

# Letters to the Editor

## EFFECT OF LOGARITHMIC SINGULARITIES ON THE PARAMETERS OF SOME RESONANCES

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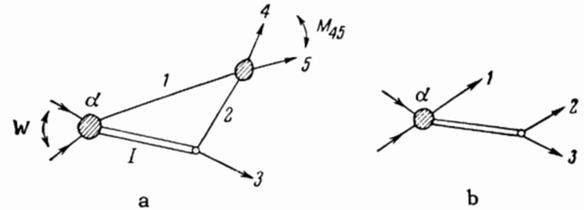


FIG. 1

IN reactions where three particles are produced, an appreciable role can be played by the process shown in Fig. 1a: particle 1 and a resonance are produced first, after which this resonance decays into particles 2 and 3, and particle 2 interacts with particle 1 to produce particles 4 and 5. The diagram shown in Fig. 1a has a logarithmic singularity<sup>[1]</sup>. The position of this logarithmic singularity in the complex  $M_{45}$  plane ( $M_{45}$  —total energy of the particles 4 and 5) depends on the total energy  $W$  of the system. It can be shown that when  $W$  satisfies the condition

$$(\text{Re } I^2 - m_2^2 - m_3^2) \frac{m_1 + m_2}{m_2} + (m_1 + m_2)^2 + m_3^2 \gtrsim W^2 \gtrsim (\text{Re } I + m_3)^2$$

( $I$  —complex mass of the resonance), the logarithmic singularity is close to the physical region of the reaction and, as noted in<sup>[2,3]</sup>, it can greatly influence the spectrum of the masses 4 and 5, and may even imitate resonances. It must be kept in mind, however, that the diagram shown in Fig. 1a can play an appreciable role only if the production of particles with masses  $m_1$ ,  $m_2$ , and  $m_3$  has an appreciable probability of proceeding via the resonance  $I$ , i.e., if the block  $\alpha$  in the diagrams of Fig. 1 is not small.

The present note is devoted to the question of resonances in the systems  $\rho\pi$  (A-resonance<sup>[4]</sup>),  $\omega\pi$  (B-resonance<sup>[5,6]</sup>), and  $\Sigma\pi$  ( $\Lambda_{1405}$  or  $Y_0^*$  resonance<sup>[7-12]</sup>). We wish to call attention to the fact that all these resonances were investigated in the overwhelming majority of cases under conditions when the spectra of the particles  $\rho\pi$ ,  $\omega\pi$ , and  $\Sigma\pi$  could be strongly influenced in the region of resonant values of energy by logarithmic singularities from diagrams of the type shown in Fig. 1a. The

presence of these logarithmic singularities could therefore lead to an incorrect determination of the parameters of the resonances indicated above.

We shall henceforth denote by  $M_{45}(I, 1, 2)$  the position of the logarithmic singularity of the diagram of Fig. 1a in the plane of complex values of the energy of particles 4 and 5.

1. The A-resonance was investigated in the reaction  $\pi^+ + p \rightarrow p + \pi^+ + \rho^0$  at an incident  $\pi^+$ -meson momentum  $p_\pi = 3.65 \text{ BeV}/c$ <sup>[4]</sup>. A broad resonance was found in the  $\rho\pi$  system, with mass  $M_{\rho\pi} = (1.2 - i 0.175) \text{ BeV}$ . However, near this resonance there are logarithmic singularities resulting from diagrams of the type of Fig. 1a. If we denote by  $\Delta_{1920}$  the pion-nucleon resonance with isotopic spin  $3/2$  and mass 1920 MeV, and by  $N_{1512}$  and  $N_{1688}$  the pion-nucleon resonances with isotopic spin  $1/2$ , then at  $p_\pi = 3.65 \text{ BeV}/c$  the logarithmic singularities resulting from diagrams 1a turn out to be located at  $M_{\rho^0\pi^+}(\Delta_{1920}, \rho^0, \pi^+) = (1040 - i 168) \text{ MeV}$ ,  $M_{\rho^0\pi^+}(N_{1688}, \rho^+, \pi^0) = (945 - i 108) \text{ MeV}$ , and  $M_{\rho^0\pi^+}(N_{1512}, \rho^0, \pi^0) = (904 - i 77) \text{ MeV}$ . An impression is gained that the observed large width of the A-resonance is in fact connected with the presence of these singularities.

2. The B-resonance was investigated in<sup>[5,6]</sup>. Different values were obtained for the width of the B-resonance:  $M_{\omega\pi} = (1215 - i 85) \text{ MeV}$ <sup>[5]</sup> and  $M_{\omega\pi} = (1220 - i 50) \text{ MeV}$ <sup>[6]</sup>. The reaction  $\pi^- + p \rightarrow p + \pi^- + \omega$  at  $p_\pi = 4.0 \text{ BeV}/c$  was investigated in<sup>[5]</sup>, where the logarithmic singularities were quite far from the B-resonance. The reaction  $\pi^+ + p \rightarrow p + \pi^+ + \omega$  at  $p_\pi = 3.54 \text{ BeV}/c$  was investigated in<sup>[6]</sup>, and a logarithmic singularity connected with the resonance  $\Delta_{1920}$  was observed near resonant values of the energy of the  $\omega\pi^+$  system, located at  $M_{\omega\pi^+}(\Delta_{1920}, \omega, \pi^+) = (1135 - i 154) \text{ MeV}$ . It is quite possible that this logarithmic singularity decreased the probability of  $\omega\pi^+$  production in the  $\sim 1150 \text{ MeV}$  mass region, and this has effectively led to a decrease in the observed resonance width.

