BROKEN ISOTOPIC SYMMETRY OF WEAK INTERACTIONS

É. M. LIPMANOV

Volgograd State Pedagogical Institute

Submitted to JETP editor February 15, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) 47, 360-365 (July, 1964)

A model for weak interactions between elementary particles is formulated, in which isotopic symmetry of the interaction between hadron currents and intermediate bosons is violated only by such free-field-Lagrangian terms which have the form of the "off-diagonal mass" of intermediate bosons. The $\Delta S < 2$ and $\Delta|T| = 1/2$ rules are some of the basic assumptions underlying the model. Violation of universality of the four-fermion interaction constant with change of strangeness is a natural consequence of the model.

1. INTRODUCTION

VARIOUS attempts were made recently in the theory of elementary-particle interaction to formulate higher symmetries (see the bibliography in [1]) which are patently violated at low energies. Gell-Mann and Zachariasen [2] noted that the violated symmetry can apparently have a definite physical meaning only if some limit in which it becomes exact exists. It is indicated that the region of high energies and large momentum transfers may serve as such a limit. This becomes possible if there exist interactions that violate the higher symmetry which are significant at low energies, but become negligible in the high-energy limit. The masses and mass differences of free particles are frequently included in this category of "interactions," particularly on grounds that in the high-energy limit the free-particle mass differences can be neglected compared with the kinetic energies of the particles (see, for example, [2-4]).

It is possible that the free-field Lagrangian of this category includes in addition to the diagonal mass terms also off-diagonal mass-like terms of the type $k^2\varphi_1^*\varphi_2$ or $k\bar{\psi}_1\psi_2$, which are bilinear relative to the operators of the different fields, having identical strictly-conserved quantum numbers. Such "interactions" were considered previously by Zel'dovich 1) and by Deo [5]. They were also used recently by Sacurai [12] to describe the violated unitary symmetry in strong interactions.2)

It will be shown below that the formulation of the broken isotopic symmetry of weak interactions between hadrons 3) and intermediate bosons, based on the representation of the existence of off-diagonal mass-like terms in the Lagrangian of the free intermediate field, makes it possible to satisfy in natural fashion the requirements $\Delta S < 2$ and $\Delta|T| = 1/2$ in non-lepton weak processes, and to explain simultaneously the violation of the universality of the four-fermion interaction constant with change of hadron strangeness.

2. VIOLATION OF SYMMETRY OF INTERACTION BETWEEN A HADRON CURRENT AND INTERMEDIATE BOSONS

Following Lee and Yang [6] we write down the elementary-particle interaction Lagrangian in the form

$$L = L_s + L_\gamma + L_l + L_a, \qquad (1)$$

where L_s denotes the strong-interaction Lagrangian, L_γ—electromagnetic interaction Lagrangian, and L_l and L_a—Lagrangians of the semi-weak interactions between intermediate bosons and leptons or hadrons, respectively. It was assumed in [6] that the invariance of the part of the Lagrangian $L_s + L_a$ is violated because of the "schizon" properties of the intermediate bosons. To formulate such a condition in the sense of "broken symmetry" it is convenient to start from the representation of the "preliminary" isotopic invariant Lagrangian [7]. We shall write out a universal isotopically-invariant "preliminary" Lagrangian of the interaction of hadron currents with the intermediate X-bosons in the form

1)I take the opportunity to thank Academician Ya. B. Zel'dovich for this communication.

2)This was pointed out to us by V. M. Shekhter.

