ANGULAR AND ENERGY DISTRIBUTIONS OF FAST MUONS PENETRATING THE EARTH FROM THE ATMOSPHERE

L. G. ZASTAVENKO and A. CHILOK

Joint Institute for Nuclear Research

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The angular and energy distributions of mesons from the atmosphere which have penetrated large depths of the earth (4×10^4 g/cm^2) are calculated for angles in the range $\cos\theta < -0.4$ ($\cos\theta = 1$ corresponds to the downward direction) and energies $k \geq 0.75$ BeV. The calculation is more exact than that performed in the single scattering approximation, as a result of use of the stationary phase method for solving the kinetic equation for μ-meson scattering.

1. INTRODUCTION

MU MESONS which have penetrated deep into the earth from the atmosphere and which have been scattered at large angles are a troublesome background in the experiment proposed by Markov and Zheleznykh for the detection of cosmic-ray neutrinos at great depths in the earth[1].

The aim of the present work is the evaluation of this background for angles $\cos\theta < -0.4$ and energies $k \geq 0.75$ BeV at a depth of 4×10^4 g/cm^2. The angular and energy distributions of high-energy μ-mesons arriving at the earth's surface from above can be written in the form[2]

$$F_0(>k,\, \theta) = N_0 k^{-1.5}(1 + k\cos\theta_e / k_\pi)^{-1}, \qquad (1)$$

where

$$k_\pi = 100\ \text{BeV}, \qquad \cos\theta_e = \max\{\cos\theta,\ 1/8\},$$

$$\theta < \pi/2, \qquad N_0 = 0.033\ \text{cm}^{-2}\,\text{sec}^{-1}\,\text{sr}^{-1}\cdot\text{BeV}^{1.5},$$

$$f_0(k,\ \cos\theta) \equiv \frac{dF_0}{dk} = N_0 k^{-2.5}\frac{1.5 + 2.5k\cos\theta_e/k_\pi}{[1 + k\cos\theta_e/k_\pi]^2} \qquad (2)$$

(the energy k is measured in BeV). We will assume that, on passing through a layer of matter 1 g/cm^2 thick, a μ meson loses an energy

$$q = 2\ \text{MeV}\cdot\text{cm}^2/\text{g} \qquad (3)$$

which is valid in the region $k < 1000$ BeV with which we are concerned.

Without taking into account scattering, the μ-meson distribution at a depth x g/cm^2 would be

$$F_0(x,\, >k,\, \theta) = F_0(>k + qx/\cos\theta,\, \theta). \qquad (4)$$

This formula gives the following downward flux at depth x:

$$2\pi \int F_0(x,\, >k,\, \theta)\cos\theta\, d\cos\theta \cong N_0 (qx)^{-1.5}.$$

For a depth of 4×10^4 g/cm^2 the flux amounts to 4.5×10^{-5} particles per cm^2 per second, i.e., 1.2×10^8 particles per 100 meter^2 per month, which is 9 orders of magnitude greater than the expected number of μ mesons from neutrinos[1].

μ-meson scattering in the earth leads to a distribution function different from (4) and which is not zero for $\theta > \pi/2$. However, since this function, as we would naturally expect, falls off sharply with increasing θ, at a given energy k there is a cone around the upward direction, $\cos\theta < \alpha(k)$, such that in it the density of μ-mesons which have arrived from above, $F(x, > k, \theta)$, is less than the density of μ-mesons from neutrinos, $F_\nu(x, > k, \theta)$.

It is our purpose to evaluate the function $F(x, > k, \theta)$; this evaluation makes it possible to determine the function $\alpha(k)$ which is essential for the experiment proposed by Markov and Zheleznykh. It is extremely simple to calculate $F(x, > k, \theta)$ in approximate form, considering only single scattering of the μ mesons. It is reasonable to expect (see the comment at the end of Sec. 2) that this approximation will provide a solution of our problem with sufficient accuracy. Moreover, the large-angle μ-meson scattering cross section which we have used in the calculation is a rough approximation and may itself introduce a larger error than the approximation of single scattering.

