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NONLINEAR EFFECTS IN AN INHOMOGENEOUS PLASMA

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We investigate nonlinear effects that arise in the interaction of a high-frequency field and a stratified inhomogeneous plasma; these effects are due to the existence of a sharp maximum in the potential that characterizes the effective average force in the vicinity of the plasma resonance point. If the external field E_0 is large enough, ($E_0 > E_0^{cr}$, $E_0^{cr} \sim \nu^{3/2}$, ν is the electron collision frequency) the density distribution becomes steplike in the resonance region. The positions of the weakly sloping (top) and steep (side) parts of the step depend on the history of the amplitude variation (hysteresis): if the amplitude is increasing the dielectric constant ϵ is positive in the weakly sloping part and the transition through the point $\epsilon = 0$ is discontinuous; if the amplitude is decreasing the discontinuity shifts to the region $\epsilon < 0$ and ϵ approaches zero in the weakly sloping part. The first case corresponds to a sharp increase in the quality factor of quasistatic resonances for objects with smeared out boundaries; in the second case these resonances disappear completely as a result of the screening effect of the layer of zero dielectric constant. Estimates show that the nonlinear effects considered here can be important both under laboratory conditions and in the reflection of radio waves from ionospheric strata.

INTRODUCTION

It is well known[1-3] that the resonance properties of plasma objects with diffuse boundaries depend quite sensitively on the slope of the electron density distribution in the region in which the plasma frequency ω_p approaches the wave frequency ω. Whether it is due to particle collisions[1,2] or to the conversion of energy into plasma waves,[3] (for an arbitrarily low collision frequency and temperature in the linear approximation) the energy loss is the same in this region and is proportional to the scale size of the density inhomogeneity. For this reason, the quality factor of electron resonances can be very sensitive to nonlinear effects associated with the presence of a sharp maximum in the high-frequency potential[4-6] in the region in which $\omega_p \approx \omega$;[1)] this potential is given by

$$\Phi = \frac{e^2}{4m\omega^2}|\mathbf{E}|^2, \qquad (1)$$

and plays the role of the potential energy for the motion averaged over a period $2\pi/\omega$ (e is the electron charge, m is the electron mass, **E** is the amplitude of the high-frequency field).

The redistribution of electron density due to the averaged force $\nabla\varphi$ and the resulting change in resonance absorption are investigated in the present work for the simple case of a plane stratified plasma structure at low field amplitudes. A similar problem, i.e., the effect of the field interaction on the plasma density for the one-dimensional case, has been treated by Gurevich and Pitaevskiĭ.[8] However, these authors assumed the plasma density to be uniform in the unperturbed state and the change in spatial distribution of density was not investigated. The analysis given below makes use of the results of [8] and is essentially a generalization of these results (and earlier results[5,6,9]) to the case of interest here, the problem of determining the change in the electron density gradient in the vicinity of the plasma resonance point.

1. DENSITY DISTRIBUTION IN THE PRESENCE OF A FIELD

We consider a plane plasma layer located in an external uniform field $\mathbf{x}_0 E_0 e^{i\omega t}$ perpendicular to the plane of constant plasma density (x = const). By virtue of the continuity of the normal compo-

1)Because of this effect the nonlinear interaction of the field with the plasma in the region of the plasma resonance becomes important at much lower fields than would follow from the estimate given by Ginzburg (cf.[7], p. 272) on the basis of spatial-dispersion phenomena.

nent of the electric induction the field in the plasma is given by

$$\varepsilon(x)E(x) = E_0. \tag{2}$$

Here, $\epsilon(x)$ is the complex dielectric constant of the plasma:[2)]

$$\varepsilon = \varepsilon_r - i\varepsilon_i, \qquad \varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2},$$

$$\varepsilon_i = \frac{\nu}{\omega}\frac{\omega_p^2}{\omega^2}, \qquad \omega_p^2 = \frac{4\pi e^2}{m} n(x), \tag{3}$$

