ON THE INTERACTION BETWEEN CHARGED PARTICLES AND A TURBULENT PLASMA

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We investigate the interaction between charged particles and a plasma consisting of cold ions and hot electrons moving relative to the ions with a velocity exceeding that of two-temperature sound. The rate of energy and momentum change due to emission and absorption of turbulent sound oscillations is determined. It is shown that the details of the turbulence spectrum do not affect the dependence of the indicated quantities on the particle velocity or on the direction of motion. The level of the turbulent fluctuations determines only the overall coefficient in the expressions for energy and momentum transfer. The energy of a particle moving in a plasma decreases if the particle velocity v satisfies the condition $\mathbf{v}\cdot\mathbf{u} > s^2$ ($\mathbf{u}$ is the mean directed electron velocity, $\mathbf{s}$ is the velocity of two-temperature sound); the particle energy increases if $\mathbf{v}\cdot\mathbf{u} < s^2$. The rate of change of the particle energy is proportional to the effective temperature T of the turbulent oscillations, provided the value of $\mathbf{v}\cdot\mathbf{u}$ lies between $s^2 \pm (u^2 - s^2)^{1/2}(v^2 - s^2)^{1/2}$. Near the boundaries of this region the energy varies as $T^{3/2}$. If $v \approx u$ or $v \approx s$ (and $\mathbf{v}\cdot\mathbf{u} \approx v$) the rate of particle energy loss is proportional to T^2. Interactions between charged particles and other types of turbulent oscillations are also considered e.g., with high-frequency-electron or short-wave-ion oscillations.

1. INTRODUCTION

As is well known, the intensity of the interaction between charged particles and a plasma is determined by the fluctuation level in the plasma. If the plasma is characterized by a very high fluctuation level, the change per unit time of the energy and momentum of a particle moving in the plasma is also very large.

The interaction between particles and a plasma whose state is close to unstable was investigated in [1]. In such a plasma the damping decrement of oscillations of any kind (for example, two-temperature sound or Langmuir oscillations) is much lower than the damping decrement of the corresponding oscillations in an equilibrium plasma. Because of this, the level of the fluctuations in a nearly-unstable plasma is much higher than the thermal level[2-4], and this can lead to very intense slowing down of a particle moving through the plasma.

In the present paper we investigate the interaction between particles and a plasma in which, neglecting nonlinear effects, oscillations of arbitrary type, do not attenuate but grow. The growth of the random oscillations continues until the nonlinear effects bring about a stationary distribution of the fluctuations, i.e., until the plasma goes over into a state of stationary turbulence[1)].

An essential feature of the steady-state spectral distributions of the fluctuations is the fact that the effective temperature of the waves is small at wave-vector values $\mathbf{q}$ corresponding to non-growing oscillations, and increases sharply near wave-vector values that separate the stability and instability regions of the oscillations. At wave-vector values corresponding to growing oscillations, the effective temperature varies smoothly with varying $\mathbf{q}$, remaining very large. This character of the spectral distribution of the fluctuations enables us to investigate the interaction between charged particles and a turbulent plasma in general form, without making use of detailed properties of the turbulence spectrum.

In the present paper we determine the variation per unit time in the energy and momentum of a charged particle, due to the emission and absorption of sound oscillations. We obtain in explicit

[1)]The limitation on the growth of the amplitude of the waves, resulting from the nonlinear interaction between the waves and the plasma particles, was investigated in[5,6]. The spectral distributions of the turbulent fluctuations are determined in[7-9]. The interaction between the particles and a plasma in which the external sources produce a nonequilibrium wave distribution was investigated in[10].

form the dependence of these quantities on the direction and on the magnitude of the particle velocity. The level of turbulent fluctuations determines only a common coefficient in the expression for the variation of the energy and momentum of the particle. We investigate also the particle-energy variation due to the emission and absorption of high-frequency electronic and short-wave ionic oscillations.

2. INTENSITY OF CERENKOV RADIATION

Let us relate first the change in the energy of a particle passing through a plasma with the effective temperature of plasma oscillations.

The probability of the transition of a particle from a state with momentum **p** into a state with momentum **p′** is connected with the correlator of the charge density fluctuations by the known relation

$$dw = (4\pi ez / \hbar q^2)^2 \langle\rho^2\rangle_{\mathbf{q}\omega} d\mathbf{p}' / (2\pi\hbar)^3, \tag{1}$$

where $\langle\rho^2\rangle_{\mathbf{q}\omega}$ —charge-density correlation function, $\hbar\omega = (2\mu)^{-1}(p'^2 - p^2)$, and $\hbar\mathbf{q} = \mathbf{p}' - \mathbf{p}$ — changes in the energy and momentum of the particle, ez —charge, and μ —mass of the particle.

