ON THE EXISTENCE OF A PSEUDOSCALAR MESON OF ZERO MASS

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Submitted to JETP editor August 8, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **47**, 116-125 (July, 1964)

Some consequences of the existence of a pseudoscalar meson of zero mass are considered: corrections to the anomalous magnetic moment of the lepton, to the Lamb shift of the energy levels of the hydrogen atom and to the shift of the energy levels of the ground states of para- and orthopositronium; the effective static fermion interaction potential at large distances; emission of long wavelength mesons. Some possibilities of observing this particle are discussed.

1. INTRODUCTION

In nonlinear field theory[1] the equation for the fermion fields is invariant under the transformation (γ_5-invariance)

$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}. \tag{1}$$

This expresses the fact that the bare particles (fermions) have no mass, while the fact that real fermions have a mass is due to interaction. An analogous situation also exists in the theory of superconductivity.

Basing themselves on the analogy with the theory of superconductivity several authors[2,3] have investigated the properties of the solutions of the nonlinear field equation and have attempted to explain the appearance of a nonzero mass. It is evident that the existence of a mass in the case of real fermions denotes violation of γ_5-invariance. It turns out that the solution of the field equation does not have several of the symmetry properties which are possessed by the equation itself.[4,5] One of these violated symmetries is the γ_5-invariance. Moreover, at the same time that the fermions have a nonvanishing mass there must exist a pseudoscalar meson with mass equal to zero[2,3]. We shall refer to this meson as the ξ-meson. In the paper by Nambu and Jona-Lasinio[3] it was also shown that this meson corresponds to a solution of the approximate Bethe-Salpeter equation.

Apart from the predictions made in [2,3], an investigation of the consequences of the existence of a ξ-meson is of independent interest. In nature there exist spinor and vector particles both with and without finite mass. There also exist pseudoscalar mesons of finite mass. The question naturally arises whether there exists a pseudoscalar meson of zero mass.

In this paper we consider certain consequences of the existence of the ξ meson. From experimental data we shall obtain estimates of the upper limit for the constants of interaction of this meson with fermions. We shall also discuss some possibilities of observing this particle. We first consider the case of interaction with leptons, and then the case of interaction with nucleons.

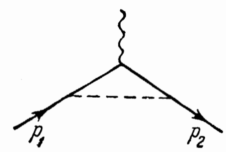

FIG. 1

2. CORRECTION TO THE ELECTROMAGNETIC VERTEX PART AND TO THE ANOMALOUS MAGNETIC MOMENT OF THE LEPTON

In lowest order the correction to the electromagnetic vertex part of the lepton due to its interaction with a ξ meson is determined by the diagram shown in Fig. 1. We denote by p_1 and p_2 the four-momenta of leptons in the initial and final states respectively, and we set $q = p_2 - p_1$. The matrix element corresponding to the diagram of Fig. 1, is equal to

$$\lambda_\mu = \frac{if^2}{(2\pi)^4}\int d^4k\gamma_5 \frac{i\hat{p}_2 - m - i\hat{k}}{(p_2-k)^2+m^2} \times \gamma_\mu \frac{i\hat{p}_1 - m - i\hat{k}}{(p_1-k)^2+m^2}\gamma_5 \frac{1}{k^2}, \tag{2}$$

where f is the constant for the interaction between the ξ meson and a lepton, m is the lepton mass.

After elementary transformations we obtain

$$\lambda_\mu = \frac{if^2}{(2\pi)^4}\gamma_\alpha\gamma_\mu\gamma_\beta J_{\alpha\beta}^1, \tag{3}$$

$$J_{\alpha\beta}^1 = \int d^4k \frac{k_\alpha k_\beta}{(k^2-2kp_2)(k^2-2kp_1)k^2}. \tag{4}$$

The expression for the integral (4) was obtained by Feynman and is given, for example, in the book of Akhiezer and Berestetskiĭ [6]. Substituting this expression into (3) and carrying out the transformation we obtain the vertex part λ_μ after charge renormalization:

$$\lambda_\mu^R = -\frac{i\beta}{8\pi m}(\gamma_\mu\hat{q} - \hat{q}\gamma_\mu)\frac{u}{\operatorname{sh} 2u},$$

$$\operatorname{sh}^2 u = \frac{q^2}{4m^2}, \qquad \beta = \frac{f^2}{4\pi}. \qquad (5)^*$$