We have also made a more exact calculation considering multiple scattering, in order to reduce the uncertainty of our result. Our calculation was performed by a method proposed by us earlier[3] for the solution of kinetic equations of the type encountered in multiple scattering theory. The results of the calculation, as expected, are close to those obtained with the single scattering approximation, but the differences are by no means insignificant. Let us consider, for example, the ratio φ/φ_1, where $\varphi(x, k, \tau)$ is the flux of μ mesons with energy k in the backward cone $\cos\theta < \tau$ (see Sec. 4), and φ_1 is the same quantity calculated in the single scattering approximation. For a depth of 4×10^4 g/cm^2 and an energy k = 0.75 BeV we obtain $\varphi/\varphi_1 = 4, 3, 2$, and $^3/_2$, respectively, for $\tau = -0.4, -0.5, -0.6$, and -0.7.

2. MUON SCATTERING

Scattering of fast μ mesons by a nucleus occurs in different ways, depending on the momentum transfer to the nucleus; for small momentum transfer,

$$\sigma(k, \cos\theta) = \sigma_1(k, \cos\theta)$$

$$= \begin{cases} 0, & \theta < \theta_{min} \\ r_0^2 (\mu/k)^2 Z^2 (1-\cos\theta)^{-2}, & \theta_{min} < \theta < \theta_{max} \end{cases};$$

$$\theta_{min} = A^{1/3}(kr_B)^{-1}, \qquad \theta_{max} = A^{-1/3}(kr_N)^{-1},$$

$$r_B = 10^{-8}\text{ cm}, \qquad r_N = 1.3 \cdot 10^{-13}\text{ cm}, \tag{5}$$

A is the mass number, Z the atomic number, μ is the rest mass of the μ-meson, and $r_0 = e^2/\mu$.

At large momentum transfer to the nucleus, disintegration occurs. For a simplified description of the complex event that takes place here, we assume that the scattering in this case occurs separately on each of the protons in the nucleus as on a free proton, which gives, on conversion to a whole nucleus[5],

$$\delta(k, \cos\theta) = \sigma_2(k, \cos\theta) = r_0^2 \left(\frac{\mu}{k}\right)^2 \frac{ZP}{(1-\cos\theta)^2}, \quad \theta > \theta_{max},$$

$$P = \left\{\cos^2\frac{\theta}{2} + 2\frac{k}{M}\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2} + \left(\frac{k}{M}\right)^2 \right.$$

$$\left. \times\left[15\sin^4\frac{\theta}{2} + 3\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\right]\right\}\left(1 + 2\frac{k}{M}\sin^2\frac{\theta}{2}\right)^{-1}. \tag{6}$$

Here the μ-meson energies k and p, before and after scattering, are connected by the relation $1/p = 1/k + 1 - \cos\theta$, in which we neglect the rest mass of the μ meson (since we are interested in energies greater than 0.75 BeV) and we take the nucleon mass to be 1 BeV.

In order to obtain the cross section expressed in cm^2/g, formulas (5) and (6) must be multiplied by n_0, the number of nuclei in one gram of material:

$$n_0 \approx 1/2MZ = 3.1 \cdot 10^{23}/Z. \tag{7}$$

On the basis of data on the composition of the earth we take Z = 12. From (5) we find the mean-square deflection angle of the μ-meson on slowing down in the earth from energy k_1 to energy k_2:

$$\bar{\theta}^2(k_1, k_2) = 8.9 \cdot 10^{-3}\ (1/k_2 - 1/k_1).$$

Thus, in our situation, multiple scattering gives a deflection of ~ 0.1 radian, so that we expect the function $F(x, > k, \theta)$ in the region of interest $\theta - \pi/2 \sim 1$ to be determined principally by single scattering alone, with some correction due to multiple scattering. Setting P = 1 in (6), we obtain an estimate of the average number of collisions with a deflection angle $> \theta > \theta_{max}$, which a μ meson undergoes in slowing down from an energy $k_1 = \infty$ to an energy k: $N(k) \cong 4 \times 10^{-5} \times (k\theta^2)^{-1}$; for $\theta = 0.1$ and k = 1 BeV this number is 4×10^{-3}. Thus, as a rule a μ meson is slowed down to 1 BeV without undergoing a large-angle scattering.