We discuss first the case of a plasma consisting of cold ions and hot electrons moving relative to the ions. In the region of large wavelengths ($aq \ll 1$) and "medium" frequencies ($q(T_i/M)^{1/2} \ll \omega \ll q(T_e/m)^{1/2}$) the correlator of the charge density in such a plasma can be represented in the form

$$\langle\rho^2\rangle_{\mathbf{q}\omega} = {}^1/_4 q^2 (aq)^2 \{T(\mathbf{q})\delta(\omega - qs) + T(-\mathbf{q})\delta(\omega + qs)\}, \tag{2}$$

where T_e and T_i —temperatures, m, M —masses of the electrons and ions, $s = (T_e/M)^{1/2}$ —two-temperature sound velocity, $T(\mathbf{q})$ —so-called effective temperature of sound oscillations, and $a = T_e^{1/2}(4\pi e^2 n)^{-1/2}$ —electronic Debye radius. Recognizing that the effective temperature of the oscillations is a function of two scalar quantities q and $\eta \equiv \mathbf{q}\cdot\mathbf{u}$, where **u** —directed velocity of the electrons, we shall henceforth denote the effective temperature by $T(q, \mathbf{q}\cdot\mathbf{u})$.

Using (1) and (2), we can determine the energy lost by a particle per unit time to excitation of sound oscillations with wave vectors in the interval ($\mathbf{q}, \mathbf{q}+d\mathbf{q}$):

$$dP = -(eza)^2 qs (2\pi\hbar)^{-1}\{T(q, \mathbf{qu})\delta(\mathbf{qv} - qs + \hbar q^2/2\mu) - T(q, -\mathbf{qu})\delta(\mathbf{qv} + gs + \hbar q^2/2\mu)\}d\mathbf{q}, \tag{3}$$

where **v** —particle velocity. The first term in this expression describes the absorption, and the second the induced emission of oscillations by the particle (we are interested in plasma states for which the number of sound waves in the plasma is very large, so that we need not, naturally introduce in (3) an additional term to account for the spontaneous emission that is independent of the number of waves).

Integrating in (3) over the angle variables, we obtain the intensity of the Cerenkov radiation per unit frequency interval:

$$\frac{dP}{d\omega} = \frac{(ez)^2 (aq)^2 u}{\pi\mu v^2} q^2 D(q); \tag{4}$$

$$D(q) = \int_0^\pi d\varphi \left\{\cos\theta - \left(\frac{v^2}{s^2} - 1\right)^{-1/2} \sin\theta\cos\varphi\right\} \frac{\partial}{\partial\eta} T(q;\eta); \tag{5}$$

θ (the angle between the vectors **v** and **u**) and η are connected with the integration variable φ by the relation

$$\eta = qu\{sv^{-1}\cos\theta + (1 - s^2 v^{-2})^{1/2}\sin\theta\cos\varphi\}. \tag{6}$$

We see that the energy lost by the particle is determined by the function $\partial T(q, \eta)/\partial\eta$ ($\eta \equiv \mathbf{q}\cdot\mathbf{u}$); to calculate the energy losses it is necessary therefore to know the character of the dependence of the effective temperature on the quantity $\mathbf{q}\cdot\mathbf{u}$. At small values of $\mathbf{q}\cdot\mathbf{u}$, when the oscillations with wave vector **q** attenuate ($\mathbf{q}\cdot\mathbf{u} < qs$), the effective temperature is determined by the formula [2-4]

$$T(q, \mathbf{qu}) = T_e(1 - \mathbf{qu}/qs)^{-1}. \tag{7}$$

When $\mathbf{q}\cdot\mathbf{u} \approx qs$ the effective temperature increases sharply. When $\mathbf{q}\cdot\mathbf{u} > qs$, the quantity $T(q, \mathbf{q}\cdot\mathbf{u})$ becomes (for fixed q) a smooth function of $\mathbf{q}\cdot\mathbf{u}$.

It follows from this dependence of the effective oscillation temperature on $\mathbf{q}\cdot\mathbf{u}$ that the derivative $\partial t/\partial(\mathbf{q}\cdot\mathbf{u})$ has a sharp maximum at some value of $\mathbf{q}\cdot\mathbf{u}$ close to qs, namely $\mathbf{q}\cdot\mathbf{u} = \eta_0 \approx qs$. The presence of such a maximum enables us to calculate the function $D(q)$ without knowing in detail the dependence of T on $\mathbf{q}\cdot\mathbf{u}$ when $\mathbf{q}\cdot\mathbf{u} > qs$.