The result obtained above shows that in the lowest order the interaction with the ξ meson does not change the electric form factor of the lepton, but gives a contribution to the magnetic form factor. In particular, the correction to the anomalous magnetic moment of the lepton is equal to

$$\mu_a^\xi = -\beta/4\pi. \tag{6}$$

We recall that in the lowest order the anomalous magnetic moment of the lepton due to the electromagnetic interaction is equal to

$$\mu_a^e = \alpha/2\pi.$$

At the present time the experimental value of the anomalous magnetic moment of the electron agrees with the theoretical calculation up to a term of the order of 2×10^{-5} [7]. The correction (6) cannot exceed this limit. Therefore, for the electron

$$\beta_e \lesssim 4\pi\cdot2\cdot10^{-5} \approx 2\cdot10^{-4}. \tag{7}$$

The experimental value of the anomalous magnetic moment of the μ-meson also agrees with experiment up to a term of the order of 5×10^{-6} [8]. Therefore for the μ-meson we have the estimate

$$\beta_\mu \lesssim 5\cdot10^{-5}. \tag{8}$$

3. CORRECTION TO THE LAMB SHIFT OF THE ENERGY LEVELS OF THE HYDROGEN ATOM

For the determination of the correction to the Lamb shift we first calculate the correction to the interaction potential between the electron and the static electric field. We denote this correction by δU. It can be easily shown that in the lowest order only the diagram with the vertex part gives a contribution. Utilizing the results of the preceding section we obtain

$$\delta U = -\frac{i\beta_e e}{8\pi m}\gamma_4\boldsymbol{\alpha}\mathbf{E}, \tag{9}$$

where $\mathbf{E}$ is the electric field intensity. From this expression for δU one can obtain the correction to the Lamb shift of the $2S_{1/2}$ and $2P_{1/2}$ energy levels of the hydrogen atom due to the interaction with the ξ meson:

$$\delta E_\xi = -\beta\alpha^4 m/24\pi. \tag{10}$$

We recall that in the lowest order the magnitude of the shift under consideration as a result of the electromagnetic interaction is equal to [9]

$$\delta E_e = \frac{\alpha^5 m}{6\pi}\left[\ln\frac{m}{2\varepsilon_0} + \frac{23}{24} - \frac{1}{5}\right], \quad \ln\frac{m}{2\varepsilon_0} = 7.6878. \tag{11}$$

The experimental value of the shift and the theoretical calculation up to the highest order agree up to a term of the order of $10^{-4}\delta E_e$ [7]. From this value and from expressions (10) and (11) we obtain an estimate of the upper limit for the constant:

$$\beta \lesssim 3\cdot10^{-3}\alpha \approx 3\cdot10^{-5}. \tag{12}$$

4. HYPERFINE STRUCTURE OF POSITRONIUM

The interaction between an electron and a ξ-meson can also give a correction to the shift of the energy levels of the ground states of para- and orthopositronium. In order to determine this correction we calculate first of all the effective potential for the interaction between the electron and the positron due to the exchange of a ξ meson. In order to do this we first of all consider the matrix element for the scattering of an electron by a positron. We denote the four-momenta

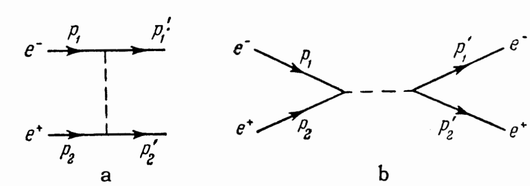

FIG. 2

of these particles before and after scattering respectively by p_1, p_1' (electron) and p_2, p_2' (positron). The scattering matrix element up to second order in f is equal to (diagrams in Fig. 2)

$$\langle f|S|i\rangle = -i(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')T, \tag{13}$$

$$T = f^2[\bar{u}(p_1')\gamma_5 v(-p_2')\bar{v}(-p_2)\gamma_5 u(p_1)(p_1+p_2)^{-2} - \bar{u}(p_1')\gamma_5 u(p_1)\bar{v}(-p_2)\gamma_5 v(-p_2')(p_1-p_1')^{-2}]. \tag{14}$$

The last term in (14) can also be written in the

*sh = sinh.