3. BASIC FORMULAS

The problem reduces to the solution of the kinetic equation

$$\mathbf{n}\nabla_x f(\mathbf{x}, \mathbf{n}, k) = -n_0\sigma(k) f$$

$$+ n_0 \int d\Omega(\mathbf{n}')\, \sigma_1(k, \mathbf{nn}') f(\mathbf{x}, \mathbf{n}', k)$$

$$+ n_0 \int dp\, d\Omega(\mathbf{n}')\, \sigma_2(p, \mathbf{nn}') f(\mathbf{x}, \mathbf{n}', p)$$

$$\times \delta[k - \psi(\mathbf{nn}', p)] + q\frac{\partial f}{\partial k};$$

$$\sigma(k) = \int d\Omega(\mathbf{n}')[\sigma_1(k, \mathbf{nn}') + \sigma_2(k, \mathbf{nn}')],$$

$$\psi(\mathbf{nn}', p) = (1/p + 1 - \mathbf{nn}')^{-1}; \tag{8}$$

the functions $\sigma_{1,2}(k, \mathbf{n}\cdot\mathbf{n}')$ are determined by formulas (5) and (6), and n_0 and q are obtained from formulas (7) and (3).

Equation (8) must be solved in the region $\mathbf{x}\cdot\mathbf{n}_0 > 0$ for the condition that f coincides with f_0 [see Eq. (2)] in the plane $\mathbf{x}\cdot\mathbf{n}_0 = 0$ for downward directions of $\mathbf{n}$: $\mathbf{n}\cdot\mathbf{n}_0 > 0$.

Equation (8) is extremely complex. We have, however, succeeded in constructing a reasonable approximation to the solution $f(\mathbf{x}, \mathbf{n}, k)$ of this equation in the interesting region of angles. This is the function

$$\tilde{f}_{\mathbf{x}}(\mathbf{n}, k) = \int d\Omega(\mathbf{n}')\, \Phi(k, \mathbf{nn}')\, \eta(\mathbf{n}'\mathbf{n}_0) f_0\left(q\frac{\mathbf{xn}_0}{\mathbf{n}'\mathbf{n}_0}, \mathbf{n}'\mathbf{n}_0\right). \tag{9}$$

In formula (9) $\Phi(k, \mathbf{nn}')$ is the solution of Eq. (8) independent of x and satisfying the condition

$$\Phi(k, \mathbf{nn}') \to \delta(\mathbf{n} - \mathbf{n}') \quad \text{for } k \to \infty;$$

$$\eta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}.$$

In writing down formula (9) we proceed from the following starting points: 1) particles which at the earth's surface have an energy appreciably less than $k(\xi, m) = q(\xi \cdot m)/(m \cdot n_0)$ do not reach a depth ξ; 2) particles with energy at the earth's surface appreciably greater than $k(\xi, m)$ make a small contribution to the function $f(\mathbf{x}, \mathbf{n}, k)$ in the interesting region $\mathbf{x} = \xi$, $\mathbf{n} \cdot \mathbf{n}_0 \sim -1$ (they leave downward without being scattered).

Thus, only those particles are important in our case which at the surface of the earth have, in a given direction m, an energy within the limits $-\Delta_1 + k(\xi, m)$ to $k(\xi, m) + \Delta_2$. For a depth $\xi n_0 \approx 4 \times 10^4$ g/cm^2, obviously $\Delta_1 \ll k(\xi, m)$. We assume also that $\Delta_2 \ll k(\xi, m)$ (this is true if only single scattering is considered). Then the function f_0 in the "significant" interval (we will denote it by $s(\xi, m)$) can be considered as independent of k. It is clear that replacing the original distribution $f_0(k, \cos\theta)$ by any other distribution $\tilde{f}_0$ which is close to f_0 in the interval $s(\xi, m)$ does not seriously change $f(\mathbf{x}, \mathbf{n}, k)$ for $\mathbf{x} = \xi$, $\mathbf{n} \cdot \mathbf{n}_0 \sim -1$.

The approximate solution (9) is obtained for a certain definite choice of the function $\tilde{f}_0$. Specifically, formula (9) is based on a function $\tilde{f}_0$ such that the corresponding solution of Eq. (8), $\tilde{f}_\xi(\mathbf{x}, \mathbf{n}, k)$, does not depend on the coordinate $\mathbf{x}$. (This superfluous argument is dropped in Eq. (9).) Accordingly, $\tilde{f}_\xi(\mathbf{x}, \mathbf{n}, k) \approx f(\xi, \mathbf{n}, k)$ for $\mathbf{x} = \xi$ (ξ corresponds to a great depth $\sim 4 \times 10^4$ g/cm^2 and $\mathbf{n} \cdot \mathbf{n}_0 \sim -1$).