In fact, let us expand the function $(\partial T/\partial\eta)^{-1}$ in powers of the quantity $\eta - \eta_0$. Noting that when $\eta = \eta_0$ the function $(\partial T/\partial\eta)^{-1}$ has a minimum, we obtain

$$\left[\frac{\partial}{\partial\eta} T(q, \eta)\right]^{-1} = \frac{qs}{T_e}\left\{\xi^2(q) + \lambda^2(q)\left(1 - \frac{\eta}{\eta_0}\right)^2\right\}, \tag{8}$$

where

$$\xi^2(q) = T_e(qs\ \partial T/\partial\eta)^{-1}_{\eta=\eta_0}$$

is a small quantity proportional to $T_e^2[T(q, \mathbf{q}\cdot\mathbf{u})]^{-2}_{\mathbf{q}\cdot\mathbf{u} > qs}$, and

$$\lambda^2(q) = T_e\eta_0^2(2qs)^{-1}\frac{\partial^2}{\partial\eta^2}\left(\frac{\partial T}{\partial\eta}\right)^{-1}_{\eta=\eta_0}$$

is a quantity of the order of unity. Substituting this expansion in (5) and recognizing that $\eta_0 \approx qs$, we see that the main contribution to the integral that determines the function T(q) is made by angles φ close to φ_0, where

$$\cos\varphi_0 = \frac{v/u - \cos\theta}{(v^2/s^2 - 1)^{1/2}\sin\theta}. \quad (9)$$

For $\cos^2\varphi_0 < 1$ (and for not too small values of $\sin\varphi_0$) we obtain for D

$$D(q) = \pi T_e(\lambda\xi qu)^{-1}\left(1 - \frac{s^2}{v^2}\right)^{-3/2}\frac{\cos\theta - s^2/uv}{\sin\theta\sin\varphi_0}. \quad (10)$$

The value of $|D|$ is proportional to the large parameter ξ^{-1} and increases sharply with decreasing $\sin\varphi_0$. At very small values of $|\sin\varphi_0|$ ($|\sin\varphi_0|^2 \ll \xi s/u(1 - s^2/v^2)^{1/2}\sin\theta$, and $\cos^2\varphi$ can be both larger and smaller than unity), we obtain by using (5) and (8)

$$D(q) = \pm\frac{\pi T_e v^{3/2}(u^2 - s^2)^{1/2}}{2q(\lambda s\sin\theta_\pm)^{1/2}(\xi u)^{3/2}(v^2 - s^2)^{3/4}}. \quad (11)$$

The plus sign in this expression corresponds to the case $\theta \approx \theta_+$ the minus sign to the case $\theta \approx \theta_-$, where $\theta_\pm$—the two values of the angle θ at which $\sin\varphi_0$ vanishes,

$$\cos\theta_\pm = (uv)^{-1}\{s^2 \pm (u^2 - s^2)^{1/2}(v^2 - s^2)^{1/2}\}. \quad (12)$$

Thus, for very small values of $|\sin\varphi_0|$, the value of D is proportional to $\xi^{-3/2}$.

If $u \to v$, then $\sin\theta_+ \to 0$, and (11) no longer holds. For $\sin\theta_+ \ll \xi s(v^2 - s^2)^{-1/2}$ we obtain in place of (11)

$$D(q) = \pi T_e\cos\theta(\xi^2 qs)^{-1}. \quad (13)$$

In this case D is particularly large and is proportional to ξ^{-2}.

We note that in deriving (10) and (11) we have assumed that the particle velocity v is not too close to the velocity of sound s. Using (5) and (7) we can verify that when $1 - s^2/v^2 \ll (\xi s/u)^2\sin^{-2}\theta$ the function D is determined by relation (13) if $\cos\theta \approx s/u$, and changes sharply when $|\cos\theta - s/u|$ increases.

The case when $\cos\varphi_0 > 1$ (and the difference $\cos\varphi_0 - 1$ is not too small) corresponds to a particle velocity such that the particle cannot interact with the turbulent sound waves. In this case we cannot use expansion (8) to determine D(q); it is easy, however, to calculate D(q) directly, by substituting in (5) the expression (7) for the effective temperature. We then obtain

$$D(q) = \pi T_e s^2(qu^3)^{-1}\left(1 - \frac{s^2}{v^2}\right)^{-3/2}\frac{\cos\theta - u/v}{\sin^3\theta|\sin\varphi_0|^3}. \quad (14)$$

In this case D does not contain the large parameter ξ^{-1}; nonetheless, if $|\sin\varphi_0| \ll 1$, this quantity is very large (although it is much smaller than when $\cos^2\varphi_0 \le 1$). This is due to the intense interaction of the particle with the sound waves, for which $qs - \mathbf{q}\cdot\mathbf{u} \ll qs$ and whose effective temperature is large, in accordance with (7).