form

$$f^2\bar{u}(p_1')\gamma_5 u(p_1)\,\bar{u}(p_2')\gamma_5 u(p_2). \qquad (15)$$

In our case the electron and the positron can be considered as nonrelativistic particles. We denote by $\chi_1, \chi_2, \chi_1', \chi_2'$, the two-component Pauli spinors. In this case we have in the center of mass system ($\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$; $\mathbf{p}_1' = -\mathbf{p}_2' = \mathbf{p}'$)

$$T = \chi_2'\chi_1'\tau(\mathbf{p}', \mathbf{p})\chi_2\chi_1; \qquad (16)$$

$$\tau(\mathbf{p}', \mathbf{p}) = -\frac{f^2}{4m^2}(\sigma_1\mathbf{q})(\sigma_2\mathbf{q})\frac{1}{\mathbf{q}^2} + \frac{f^2}{4m^2}(2 - S^2), \qquad (17)$$

$$\mathbf{q} = \mathbf{p} - \mathbf{p}', \qquad S = \tfrac{1}{2}(\sigma_1 + \sigma_2).$$

In the lowest order the effective interaction potential is equal to $\tau(\mathbf{p}', \mathbf{p})$. On going over into the coordinate representation we obtain

$$U(r) = \frac{\beta}{4m^2}[3(\sigma_1\nu)(\sigma_2\nu) - \sigma_1\sigma_2]\frac{1}{r^3} + \frac{\beta\pi}{m^2}(2 - S^2)\,\delta(\mathbf{r}),$$

$$\nu = \frac{\mathbf{r}}{r}. \qquad (18)$$

In order to determine the correction to the shift of the energy levels of the ground states of para- and orthopositronium it is sufficient to evaluate the average values of the potential (18) in these states. We note that the average value of the first term is equal to zero in all the S states. The second term differs from zero only for singlet states. For the ground state of para-positronium we have

$$\bar{U} = \beta\alpha^3 m / 4.$$

Thus, the correction to the shift of the energy levels of the ground states of para- and orthopositronium is equal to

$$\delta E_\xi = \beta_e\alpha^3 m / 4. \qquad (19)$$

In the lowest order in the electromagnetic interaction the value of the shift amounts to [6]

$$\delta E_e = {}^7/_{12}\alpha^4 m. \qquad (20)$$

This quantity was also calculated in the highest order[10]. It agrees with experiment up to terms of order $2 \times 10^{-4}\delta E_e$ [11]. Thus,

$$\delta E_\xi / \delta E_e \approx \beta_e / 2\alpha \lesssim 2\cdot 10^{-4}, \qquad \beta_e \lesssim 3\cdot 10^{-6}. \qquad (21)$$

5. THE EFFECTIVE POTENTIAL OF THE INTERACTION BETWEEN FERMIONS AT LARGE DISTANCES

We consider first the interaction between two nonrelativistic fermions separated by a large distance. In the lowest order the interaction potential due to the exchange of a ξ meson is determined by the first term on the right hand side of (18). We note certain peculiarities of this potential. Firstly, the interaction potential between two particles separated by a large distance is equal to the interaction potential between a particle and an antiparticle, in contrast to the case of electromagnetic interaction. This is due to the fact that the pseudoscalar quantity $\bar{\psi}\gamma_5\psi$ does not change sign under charge conjugation, while the vector quantity $\bar{\psi}\gamma_\mu\psi$ does change sign. Secondly, this potential depends on the spin of the particles even in the lowest order, and falls off with increasing distance like $1/r^3$, in contrast to the case of interaction resulting from the exchange of a vector or scalar particle of zero mass.

If there exist two macroscopic bodies containing electrons and nucleons, then the correction to the potential energy due to the interaction with a ξ meson is equal to the sum of the potentials (18), with the summation taken over all pairs of particles. It is evident that this sum is equal to zero independently of the masses of the particles and of the interaction constants. This is due to the fact that the potential for the interaction between two particles depends on the components of the spin of each particle. In order to determine the correction to the potential energy of a system of two macroscopic bodies it is necessary to evaluate the potential for the interaction between two particles in fourth order in f, since only in this order can there appear a term independent of the spin.

We note that in higher orders the interaction potential does not coincide with the scattering amplitude, but if we know the scattering amplitude we can then reconstruct the potential. For our purposes it is sufficient to consider the part of the potential $U_0(r)$ which does not depend on the spin. It can be easily shown that only fourth order diagrams in Fig. 3 give a contribution to $U_0(r)$. On calculating the scattering amplitude and reconstructing the potential by the method used in references [12,14] we obtain

$$U_0(r) = \beta^2 / 2\pi m^2 r^3. \qquad (22)$$

If the value (21) is taken for the constant β_e the potential (22) for electrons is less than the gravitational potential for two electrons at macroscopic distances. Later we shall see that for nu-

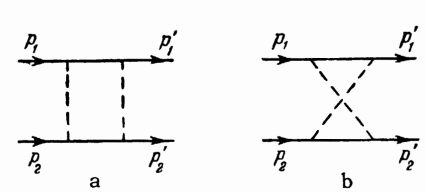

FIG. 3

cleons $\beta_N \lesssim 10^{-2}$. From these estimates it follows that corrections to the potential energy of interaction of macroscopic bodies due to the interaction with a ξ meson is negligibly small in comparison with the gravitational potential.