Of course, the justification given for formula (9) contains the implicit assumption that the order of magnitude of $f(\mathbf{x}, \mathbf{n}, k)$ is determined by single scattering.

Thus, the meaning of formula (9) is that, for a given depth in the earth ξ, a distribution function $\tilde{f}_\xi(\mathbf{n}, k)$ is constructed: a solution of Eq. (8) independent of the coordinate, the flux corresponding to this distribution (more accurately, the part of it existing at depth ξ) being close to f_0 of Eq. (2).

Table I. The function: $-\log_{10}\Phi(k, \cos\theta)$

$1+\cos\theta$	1/k: 4/3	1	1/2	1/3	1/5
1.9	+1,33	1.50	2.01	2.48	3.32
1.8	+2.07	2.28	3.37	4.21	5.50
1.7	+2.63	3.06	4,40	6.10	8.00
1.6	+3.13	3.72	4.67	7.90	11.00
1.5	+3.62	4.29	6.16	10.00	14.00
1.4	+4.07	4.81	7.67	12.00	17.00
1.3	+4.48	5.17	9.40	14,00	20
1.2	4.89	5.51	11,40	16.00	
1,1	5.29	6.78	13.50	18	
1,0	5.50	7.20	15.50	20	
0,9	5.70	8.70	17.50		
0.8	6.10	10.20	19.50		
0.7	7,10	11.70	20		
0,6	8.20	13.20			
0,5	9.30	14.70			

We remark further that only μ mesons with an energy $k \gg 100$ BeV at the earth's surface penetrate to a depth $x \gg 4 \times 10^4$ g/cm^2; at this energy radiation losses become important, and the quantity q [see Eq. (3)] is a function of energy.

With the aid of a table of the function $\Phi(k, \cos\theta)$, the μ-meson distribution at such a depth can be calculated in two steps: 1) we determine the distribution at a depth $x - 4 \times 10^4$ g/cm^2 using a formula similar to (4) but with account of the energy dependence of q; 2) taking this distribution as the parent distribution, we then find the distribution at a depth x from a formula similar to (9). Conversion to Z values different from the value Z = 12 used by us can be carried out by assuming a linear dependence of Φ on Z.

4. RESULTS OF THE CALCULATION

Calculation of the function $\Phi(k, \cos\theta)$ is most complicated; it was done by the method of steepest descents, which has been described by us[3]. Table I gives values of the function $-\log_{10}\Phi(k, \cos\theta)$.

Table II lists the function $K(k, \tau)$. The important quantity

$$\varphi(x, k, \tau) = -\int_k^\infty dp \int_{\mathbf{n}'\mathbf{n}_0<\tau} d\Omega(\mathbf{n}')\, \mathbf{n}'\mathbf{n}_0 f(x, \mathbf{n}', p)$$

the flux of μ mesons with energy greater than k BeV in the backward cone $\mathbf{n}_0 \cdot \mathbf{n} < \tau$, per cm^2 per second at a depth $(x/80) \times 4 \times 10^4$ g/cm^2, is expressed by K in the following way:

$$\varphi(x, k, \tau) \cong 4\pi N_0 x^{-2.5} K(k, \tau).$$

Here N_0 is the constant entering into formula (1). For our depth of 4×10^4 g/cm^2 the value of x is 80 and $4\pi N_0 x^{-2.5} = 7.2 \times 10^{-6}$ cm^{-2} sec^{-1}.

For example, for k = 1 BeV and $\tau = -0.7$ we obtain $\varphi = 3 \times 10^{-14}$ μ mesons/cm^2 sec, which corresponds to 8×10^{-2} μ mesons/month/100 meter^2.

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Table II. The function $K(k, \tau)$

τ	1/k: 4/3	1	1/2	1/3
—0.4	8.29^{-7}	$3{,}65^{-7}$	$2{,}94^{-8}$	4.61^{-9}
—0,5	2.52^{-7}	$9{,}47^{-8}$	4.28^{-9}	3.20^{-10}
—0,6	$7{,}13^{-8}$	2.21^{-8}	4.96^{-10}	1.16^{-11}
—0.7	1.78^{-8}	$4{,}24^{-9}$	3.52^{-11}	$1{,}71^{-13}$
—0,8	3.49^{-9}	5.39^{-10}	$9{,}00^{-13}$	5.65^{-16}
—0.9	2.90^{-10}	1.16^{-11}	3.23^{-16}	

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