The case $\cos\varphi_0 < -1$ corresponds to a particle moving in such a way that it interacts effectively with the turbulent sound waves with $\mathbf{q}\cdot\mathbf{u} > qs$. As already noted, $\partial T/\partial(\mathbf{q}\cdot\mathbf{u})$ is small in the region $\mathbf{q}\cdot\mathbf{u} > qs$, so that when $\cos\varphi_0 < -1$ the function D(q) has the same order of magnitude as when $\cos\varphi_0 > 1$.

We note that the main results of this section (and consequently also the expressions for the change in the energy and momentum of the particle) remain in force even if $\partial T/\partial\eta$ is not small in the region $\eta > \eta_0$. In fact, the contribution of $\partial T/\partial\eta$ with $\eta > \eta_0$ to the expression for D can only change the function D when $\theta_+ < \theta < \theta_-$, without changing the value of D at $\theta \approx \theta_\pm$, and consequently without changing the character of the function D when $\theta_+ \le \theta \le \theta_-$. In particular, the function D is positive for $\theta = \theta_+$ and negative for $\theta = \theta_-$, so that it must vanish at some value of the angle $\theta = \theta_0$, $\theta_+ < \theta_0 < \theta_-$ [θ_0 can differ here somewhat from $\cos^{-1}(s^2/uv)$].

3. CHANGE IN PARTICLE ENERGY

The function D, in accordance with (4), determines the spectral distribution of the energy radiated by the particle. Substituting D in (4) and integrating over the frequencies, we obtain an expression for the change in particle energy (per unit time) P, due to its interaction with the sound waves.

If the angle θ between $\mathbf{v}$ and $\mathbf{u}$ lies in the interval $\theta_+ < \theta < \theta_-$, where the angles $\theta_\pm$ are determined by the formula (12), then P takes the form

$$P = \frac{(ez\Omega)^2}{s}\frac{m}{\mu}Af(\mathbf{v}), \quad (15)$$

where A is a large quantity, proportional to the ratio of the effective temperature of the turbulent sound waves to the temperature of the plasma electrons

$$A = a\int(aq)^3(\lambda\xi)^{-1}dq, \quad (16)$$

$\Omega^2 = 4\pi e^2 n/m$ —square of electronic plasma frequency and

$$f(\mathbf{v}) = \frac{\mathbf{uv} - s^2}{v^2 - s^2}\left\{\left(\frac{u^2}{s^2} - 1\right)\left(\frac{v^2}{s^2} - 1\right) - \left(\frac{\mathbf{uv}}{s^2} - 1\right)^2\right\}^{-1/2}. \quad (17)$$

Formulas (15) and (17) determine in explicit form the dependence of the variation of the particle energy on the magnitude and direction of its velocity. It is easy to see that when $\theta_+ < \theta < \theta_-$, where $\cos\theta_0 = s^2/uv$, the particle loses energy. When $\theta_0 < \theta < \theta_-$, the interaction with the sound oscillations leads to an increase in the particle energy. When $\theta = \theta_0$, the change in particle energy vanishes. As $\theta \to \theta_\pm$, the value of $|P|$ increases sharply.

At angles θ very close to $\theta_\pm$, formula (15) ceases to be correct. When

$$|\theta - \theta_\pm| \ll \bar{\xi}(s/u)(1 - s^2/v^2)^{1/2}$$

($\bar{\xi}$ —quantity of the order of the ratio of the electron temperature to the effective wave temperature in the turbulence region) we obtain, substituting (11) in (4) and integrating over the frequencies,

$$P = \frac{(ez\Omega)^2}{s}\frac{m}{\mu}A_1 f_\pm, \quad (18)$$

where A_1 —large quantity proportional to $\bar{\xi}^{-3/2}$

$$A_1 = \frac{1}{2}a\int (aq)^3 \xi^{-3/2}\lambda^{-1/2}dq, \quad (19)$$

(the plus and minus signs correspond to the cases $\theta \approx \theta_\pm$). We see that when $\theta \approx \theta_\pm$ the change in particle energy per unit time is especially large.