6. EMISSION OF LONG WAVELENGTH MESONS

As is well known, accelerated charged particles radiate electromagnetic waves-photons of long wavelengths. We denote by $d\mathscr{E}$ the energy of the radiation of frequencies lying between ω and $\omega + d\omega$. For the existence of electromagnetic waves it is necessary that $d\mathscr{E}/d\omega$ should tend to a finite limit, and not to zero as $\omega \to 0$. From this it follows that the probability of emission of a photon tends to infinity. This is related to the so called infrared catastrophe in electrodynamics.

We consider the emission of ξ mesons of long wavelengths. For the sake of definiteness we investigate the process of bremsstrahlung emission of ξ-mesons when an electron is scattered by a Coulomb field. In lowest order the process is described by the diagrams shown in Fig. 4. The matrix element of the process under consideration is equal to

$$M = 2\pi\delta(E_1 - E_2 - \omega)\frac{Ze^2f}{\sqrt{2\omega}}\frac{1}{(p_1 - p_2 - q)^2}\bar{u}(p_2)$$
$$\times\left[\gamma_5\frac{i(\hat{p}_2 + \hat{q}) - m}{2p_2q}\gamma_4 + \gamma_4\frac{i(\hat{p}_1 - \hat{q}) - m}{2p_1q}\gamma_5\right]u(p_1), \quad (23)$$

where p_1 and p_2 are the four-momenta of the electron in the initial and final states, q is the four-momentum of the meson, $E_1 = p_1^0$, $E_2 = p_2^0$, and $\omega = q^0$.

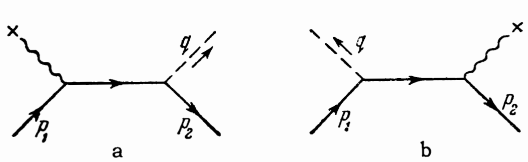

FIG. 4

The expression for the cross section is quite awkward. However, for our purposes it is sufficient to consider the case of nonrelativistic particles ($|p_1| \ll m$). In this case the cross section is equal to

$$d\sigma = \frac{Z^2\alpha^2\beta}{4\pi^2 m^2}\frac{p_2}{p_1}\frac{[(\mathbf{p}_1 - \mathbf{p}_2)\mathbf{n}]^2}{[(\mathbf{p}_1 - \mathbf{p}_2)^2]^2}\omega\, d\omega\, d\Omega_{\mathbf{q}}\, d\Omega_{\mathbf{p}_2}, \quad n = \frac{\mathbf{q}}{q}. \quad (24)$$

We denote by $d\sigma_S$ the cross section fot the elastic scattering of an electron by a Coulomb field:

$$d\sigma_s = \frac{4m^2Z^2\alpha^2}{[(\mathbf{p}_1 - \mathbf{p}_2)^2]^2}d\Omega_{\mathbf{p}_2}. \quad (25)$$

It follows from (24) and (25) that the cross section for bremsstrahlung is equal to the product of the elastic scattering cross section $d\sigma_S$ and the probability of emission of a ξ meson dW_ξ, with

$$dW_\xi = \frac{\beta}{(4\pi)^2}[(\mathbf{v}_1 - \mathbf{v}_2)\mathbf{n}]^2\frac{\omega\, d\omega}{m^2}d\Omega_{\mathbf{q}}. \quad (26)$$

We recall that the cross section for the bremsstrahlung emission of a photon is also equal in the nonrelativistic limit to the product of the elastic scattering cross section $d\sigma_S$ and the probability of emission of a photon dW_γ, with the expression for the probability

$$dW_\gamma = \frac{\alpha}{(2\pi)^2}[(\mathbf{v}_1 - \mathbf{v}_2)\mathbf{n}]^2\frac{d\omega}{\omega}d\Omega_{\mathbf{q}} \quad (27)$$

being in correspondence with classical radiation theory[6]. As has been noted, $dW_\xi/d\omega$ tends to infinity as $\omega \to 0$. This is the condition for the existence of electromagnetic waves—long wavelength photons.