3)The term "hadron" was introduced by L. B. Okun' to denote strongly-interacting particles, in distinction from leptons, photons, and intermediate bosons.

$$L_{\alpha X} = g\,[\mathbf{j}_\alpha^v \mathbf{X}_\alpha^v + 2^{-1/2}(j_\alpha^{S(-)} X_\alpha^{S(-)} + j_\alpha^{S(0)} X_\alpha^{S(0)}) + \text{c.c.}]. \quad (2)$$

Here $\mathbf{X}_\alpha^\nu$ isotopic-vector boson, X_α^S—isopinor boson, and j_α—corresponding hadron currents. The interaction between the lepton current and the X-bosons is written in the form

$$L_{lX} = g2^{-1/2}(l_\alpha^{(+)} X_\alpha^{v(+)} + l_\alpha^{(-)} X_\alpha^{v(-)}), \quad (3)$$

where $l_\alpha^{(\pm)}$—charged lepton current[4]. The masses of all the X bosons are assumed equal:

$$M_v = M_S = M. \quad (4)$$

We consider charged currents first and assume that the free-field Lagrangian contains an off-diagonal mass-like term which violates the isotopic symmetry of the Lagrangian (2) and which has the form

$$L_1^{vS} = k^2 (X_\alpha^{v(+)} X_\alpha^{S(-)} + X_\alpha^{v(-)} X_\alpha^{S(\pm)}). \quad (5)$$

It is easy to verify that the system of boson fields $X_\alpha^{V(\pm)}$ and $X_\alpha^{S(\pm)}$, connected with each other in accordance with (5), is equivalent to the following system of free (non-interacting) fields[5]:

$$W_\alpha^{(\pm)} = (X_\alpha^{v(\pm)} + X_\alpha^{S(\pm)})/\sqrt{2},$$
$$W_\alpha^{\prime(\pm)} = (X_\alpha^{v(\pm)} - X_\alpha^{S(\pm)})/\sqrt{2}, \quad (6)$$

which have displaced masses

$$M_{W}^2 = M^2 + k^2, \qquad M_{W'}^2 = M^2 - k^2. \quad (7)$$

The requirement of stability of the W particles in the absence of interaction imposes the condition

$$k^2 \leqslant M^2. \quad (8)$$

The possibility of $k^2 \cong M^2$ is excluded in this case, since the mass of the intermediate boson cannot be small.

It is also obvious that in view of the universality of the electric charge the total electromagnetic current of the charged X-fields goes over in this case into the corresponding total current for the charged W-fields.

Inverting (6), we get

$$X_\alpha^{v(\pm)} = (W_\alpha^{(\pm)} + W_\alpha^{\prime(\pm)})/\sqrt{2},$$
$$X_\alpha^{S(\pm)} = (W_\alpha^{(\pm)} - W_\alpha^{\prime(\pm)})/\sqrt{2}. \quad (9)$$

Thus, the isotopically non-invariant connection (5) between the fields leads to a mixing of the charged bosons from the different isotopic multiplets in the Lagrangian (2), and to the appearance of "schizons." Substituting (9) in (2) and (3) we obtain the following final Lagrangian for semi-weak interactions of intermediate W-bosons with charged hadron and lepton currents:

$$L_W^{(\pm)} = \frac{1}{2}\, g\,[(j_\alpha^{v(+)} + l_\alpha^{(+)} + j_\alpha^{S(+)}) W_\alpha^{(+)} + (j_\alpha^{v(+)} + l_\alpha^{(+)} - j_\alpha^{S(+)}) W_\alpha^{\prime(+)} + \text{c.c.}]. \quad (10)$$

The interaction (10) has several important properties. It is easy to verify that in four-fermion diagrams without change of hadron strangeness constructive interference takes place between the W and W′ channels of the transition, and the effective intermediate-boson propagator takes the form

$$\frac{-i}{M^2 + k^2 + q^2}\left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{M^2 + k^2}\right) + \frac{-i}{(M^2 - k^2 + q^2)}\left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{M^2 - k^2}\right), \quad (11)$$

where q^2—invariant square of the four-momentum transfer. On the other hand, in four-fermion processes with change of hadron strangeness, destructive interference takes place between the W and W′ channels of the transition, while the effective propagator of the intermediate bosons takes the form

$$\frac{-i}{(M^2 + k^2 + q^2)}\left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{M^2 + k^2}\right) - \frac{-i}{(M^2 - k^2 + q^2)}\left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{M^2 - k^2}\right). \quad (12)$$