At very small values of θ_+ (which corresponds to $u \approx v$), relation (18) ceases to be valid. For $\theta_+ \ll \bar{\xi}s(v^2 - s^2)^{-1/2}$, we have in place of (18)

$$P = \frac{(ez\Omega)^2}{v}\frac{m}{\mu}A_2, \quad A_2 = a\int (aq)^3\xi^{-2}dq. \quad (20)$$

We emphasize that a very large change in the particle energy per unit time (proportional to $\bar{\xi}^{-1}$, $\bar{\xi}^{-3/2}$, or $\bar{\xi}^{-2}$) takes place when the angle between $\mathbf{v}$ and $\mathbf{u}$ lies in the region $\theta_+ \le \theta \le \theta_-$. In order for the angle region to exist it is necessary to satisfy the inequalities $v > s$ and $u > s$. The first of these inequalities is the condition for the effective interaction between the particle and the sound oscillations (Cerenkov condition), while the second inequality ensures the existence of turbulence sound oscillations.

As $u \to s$ we have $\theta_+ \to \theta_- \to \cos^{-1}(s/v)$. Expressions (15) and (18) for P vanish when $\mathbf{v}\cdot\mathbf{u} = s^2$ and $u = 2$, for in this case the energy loss is proportional to $1/\bar{\xi}$ raised to a power lower than the first. In fact, substituting (8) in (5) and (4) we obtain

$$P = \frac{(ez\Omega)^2}{s}\frac{m}{\mu}A_0\frac{s^3}{v^{3/2}(v^2 - s^2)^{3/4}},$$

$$A_0 = \frac{1}{2}a\int (aq)^3\xi^{-1/2}\lambda^{-3/2}dq. \quad (21)$$

In the derivation of (15)—(21) it was assumed that the particle velocity $\mathbf{v}$ is not too close to the velocity of sound s $(1 - s^2/v^2 \gg \bar{\xi}^\nu$; in the general case described by formula (15) $\nu = 1$; in the case when $\theta \approx \theta_\pm$ we have $\nu = 2/3$). If $1 - s^2/v^2 \ll \bar{\xi}^\nu$, then for $\theta = \cos^{-1}(s/u)$ the energy lost by the particle is determined by formula (20) and is proportional to $\bar{\xi}^{-2}$. At other values of θ, P does not contain the large parameter $1/\xi$.

If $\theta > \theta_-$, and also if $\theta < \theta_+$ and $u < v$, then the change in particle energy can be determined by substituting (7) in (5). Integrating over frequencies lower than some maximum frequency $s/\tilde{a}$, where $\tilde{a}$ is on the order of several Debye radii, we obtain

$$P = \frac{(ez\Omega)^2}{s}\frac{m}{\mu}\alpha g(\mathbf{v}), \quad (22)$$

where $\alpha = \frac{1}{4}(a/\tilde{a})^4$ and

$$g(\mathbf{v}) = s^{-2}(\mathbf{uv} - u^2)\left\{\left(\frac{\mathbf{uv}}{s^2} - 1\right)^2 - \left(\frac{u^2}{s^2} - 1\right)\left(\frac{v^2}{s^2} - 1\right)\right\}^{-3/2}. \quad (23)$$

We note that this formula takes into account the induced emission and absorption of sound oscillations by the particle, and does not take into account other processes that change the particle energy (short-range collisions, spontaneous emission of sound oscillations, emission and absorption of other types of plasma oscillations). Therefore expression (22) determines the total particle energy loss only at angles θ sufficiently close to $\theta_\pm$, when the relative contribution of the other processes to the change in its energy is small.

If $\theta < \theta_+$ and $u > v$, then the change in the particle energy per unit time is much smaller than if $\theta_+ < \theta < \theta_-$. In this case, in order to establish the dependence of P on $\mathbf{v}$, it is necessary to know in detail the behavior of the function $T(q, \mathbf{q}\cdot\mathbf{u})$ for $\mathbf{q}\cdot\mathbf{u} > qs$.

In concluding this section, let us estimate the coefficients A and $A_{0,1,2}$ in expressions (15), (18), (20), and (21) for the change in the particle energy per unit time; these coefficients are independent of the particle velocity. We use for this purpose the expression obtained for the effective temperature in [7] under the assumption that the growth of the amplitude of the sound waves with $\mathbf{q}\cdot\mathbf{u} > qs$ is limited by nonlinear Landau damping by the ions,

$$T(q, \mathbf{qu}) \sim \frac{T_e}{(aq)^3} R, \quad R = \frac{T_e}{e^2/a}\left(\frac{msu}{T_i\Omega\tau_e}\right)^{1/2} \quad (\mathbf{qu} > qs), \quad (24)$$

where τ_e —electron mean free path time. Putting $\xi \sim T_e/T$ and $\lambda \sim 1$, and integrating with respect to q from $q \sim s/\tau_i$ to $q \sim a^{-1}$ (τ_i —ion mean free path time), we obtain

$$A \sim R, \quad A_0 \sim R^{1/2}, \qquad A_1 \sim R^{3/2}(s\tau_i/a)^{1/2},$$
$$A_2 \sim R^2(s\tau_i/a)^2. \qquad (25)$$