Expression (26) shows that $dW_\xi/d\omega$ tends to zero as $\omega \to 0$. Thus, for the process of emission of a ξ meson there exists no classical limit, in contrast to the case of the photon. For the pseudoscalar meson of zero mass there is no analogy with a radio wave.

7. POSITRONIUM DECAY

We have shown above that if the ξ meson interacts with an electron, then the interaction must be very weak. Moreover, for this meson there exists no classical limit (long range forces and infrared radiation). Therefore, experimental observation of this particle turns out to be very difficult. However, the possibility of observing the ξ meson has not yet been excluded. We consider a process which might enable us to observe the ξ meson if the constant is $\beta_e \approx 3 \times 10^{-6}$, viz., positronium decay.

As is well known, parapositronium $(e^+e^-)_0$ can decay only into an even number of photons, while orthopositronium $(e^+e^-)_1$ can decay only into an odd number of photons. Mainly they decay into 2γ and 3γ. If a ξ-meson exists, then in addition it is possible to have $(e^+e^-)_0$ decay into an even number of photons and a ξ meson, and $(e^+e^-)_1$ decay into an odd number of photons and a ξ meson. In particular, the following decay is possible

$$(e^+e^-)_1 \to \gamma + \xi. \quad (28)$$

The probability of this decay can be obtained from the cross section of the process

$$e^+ + e^- \to \gamma + \xi \quad (29)$$

as the velocity of the particles tends to zero[6]. The matrix element of the process (29) is equal to

$$M = \frac{ef}{2\sqrt{\omega\omega'}} \bar{v}(-p_+)\left[\hat{\varepsilon}\frac{i(\hat{p}_- - \hat{q}) - m}{2p_-q}\gamma_5 + \gamma_5\frac{i(\hat{p}_+ - \hat{q}) - m}{2p_+q}\hat{\varepsilon}\right]u(p_-), \quad (30)$$

where ω' and ϵ_α are respectively the energy of the photon and the vector which characterizes its polarization state, ω and q are the energy and the four-momentum of the ξ meson, p_+ and p_- are the four-momenta of the positron and the electron. From this matrix element follows the probability for the decay (28). The probability of the decay

$$(e^+e^-)_1 \to 3\gamma \quad (31)$$

was calculated in [15]. The ratio of the probabilities of the decays (28) and (31) is equal to

$$\frac{(e^+e^-)_1 \to \gamma + \xi}{(e^+e^-)_1 \to 3\gamma} = \frac{3\pi}{4(\pi^2 - 9)}\frac{\beta_e}{\alpha^2}. \quad (32)$$

If $\beta_e \approx 3 \times 10^{-6}$, then this ratio is equal to $1/10$. The decay (28) can in principle be distinguished from the decay of parapositronium into 2γ, since in the latter decay both photons can be recorded, while in the decay into $\gamma + \xi$ only one photon can be recorded. For this purpose, of course, it is necessary to distinguish the process under consideration from background. We note that the number of events of orthopositronium decay is approximately three times greater than the number of events of parapositronium decay, in spite of the fact that the probability of decay of the former is smaller by a factor of approximately 372. Therefore, if the ratio (32) is equal to $1/10$, then the observation of the decay (28) is not exceptionally difficult.

8. INTERACTION WITH NUCLEONS AND THE DECAYS $K \to \pi + \xi$, $\pi^0 \to e^+ + e^-$

If the interaction of the ξ meson with fermions is universal, then the constant for the interaction with a nucleon is also equal to β_e in (21). We consider the case when the interaction is nonuniversal, and denote the constant β for nucleons by β_N. If the ξ meson interacts with the nucleon (or generally with baryons), then the following nonleptonic decays of hyperons and of K mesons are also possible:

$$\Sigma^+ \to p + \xi, \quad (33a)$$

$$\Lambda \to n + \xi, \quad (33b)$$

$$K_1{}^0 \to \pi^0 + \xi, \quad (33c)$$

$$K^\pm \to \pi^\pm + \xi. \quad (33d)$$

The first decay would appear as a radiative decay

$$\Sigma^+ \to p + \gamma. \quad (34)$$

From the experimental data[16-18] it follows that the probabilities of the decays (33a) and (34) amount to approximately 1% of the probability of the decay

$$\Sigma^+ \to p + \pi^0. \quad (35)$$

This means that the constant β_N cannot exceed a limit of the order of 1% of the constant of strong interaction $g^2/4\pi \approx 15$. This estimate is also obtained from the experimental data on the decays of K mesons. We note that the decay $K \to \pi + \gamma$ is forbidden. Therefore, for the observation of a ξ-meson it is very desirable to look for the decay (33d).