3. The resonance  $\Lambda_{1405}$  was investigated by many authors. We shall only refer here to the papers dealing with this resonance, and indicate the position of the corresponding logarithmic singularity:

$$\begin{aligned}
 &K^- + p \rightarrow \Sigma^\pm + \pi^\mp + \pi^0: \\
 p_K = 1.51 \text{ BeV/c} \text{ [7]}, & M_{\Sigma\pi}(\rho, \Sigma, \pi) = (1400 - i 56) \text{ MeV}, \\
 & M_{\Sigma\pi}(N_{1512}, K, N) = 1400 \text{ MeV}; \\
 p_K = 2.24 \text{ BeV/c} \text{ [8]}, & M_{\Sigma\pi}(\rho\Sigma\pi) = (1350 - i 13) \text{ MeV}; \\
 p_K = 1.41 \text{ BeV/c} \text{ [9]}, & M_{\Sigma\pi}(\sigma\Sigma\pi) = (1410 - i 70) \text{ MeV}; \\
 p_K = 0.76 \text{ BeV/c} \text{ [10]}, & M_{\Sigma\pi}(\Lambda_{1520}\Sigma\pi) = (1330 + i 4) \text{ MeV}; \\
 & M_{\Sigma\pi}(K^*, N, K) = (1350 - i 120) \text{ MeV}; \\
 p_K = 0.85 \text{ BeV/c} \text{ [10]}, & M_{\Sigma\pi}(K^*, N, K) = (1387 - i 100) \text{ MeV}.
 \end{aligned}$$

An examination of the foregoing experimental data shows that almost in all the experiments the logarithmic singularities are located near the  $\Lambda_{1405}$  resonance. The only exceptions are the cases with  $p_K = 0.76 \text{ BeV/c}$  and  $0.85 \text{ BeV/c}$ <sup>[10]</sup>, where near the physical region there are only logarithmic singularities on the second sheet, connected with the reaction  $KN \rightarrow \Sigma\pi$ . Therefore their influence is possibly weaker:

$$\begin{aligned}
 &\pi^- + p \rightarrow \Sigma^\pm + K^0 + \pi^\mp: \\
 p_\pi = 1.89 - 2.04 \text{ BeV/c} \text{ [11]}, \\
 M_{\Sigma\pi}(K^*\Sigma\pi) &= (1390 - i 25) \text{ MeV}; \\
 p_\pi = 2.16 - 2.24 \text{ BeV/c} \text{ [11]}, \\
 M_{\Sigma\pi}(K^*\Sigma\pi) &= (1350 - i 10) \text{ MeV}.
 \end{aligned}$$

In <sup>[11]</sup> there was observed a maximum in the  $\Sigma\pi$  system with mass 1400 MeV at  $p_\pi = 1.89 - 2.04 \text{ BeV/c}$ . When  $p_\pi$  is increased to  $2.16 - 2.24 \text{ BeV/c}$ , the maximum decreases noticeably. This may be due to the fact that the logarithmic singularity at  $p_\pi = 1.89 - 2.04 \text{ BeV/c}$  is located close to the 1400 MeV region and when  $p_\pi = 2.16 - 2.24 \text{ BeV/c}$  it moves towards lower masses.

The  $\Lambda_{1405}$  resonance was observed also in reactions where four particles were produced  $K^- + p \rightarrow \Sigma^0 + \pi^0 + \pi^+ + \pi^-$ <sup>[12]</sup> at  $p_K = 1.22 \text{ BeV/c}$ . The logarithmic singularities may affect the  $\Sigma\pi$  spectrum in the reactions studied in <sup>[11]</sup>, owing to the diagrams shown in Fig. 2. Of course, the influence of the logarithmic singularities should be decreased in this case by the fact that the energy of the particles denoted in Fig. 2 by the indices 1, 2, and 3, is not fixed in this experiment.

In 1400 MeV region, the  $\Sigma\pi$  spectrum may be influenced not only by the logarithmic singularities but also by the resonance  $\Sigma_{1385}(Y_1^*)$  (an exception is, of course, the  $\Sigma^0\pi^0$  spectrum). The presence of this resonance in the logarithmic singularities greatly complicates the interpretation of the aforementioned experimental data on the  $\Sigma\pi$  spectrum

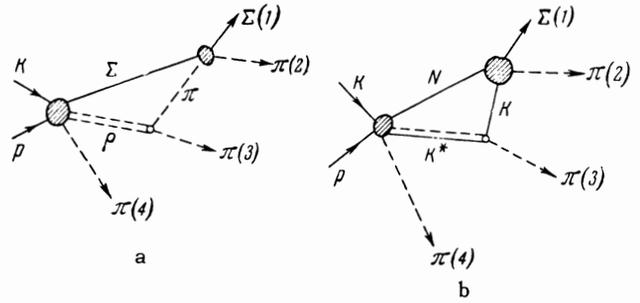


FIG. 2

in the 1400 MeV energy region. Consequently there are even grounds for doubting the existence of the  $\Lambda_{1405}$  resonance.

In conclusion, the authors are deeply grateful to V. M. Shekhter for a discussion of many problems and to N. B. Borovtynsya for carrying out the numerical calculations.

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<sup>10</sup>Bastien, Ferro-Luzzi, and Rosenfeld, Phys. Rev. Lett. **6**, 702 (1961).

<sup>11</sup>Alexander, Jakobs, Kolbflisch, Miller, Smith, and Schwartz, Proc. of the Intern. Conf. on High Energy Phys. in Geneva (1962), CERN, p. 320.

<sup>12</sup>Ferro-Luzzi, Tripp, and Watson, Phys. Rev. Lett. **8**, 28 (1962).