4. CHANGE IN PARTICLE MOMENTUM

We now see how emission and absorption of sound waves changes the momentum of a charged particle. Using (1) and (2), we can determine the momentum transferred to the particle per unit time by the sound with wave vectors in the interval $(\mathbf{q}, \mathbf{q}+d\mathbf{q})$:

$$d\mathbf{Q} = -(eza)^2(2\pi\hbar)^{-1}\mathbf{q}\{T(q, \mathbf{qu})\delta(\mathbf{qv} - qs + \hbar q^2/2\mu) + T(q, -\mathbf{qu})\delta(\mathbf{qv} + qs + \hbar q^2/2\mu)\}d\mathbf{q}.$$

Integrating with respect to $\mathbf{q}$ in this relation, we obtain

$$\mathbf{Q} = \frac{(eza)^2 us}{\pi\mu v^2}\int dq\, q^4\{\mathbf{I}D(q) + \mathbf{i}_u d_u(q) + \mathbf{i}_v d_v(q)\}, \quad (26)$$

where

$$\mathbf{I} = (uv\sin\theta)^{-2}\{\mathbf{u}(v^2 - \mathbf{uv}) + \mathbf{v}(u^2 - \mathbf{uv})\},$$
$$\mathbf{i}_u = (uv\sin\theta)^{-2}\{\mathbf{u}v^2 - \mathbf{v}(\mathbf{uv})\},$$
$$\mathbf{i}_v = (uv\sin\theta)^{-2}\{\mathbf{v}u^2 - \mathbf{u}(\mathbf{uv})\}, \qquad (27)$$

the function D is determined by formula (5), and

$$d_u(q) = (qs)^{-1}\int_0^\pi d\varphi\left[\cos\theta - \left(\frac{v^2}{s^2} - 1\right)^{-1/2}\sin\theta\cos\varphi\right] \times [T + (\eta - qs)\,\partial T/\partial\eta],$$

$$d_v(q) = v\,(qsu)^{-1}\int_0^\pi d\varphi T\,(q, \eta) \qquad (28)$$

[the quantity η is connected with the integration variable φ by relation (6)].

To determine D, as shown in Sec. 1, it is sufficient to know the behavior of the function $T(q, \mathbf{q}\cdot\mathbf{u})$ near $\mathbf{q}\cdot\mathbf{u} = qs$; to calculate d_u and d_v, on the other hand, it is necessary to know also the explicit form of the function T at $\mathbf{q}\cdot\mathbf{u} \neq qs$. It is easy to verify, however, that if $|\theta - \theta_\pm| \ll \sin\theta_\pm$, where the angles $\theta_\pm$ are determined by (12), then $|\mathbf{i}_u d_u| \ll |\mathbf{I}D|$ and $|\mathbf{i}_v d_v| \ll |\mathbf{I}D|$. Taking (4) into account, we can thus relate the change in the momentum of the particle Q at $\theta \sim \theta_\pm$ with the change in its energy P:

$$\mathbf{Q} = \mathbf{I}P. \qquad (29)$$

This relation, together with formulas (15)–(22) for P, enables us to determine the dependence of $\mathbf{Q}$ on the magnitude and direction of the particle velocity $\mathbf{v}$.

According to (29), when the particle moves, the fastest to change is the projection of its momentum on the direction of $\pm\mathbf{I}$,

$$\mathbf{QI}|\mathbf{I}|^{-1} = (uv\sin\theta)^{-1}|\mathbf{u} - \mathbf{v}|P.$$

The projection of the particle momentum on the direction of $(\mathbf{v} - \mathbf{u})$ does not change when the particle moves, $\mathbf{Q}(\mathbf{v} - \mathbf{u}) = 0$.

We note that in the interaction between a particle and a non-equilibrium plasma, the case $|\theta - \theta_\pm| \ll \sin\theta_\pm$ (to which formula (29) pertains) is of greatest interest, for in this case the change in the particle momentum per unit time (together with its energy) is particularly large.

We present one other expression for the function $\mathbf{Q}$ in the case when the particle velocity is close to the beam velocity both in magnitude and in direction ($s|v - u|u^{-2} < \theta \ll \bar{\xi}s(v^2 - s^2)^{-1/2}$). Here $\mathbf{Q} = \mathbf{v}Pv^{-2}$, where P is determined by (20). In this case the change in particle momentum per unit time is proportional to ξ^{-2} and is therefore especially large.