If the interaction of the ξ meson with a nucleon is isotopically invariant, then the ξ meson has isotopic spin I = 0. It can happen that the ξ meson exists, and the interaction of this meson with the nucleon fully destroys isotopic invariance. This does not lead to a contradiction with experiment, since the constant of the interaction with a ξ meson is small compared to the strong interaction constant.

We consider this case in detail. If the interaction of the ξ meson destroys isotopic invariance, then the ξ meson can be created in the decay

$$\Sigma^0 \to \Lambda^0 + \xi, \quad (36)$$

with this decay occurring with a probability comparable to, or smaller than, the probability of radiative decay

$$\Sigma^0 \to \Lambda^0 + \gamma. \quad (37)$$

The decays (36) and (37) can, in principle, also be distinguished, although this is difficult. Moreover, the virtual transition $\pi^0 \rightleftarrows \xi$ is possible, for example, through a nucleon-antinucleon pair. This transition could lead to the decay

$$\pi^0 \to e^+ + e^-. \quad (38)$$

The corresponding diagram is shown in Fig. 5a. A rough estimate shows that the ratio of the probabilities of decay (38) and of radiative decay

$$\pi^0 \to \gamma + \gamma \quad (39)$$

is approximately equal to

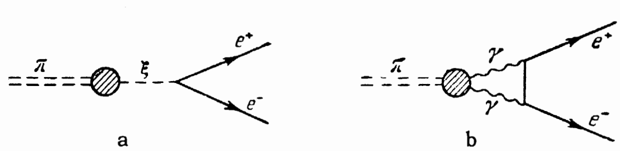

FIG. 5

$$\frac{W(\pi^0 \rightarrow \xi \rightarrow e^+ + e^-)}{W(\pi^0 \rightarrow \gamma + \gamma)} \sim \frac{\beta_N \beta_e}{\alpha^2}. \qquad (40)$$

If $\beta_N \approx 1/10$ and $\beta_e \approx 3 \times 10^{-6}$, then this ratio has a value of the order of 10^{-3}.

We note that the process (38) can occur in a higher approximation in the electromagnetic interaction. The corresponding diagram is shown in Fig. 5b. The matrix element of this diagram is proportional to e^4. Moreover, because of the γ_5-invariance of the electromagnetic interaction this matrix element is proportional to the electron mass and is equal to zero if we set $m_e = 0$. Therefore, if the process (38) takes place mainly through a pair of intermediate virtual photons (in accordance with diagram 5b), we then have

$$\frac{W(\pi^0 \rightarrow 2\gamma \rightarrow e^+ + e^-)}{W(\pi^0 \rightarrow \gamma + \gamma)} \approx \alpha^2 \left(\frac{m_e}{m_\pi}\right)^2 \approx 10^{-8}. \qquad (41)$$

This result was also obtained by Drell[19] by means of specific calculations on the basis of a certain assumption with respect to the form factor.

9. CONCLUSION

The results obtained above show that if the ξ meson exists, then the constants of interaction with leptons and nucleons must be small:

$$\beta_e \lesssim 3\cdot 10^{-6}, \qquad \beta_\mu \lesssim 3\cdot 10^{-5}, \qquad \beta_N \lesssim 10^{-1}. \qquad (42)$$

Moreover, the existence of this meson with mass equal to zero does not lead to the appearance of classical effects of the type of radiation of long wavelength mesons and of long range forces.

If the interaction constants β_e and β_N take on values of the order of the upper limit in (42), then the ξ meson can be observed in the decay of positronium and of K mesons. If the interaction of the ξ meson with a nucleon completely destroys isotopic invariance (for example, the ξ meson interacts with the proton, but does not interact with the neutron, or the constants of the interaction with p and n have opposite signs but the same magnitude) and the constants β_e and β_N take on values of the order of the upper limit in (42), or smaller than these limits by an order of magnitude, then such a ξ meson could be indirectly observed in the decay $\pi^0 \rightarrow e^+ + e^-$

The author expresses his deep gratitude to Professor M. A. Markov for constant attention and valuable advice, and to B. N. Valuev, S. S. Gershteĭn, and V. I. Ogievetskiĭ for interesting discussions.

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Translated by G. Volkoff
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