ν is the electron collision frequency, which is assumed to be small compared with ω ($\nu \ll \omega$); $n(x)$ is the electron density distribution, which obeys the Boltzmann distribution and, under quasineutral conditions, is given by (cf. for example [10])

$$n(x) = n_0 \exp\left[-\frac{\Phi(x) + \Psi(x)}{2kT}\right], \tag{4}$$

$\Psi(x)$ is the potential of the external free forces providing stability of the plasma, for a given total number of electrons N (per unit area of layer) the constant n_0 is determined by the normalization condition

$$n_0 \int_{-\infty}^{+\infty} \exp\left[-\frac{\Phi(x) + \Psi(x)}{2kT}\right] dx = N \tag{5}$$

and is thus a functional of the field amplitude:

$$n_0 = n_0\{|E|^2\}.$$

Multiplying Eq. (2) by the complex conjugate, and using Eqs. (1), (3)–(5) with the notation

$$\mathscr{E} = \frac{|E|^2}{E_0^2}, \qquad \alpha = \frac{e^2 E_0^2}{8m\omega^2 kT}, \qquad f(x) = \frac{4\pi e^2}{m\omega^2} N e^{-\Psi/2kT},$$

$$J\{\mathscr{E}\} = \int_{-\infty}^{+\infty} \exp\left\{-\frac{\Psi}{2kT} - \alpha\mathscr{E}\right\} dx, \qquad \delta = \frac{\nu}{\omega}, \tag{6}$$

we obtain the following transcendental equation for $\mathscr{E}(x)$

$$\left[\left(1 - \frac{f(x)}{J\{\mathscr{E}\}} e^{-\alpha\mathscr{E}}\right)^2 + \left(\delta \frac{f(x)}{J\{\mathscr{E}\}} e^{-\alpha\mathscr{E}}\right)^2\right] \mathscr{E} = 1. \tag{7}$$

We shall limit ourselves here to an investigation of Eq. (7) for the weak field case, requiring that $\alpha\mathscr{E}_{max} \approx \alpha/\delta^2 \ll 1$ and replacing the exponential factor $\exp(-\alpha\mathscr{E})$ by $1 - \alpha\mathscr{E}$. Assuming that the deviation of this factor from unity can be of importance only in the region where $|\epsilon(x)| \ll 1$ (to which all the subsequent analysis pertains) and that the total number of particles within this region is small compared with N, for the accuracy required here we can neglect the functional dependence $n_0\{|E|^2\}$ (i.e., $J\{\mathscr{E}\}$) and assign to the constant n_0 its value in the unperturbed state (in the absence of field):

$$n_0 = N/J, \qquad J = \int_{-\infty}^{+\infty} e^{-\Psi/2kT} dx. \tag{8}$$

Then, from Eq. (7) $|\epsilon(x)| \ll 1$ we find

$$\{[\varepsilon_0(x) + \alpha\mathscr{E}(x)]^2 + \delta^2\}\mathscr{E}(x) = 1, \tag{9}$$

which has been investigated earlier [8] for the case $\delta = 0$, $\epsilon_0 = \text{const}$ ($\epsilon_0(x)$ denotes the real part of the dielectric constant of the unperturbed plasma).

Now we substitute the value $\mathscr{E} = |E|^2/E_0^2 = 1/(\epsilon_r^2 + \delta^2)$ in Eq. (9) and assume a linear dependence for $\epsilon_0(x)$

$$\varepsilon_0(x) = -x/l_0, \qquad |\varepsilon_0(x)| \ll 1, \tag{10}$$

thereby obtaining an equation that describes the spatial distribution of the real part of ϵ in the presence of the field:

$$x/l_0 + \varepsilon_r(x) - \alpha/(\varepsilon_r^2(x) + \delta^2) = 0. \tag{11}$$

(This equation obviously can be obtained starting directly from the expression for $\epsilon_r(x) = 1 - (4\pi e^2/m\omega^2)/(x, E^2)$.)

