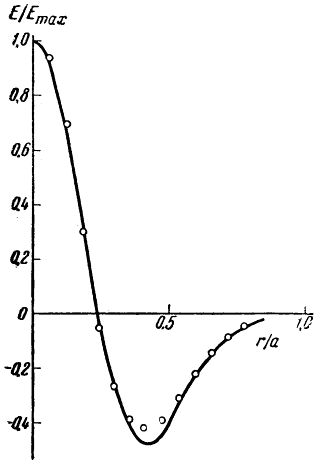

FIG. 4. Radial distribution of the field for TEM$_{01}$. The solid line is the theoretical curve, the circles are the experimental points.

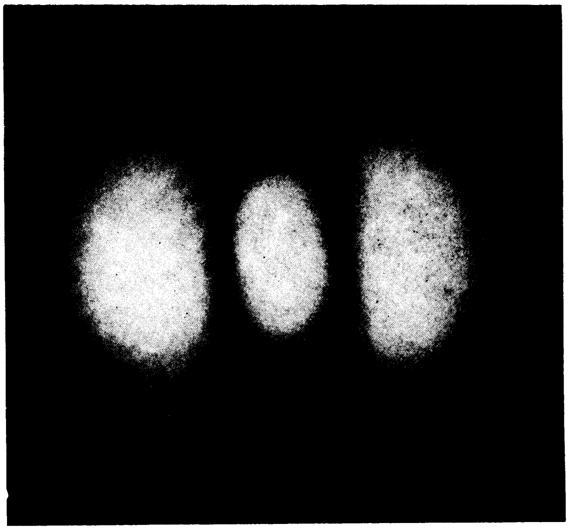

FIG. 5. Intensity distribution of light for the TEM$'_{01}$ mode of a gas laser.

one to obtain an even-symmetry mode having circular symmetry. Figure 4 shows the total radial field distribution for TEM$'_{01}$ and TEM$''_{02}$ as well as a plot of formula (1). It should be noted that the resonance frequencies for TEM$'_{01}$ and TEM$''_{02}$ could not be made to coincide either by alignment or by translation of the mirrors. Degeneracy could be obtained only by introducing into the cavity a thin foamed-polystyrene film which altered the frequency of one of the modes. The radial distribution then observed was in good agreement with the points in Fig. 4.

The modes described are very characteristic of open resonators with spherical round mirrors. These modes are observed in carefully aligned cavities with considerably smaller apertures as well as in gas lasers with non-confocal cavities (Fig. 5). The higher order even-symmetry modes also exhibit a similar splitting.

Zimmerer[5] recently described a similar measurement method, also using a small absorbing probe; however our method allows one to obtain the distribution of each mode separately, provided only that the frequencies of the modes are separated by at least the width of the resonance. On the other hand the method of Zimmerer allows one to obtain only the total field distribution. Moreover the curves obtained by the method of Zimmerer reflect less accurately the actual field distribution since they do not take into account the nonlinear relationship described by the function $f(\xi)$.

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