5. RADIATION OF SHORT-WAVE IONIC AND HIGH-FREQUENCY ELECTRONIC PLASMA OSCILLATIONS

As is well known, a plasma consisting of hot electrons and cold ions can support not only sound oscillations with linear dispersion but also short-wave ionic oscillations. The charge-density correlator in the region of "medium" frequencies $[q(T_i/M)^{1/2} \ll \omega \ll q(T_e/m)^{1/2}]$ is given by an expression that takes the cold-wave oscillations into account and takes the form

$$\langle\rho^2\rangle_{\mathbf{q}\omega} = {}^1\!/\!_4 q^2(aq)^2(1 + a^2q^2)^{-1}\{T(q, \mathbf{qu})\delta(\omega - \omega_q) + T(q, -\mathbf{qu})\delta(\omega + \omega_q)\} \quad (a^2q^2 \ll T_e/T_i), \qquad (30)$$

where $\omega_q = qV_s(1 + a^2q^2)^{-1/2}$ —frequency and $T(q, \mathbf{q}\cdot\mathbf{u})$ —effective oscillation temperature (we denote in this section the velocity of the two-temperature sound, which is equal to $(T_e/M)^{1/2}$, by V_s, retaining the symbol s for the phase velocity of the oscillations, $s(q) = \omega_q/q$). When $\mathbf{q}\cdot\mathbf{u} < \omega_q$, the effective temperature of the oscillations is determined by the formula[3] $T(q, \mathbf{q}\cdot\mathbf{u}) = T_e(1 - \mathbf{q}\cdot\mathbf{u}/\omega_q)^{-1}$. When $\mathbf{q}\cdot\mathbf{u} > \omega_q$, the value of T increases sharply. In the region $\mathbf{q}\cdot\mathbf{u} > \omega_q$, the effective temperature changes smoothly with $\mathbf{q}\cdot\mathbf{u}$.

Using (1) and (30) we obtain an expression for the change in particle energy per unit time, due to the excitation and absorption of ionic oscillations with wave vectors in the interval (q, q+dq):

$$\frac{dP}{dq} = \frac{(ez)^2 u}{\pi\mu v^2} \frac{a^2 q^3 \omega_q}{1 + a^2 q^2} \left\{ D(q)\,\Theta\left(1 - \frac{\omega_q}{qv}\right) - \frac{\pi}{u} T(q, qu\cos\theta)\,\delta\left(q - \frac{\omega_q}{v}\right) \right\},$$

where the function D is determined by (5) with $s \equiv s(q) = \omega_q/q$ and $\Theta(x) = (1 + \operatorname{sign} x)/2$.

Substituting the expression (10) [or (11)—(14)] for the function D in (31), we readily see that for all values of the wave vector (except for the value satisfying the equation $\omega_q = qv$) the value of dP/dq does not depend on the detailed behavior of the function T when $\mathbf{q}\cdot\mathbf{u} > \omega_q$. (In the case of $\omega_q = qv$ and $v > u\cos\theta$, when the second term in (31) can be disregarded, we can again determine dP/dq without knowing the exact dependence of T on $\mathbf{q}\cdot\mathbf{u}$ when $\mathbf{q}\cdot\mathbf{u} > \omega_q$.)

Integrating (31) with respect to q, we can determine the variation of the particle energy per unit time P, due to its interaction with the plasma oscillations. In the case of oscillations with a nonlinear dispersion law, in order to establish the explicit form of the dependence of P on the particle velocity $\mathbf{v}$ it is necessary, generally speaking, to know the explicit form of the functions $\xi(q)$ and $\lambda(q)$. It is possible, nonetheless, to draw several conclusions concerning the value of P, conclusions which do not depend on the explicit form of the functions ξ and λ.

First, the variation of the particle energy is large (proportional to $1/\bar{\xi}$ raised to a power not smaller than the first), if the condition $\sin^2\theta \gg T_i(\mathbf{v} - \mathbf{u})^2(Mv^2u^2)^{-1}$ is satisfied. This condition is the condition for the existence of oscillations whose wave vectors satisfy the relations $\mathbf{q}\cdot\mathbf{v} = \omega_q$ and $\mathbf{q}\cdot\mathbf{u} > \omega_q$.