It follows therefore that for real four-fermion lepton processes without change in hadron strangeness, at small momentum transfers (β-decay, μ decay, etc) the effective four-fermion coupling constant is equal to

$$G^{(\pm)} \cong \frac{g^2}{2}\frac{M^2 + q^2}{(M^2 + q^2)^2 - k^4} \cong \frac{g^2}{2}\frac{M^2}{M^4 - k^4}. \quad (13)$$

In the case of real lepton processes with change in hadron strangeness (lepton decays of hyperons, etc.) the effective four-fermion coupling "constant is

$$G_1^{(\pm)} \cong -\frac{g^2}{2}\frac{k^2}{(M^2 + q^2)^2 - k^4}. \quad (14)$$

Hence, with account of condition (8), it follows that the four-fermion coupling constant for processes with change of hadron strangeness should be smaller than the corresponding coupling constant for processes without change in this strangeness, $G_1 < G$, in accordance with the experimental results. A satisfactory quantitative agreement with the experimental data can be obtained by putting

[4]The absence of neutral lepton currents is an experimental fact. The possibility of exclusion of neutral lepton currents from the universal weak interaction scheme in which "schizon" properties are possessed also by neutrinos was indicated in[7].

$$k \sim M/2. \quad (15)$$

If the intermediate-boson mass M is equal in order of magnitude to the nucleon mass, then the condition $q \sim M$ can take place in the lepton decays of hyperons, and this, in accordance with (14), results in an additional decrease in the "coupling constant" with increasing hyperon mass.

It must be noted that according to (12) the coupling constant decreases only for four-fermion ("Fermi") processes with change of hadron strangeness, while the coupling constant of the "Yukawa" processes remains the same following a change in the hadron strangeness (production of real intermediate bosons), as in the corresponding processes without change of this strangeness (universality). For example, the cross sections of the possible processes

$$p + Z \to Z + n + W^+, \qquad p + Z \to Z + \Lambda + W^+ \quad (16)$$

should not differ noticeably in the absence of renormalization by strong interactions. On the other hand, in experiments with high energy antineutrinos, for example in the reactions

$$\bar{\nu}_\mu + p \to n + \mu^+, \qquad \bar{\nu}_\mu + p \to \Lambda + \mu^+, \quad (17)$$

it is possible to check experimentally (of course, if the intermediate boson mass is not very large) the deduction, which follows from (13) and (14), that the ratio of the probabilities of the processes (17) at large momentum transfers should be reduced even more than in the case of small momentum transfers (leptonic decays of Λ and n).

As in the case of charged bosons, we can introduce off-diagonal mass-like terms and the free Lagrangian of the neutral X-bosons

$$L_2^{vS} = -k^2 X_\alpha^{v(0)} \cdot 2^{-1/2} (X_\alpha^{S(0)} + X_\alpha^{S(0)*}) = k^2 X_\alpha^{v(0)} X_{1\alpha}^{S(0)}, \quad (18)$$

$$L^{uS} = k^2 X_\alpha^{u(0)} \cdot 2^{-1/2} i^{-1} (X_\alpha^{S(0)} - X_\alpha^{S(0)*}) = k^2 X_\alpha^{u(0)} X_{2\alpha}^{S(0)}, \quad (19)$$

where the constant k^2 has the same meaning as in (5), and it is convenient to consider in place of one complex field $X_\alpha^{S(0)}$ two real fields $X_{1\alpha}^{S(0)}$ and $X_{2\alpha}^{S(0)}$ [8]

$$X_\alpha^{S(0)} = -(X_{1\alpha}^{S(0)} + iX_{2\alpha}^{S(0)}) / \sqrt{2},$$

$$X_\alpha^{S(0)*} = -(X_{1\alpha}^{S(0)} - iX_{2\alpha}^{S(0)}) / \sqrt{2}. \quad (20)$$

In (19) we have also introduced the isotopically scalar boson $X_\alpha^{u(0)}$, which does not enter into the Lagrangian (2).