Further, if the angle between the direction of particle motion and the flow direction is small, then intense interaction between the particle and the oscillations takes place only when $v \approx u$. If the conditions $V_s|u - v|u^{-2} < \theta \ll \bar{\xi} V_s u^{-1}$ are satisfied, the particle energy loss is particularly large and proportional to $\bar{\xi}^{-2}$,

$$P = \frac{(ez\Omega)^2}{v}\frac{m}{\mu}B_2, \quad B_2 = a\int (aq)^3(1 + a^2q^2)^{-1}\xi^{-2}\,dq. \quad (32)$$

Finally, when $v \gg u$, it is easy to determine the sign of P. The function P is positive (this corresponds to a decrease in particle energy), if $\theta < \pi/2$, and negative (corresponding to an increase in its energy) if $\theta > \pi/2$.

When $v \gg V_s$ and $u \gg V_s$, the dependence of the change in the particle energy on the magnitude and direction of its velocity can be obtained in explicit form. If $\sin\theta$ is not too small ($\theta \gg V_s|u - v|(uv)^{-1}$, $\pi - \theta \gg V_s(u + v)(uv)^{-1}$), then, using (31) and (10), we obtain

$$P = (ez\Omega)^2 m V_s(\mu v^2)^{-1} B\cot\theta, \quad (33)$$

where B is a large quantity proportional to $\bar{\xi}^{-1}$,

$$B = a\int (aq)^3(1 + a^2q^2)^{-3/2}(\lambda\xi)^{-1}\,dq.$$

When $\sin\theta \ll \min\{(\xi V_s/u)^{1/2}, \xi u/V_s\}$, we have in accordance with (11)

$$P = \pm(ez\Omega)^2 u m v^{3/2}\mu^{-1}|u \mp v|^{-1/2}B_1,$$

$$B_1 = {}^1\!/_2\, a\int (aq)^3(1 + a^2q^2)^{-1}\xi^{-3/2}\lambda^{-1/2}\,dq, \quad (34)$$

where the upper (lower) sign pertains to the case of small angles θ (close to π). We see that the energy of a particle moving in the beam direction (or in the opposite direction) changes in proportion to $\bar{\xi}^{-3/2}$ and is therefore especially large. If the particle is close to the beam velocity not only in direction but also in magnitude, then the energy loss [determined in this case by (32)] is proportional to $\bar{\xi}^{-2}$.

Let us stop to discuss also the case of a plasma through which a hot beam travels with velocity u larger than the thermal velocity of the plasma electrons. The charge-density correlator in such a plasma, at high frequencies ($\omega \gg q(T_e/m)^{1/2}$) and large wavelengths ($aq \ll 1$) is of the form

$$\langle\rho^2\rangle_{\mathbf{q}\omega} = {}^1\!/_4 q^2\{T(q, \mathbf{qu})\,\delta(\omega - \Omega) + T(q, -\mathbf{qu})\,\delta(\omega + \Omega)\}, \quad (35)$$

where T—effective temperature of the electronic Langmuir oscillations. When $\mathbf{q}\cdot\mathbf{u} < \Omega$, the temperature T is determined by the temperature T_1 of the beam electrons[3], $T(q, \mathbf{q}\cdot\mathbf{u}) = T_1(1 - \mathbf{q}\cdot\mathbf{u}/\Omega)^{-1}$. When $\mathbf{q}\cdot\mathbf{u} > \Omega$, the effective temperature is determined by the level of the turbulent fluctuations and greatly exceeds T_1.

Using (1) and (35), we find the energy transferred to the particle per unit time by the electronic Langmuir oscillations with wave vectors lying in the interval (q, q + dq):

$$\frac{dP}{dq} = \frac{(ez)^2 u}{\pi\mu v^2} q\Omega \left\{ D(q)\,\Theta\left(1 - \frac{\Omega}{qv}\right) - \frac{\pi}{u} T(q, qu\cos\theta)\,\delta\left(q - \frac{\Omega}{v}\right) \right\}, \quad (36)$$

where the function D is determined by (5) with $s = \Omega/q$.

It is easy to see that all the deductions regarding the interaction between the particle and the ionic oscillations [except, of course, relations (33) and (34)] can be generalized to the case of electronic oscillations, if we make the substitutions

$T_e \to T_1$, $\omega_q \to \Omega$, $T_i \to T_e$, and $M \to m$. In particular, the energy loss is especially large (proportional to $\bar{\xi}^{-2}$) if the conditions $|1 - v/u| < \theta \ll \bar{\xi}$ are satisfied. Then

$$P = \frac{(ez\Omega)^2}{v} \frac{m}{\mu} \widetilde{B}_2, \qquad \widetilde{B}_2 = a^2 \int \xi^{-2} q dq. \tag{37}$$

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96