As before, we obtain the following final Lagrangian of semi-weak interactions of neutral hadron currents with intermediate $W^{(0)}$ bosons

$$L_W^{(0)} = \frac{1}{2} g[(2^{-1/2} j_\alpha^{v(0)} + j_\alpha^{S(0)} + j_\alpha^{S(0)*}) W_{1\alpha}^{(0)} + (2^{-1/2} j_\alpha^{v(0)} - j_\alpha^{S(0)} - j_\alpha^{S(0)*}) W_{2\alpha}^{(0)} + i (j_\alpha^{S(0)} - j_\alpha^{S(0)*}) (W_{3\alpha}^{(0)} - W_{4\alpha}^{(0)})], \quad (21)$$

where

$$W_{1\alpha}^{(0)} = (X_\alpha^{v(0)} + X_{1\alpha}^{S(0)}) / \sqrt{2}, \quad W_{2\alpha}^{(0)} = (X_\alpha^{v(0)} - X_{1\alpha}^{S(0)}) / \sqrt{2},$$

$$W_{3\alpha}^{(0)} = (X_\alpha^{u(0)} + X_{2\alpha}^{S(0)}) / \sqrt{2}, \quad W_{4\alpha}^{(0)} = (X_\alpha^{u(0)} - X_{2\alpha}^{S(0)}) / \sqrt{2}.$$

The masses of the $W^{(0)}$ bosons coincide with the corresponding values for the charged bosons:

$$M_{W_1}^2 = M_{W_3}^2 = M^2 + k^2, \qquad M_{W_2}^2 = M_{W_4}^2 = M^2 - k^2. \quad (22)$$

The Lagrangian (21) corresponds to four-fermion diagrams without change of hadron strangeness, $\Delta S = 0$, and with unity change of hadron strangeness $\Delta S = \pm 1$. An important fact, easily verified, is that transitions with $\Delta S = \pm 2$ are strictly forbidden, in view of the exact destructive interference of all four $W^{(0)}$ channels. It is to obtain this result that the isotopic boson $X_\alpha^{u(0)}$ was introduced in (19). The boson $X_{2\alpha}^{S(0)}$ has negative combined parity. The requirement of PC invariance means that $X_\alpha^{u(0)}$ also has PC = −1. It is not included in the Lagrangian (2), since it is difficult to construct an isotopic scalar hadron current with PC = −1 (see also [6]).

From a comparison of the Lagrangians (21) and (10) we see that the effective boson propagator connecting the neutral isospinor and isovector hadron currents, and the corresponding propagator for the charged currents, differ from each other only by the constant factor −1/2. It follows therefore, as can be readily verified by direct substitution, that the exact rule $\Delta|T| = 1/2$ is satisfied in the g^2 approximation for non-lepton processes with change of hadron strangeness.

3. DISCUSSION

1. In the weak-interaction model considered above, an important role is played by the "preliminary" Lagrangian (2), which has exact isotopic symmetry. What is its physical meaning? In the spirit of the ideas of Gell-Mann and Zachariasen [2] we can hope that, within the limit of high energies and large momentum transfers, the role of the off-diagonal mass terms (5), (18), and (19) in the Lagrangian of the free intermediate fields will become negligibly small, and the weak interactions will be exactly described by a symmetrical "preliminary" Lagrangian. Formally such a premise

would find an exact dynamic expression in the present model if it were possible to neglect in the boson propagators (11) and (12) the terms with $q_\nu q_\mu$ in the limit of large momentum transfers (in the limit—conserved currents). In this case, the effective boson propagator for processes without change in hadron strangeness would be of the order of $1/q^2$, while for processes with change in this strangeness the order would be $1/q^4$ as $q^2 \to \infty$.

Actually, however, the same difficulty which the presence of terms with $q_\nu q_\mu$ in the propagator raises in the field theory of weak interactions with intermediate vector bosons[9] is raised here, too[5)]. On the other hand, it is of interest in itself to investigate the possibility of experimentally verifying the hypothesis of isotopic symmetry of weak hadron interactions in the high-energy limit and in the limit of large momentum transfers. Such a possibility can arise at presently feasible particle energies only if the intermediate boson mass is actually not very large compared with the nucleon mass[10].

2. The rules $\Delta S < 2$ and $\Delta|T| = 1/2$ are among the main premises on which the present model is constructed. Indeed, the condition that the violation of the isotopic symmetry be governed by these rules is the reason why isotopic invariance of weak interactions in some limit calls for the existence of a reduced set of isotopic intermediate boson multiplets. In this model the total number of isotopic states of intermediate fields is eight. This apparently is the minimum number of intermediate bosons compatible with the isotopic symmetry in some limit and with the rules $\Delta S < 2$ and $\Delta|T| = 1/2$.

3. An independent requirement imposed on the "preliminary" Lagrangian (2) is universality of the coupling constant. Indeed, the rules $\Delta S < 2$ and $\Delta|T| = 1/2$ would be satisfied as before if the isovector and isospinor currents had different coupling constants with the intermediate bosons. The requirements of universality of the coupling constant g and equality of the masses (4) signify essentially that the "preliminary" Lagrangian has a higher symmetry than isotopic. For the Lagrangian (2) such a symmetry can be G_2: the isovector X_α^V in conjunction with the isospinor X_α^S form a seven-dimensional representation, and the isoscaler $X_\alpha^{u(0)}$—a one dimensional representation. However, the concrete expression (2) for the "preliminary" Lagrangian cannot be regarded as final.

It is possible that some additional physical requirements (for example, the formulation of an analogy between weak and electromagnetic interactions[7]) will lead to the necessity of including in the "preliminary" Lagrangian (2) a term for the interaction between the isoscalar hadron current and an isoscalar boson $X_\alpha^{S(0)}$ possessing combined-parity. In this case a higher symmetry for the "preliminary" Lagrangian can be the Gell-Mann and Neeman[11] unitary symmetry $SU_3(8)$. Then X_α^V, X_α^S and $X_\alpha^{S(0)}$ will form a unitary octet, and $X_\alpha^{u(0)}$ a unitary singlet. All the deductions obtained above remain in force in this more general case (see[7]).

I am sincerely grateful to L. B. Okun' for a discussion and valuable advice, and to V. M. Shekhter and A. A. Komar for interest in the work.

[5)]From the mathematical point of view it is interesting to note that such a difficulty would not arise in the case of scalar intermediate bosons.

[1] V. B. Mandel'tsveĭg, Bibliografiya rabot po vysshim simmetriyam élementarnykh chastits (Bibliography on Higher Symmetries of Elementary Particles), Preprint, Inst. Theoret. Exptl. Phys., 1963.

[2] M. Gell-Mann and F. Zachariasen, Phys. Rev. **123**, 1065 (1961).

[3] Ya. B. Zel'dovich, JETP **40**, 637 (1961) and **41**, 912 (1961), Soviet Phys. JETP **13**, 444 (1961) and **14**, 651 (1962).

[4] I. Yu. Kobzarev and L. B. Okun', JETP **42**, 1400 (1962), Soviet Phys. JETP **15**, 970 (1962).

[5] B. B. Deo, Nucl. Phys. **28**, 135 (1961).

[6] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

[7] É. M. Lipmanov, JETP **43**, 893 (1962), Soviet Phys. JETP **16**, 634 (1963).

[8] G. Wentzel, Quantum Theory of Fields, Interscience, N. Y., 1949.

[9] A. Pais, Weak Interactions at High Energies. Theor. Phys. Lect., Vienna, (1963).

[10] CERN Neutrino Experiment, Preprint (1963).

[11] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962). Y. Neeman, Nucl. Phys. **26**, 222 (1961).

[12] J. J. Sacurai, Phys. Rev. **132**, 434 (1963).

Translated by J. G. Adashko