The hysteresis effect in the dependence of $\mathscr{E}(\epsilon_0)$ noted by Gurevich and Pitaevskiĭ[8] corresponds, in the present case, to the ambiguity in the functions $\mathscr{E}(x)$ and $\epsilon_r(x)$; this ambiguity is easily eliminated from Eq. (11). Rewriting (11) in the form

$$\frac{x}{l_0\delta} + \frac{\varepsilon_r}{\delta} - \frac{\alpha}{\delta^3}\frac{1}{1 + (\varepsilon_r/\delta)^2} = 0, \tag{12}$$

we note that the nature of the solution depends primarily on the parameter

$$\eta = \frac{\alpha}{\delta^3} = \frac{e^2 E_0^2}{8m\omega^2 kT\delta^3} = \frac{\Phi_{max}}{2kT\delta}. \tag{13}$$

The curves in Fig. 1 show the function $\epsilon_r(x/l_0\delta)/\delta$ for various values of η; starting at some critical value ($\eta > \eta_{cr}$) there is a finite range $x_1 \leq x \leq x_2$ for which the solution of (12) becomes ambiguous and gives three equilibrium states. The intermediate state (dashed curve in Fig. 1) is unstable and cannot be realized. Since the other two (stable) equilibrium states are separated everywhere by a finite range of values of ϵ_r it is evident that at any point $\tilde{x}$ in the interval $x_1 \leq x \leq x_2$ the functions

[2)]We assume that the resonance value of the field E_{max} (in the plane $\epsilon_r(x) = 0$) is determined by the frequency of electron collisions ν while effects associated with spatial dispersion are unimportant (for this statement to hold the inequalities $eE_{max}/m \ll \nu\omega l$ and $(D/l)^{2/3} \ll \nu/\omega$ must be satisfied where l is the characteristic scale length of the density inhomogeneity and D is the Debye radius; (cf.[7] pp. 272, 279).

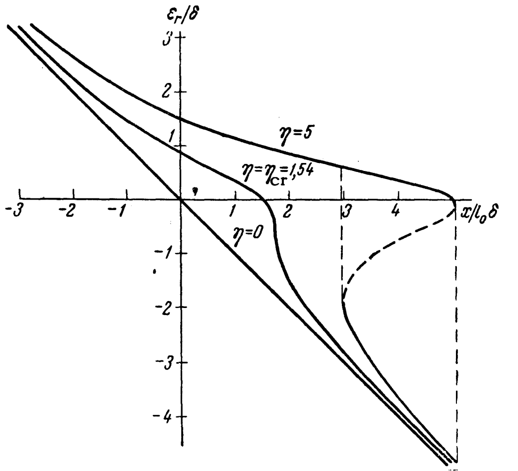

FIG. 1. The function $\epsilon_r(x/l_0\delta)/\delta$ for various values of the parameter η.

$e_r(x)$, $n(x)$, $\mathscr{E}(x)$ must be discontinuous.

Under actual physical conditions this discontinuity obviously cannot be infinitesimally sharp; its detailed structure is determined basically by two effects that have not been considered: 1) spatial dispersion, and 2) strong electric fields in the vicinity of the discontinuity due to the violation of plasma neutrality. Because of these effects the width of the sharp density drop becomes finite; in any case it is at least as large as the Debye radius.

The position of the point $\tilde{x}$ within the interval (x_1, x_2) is determined by the history of the variation in field amplitude E_0, that is to say, it depends on the way in which a given equilibrium state is approached.[3] To demonstrate this point it is sufficient to investigate the function $\epsilon_r(\eta)$ given by Eq. (12) at various points. Direct analysis of the family of curves in Fig. 1 (curves corresponding to higher values of η lie above and to the right) leads to the following conclusion. As E_0 increases monotonically from the critical value E_0^{cr} $(\eta = \eta_{cr})$ a discontinuity is formed at the point $\tilde{x} = x_1$ (i.e., at the lower boundary of the interval of inhomogeneity) and is displaced with it in the direction of positive x. When E_0 is reduced the interval (x_1, x_2) is displaced in the opposite direction and the coordinate of the discontinuity $\tilde{x}$ does not change so long as it does not coincide with the point x_2; when it does coincide both move together.

Certain numerical results of the analysis of the cubic equation (12) are given below.

1. $\eta_{cr} = 8\sqrt{3}/9 \approx 1.54$; $\eta = \eta_{cr}$ the interval (x_1, x_2) degenerates into a point $\tilde{x}_{cr} = x_1 = x_2 = \sqrt{3}\,\delta l_0$, at which $\epsilon_r = -\delta/\sqrt{3}$ and $d\epsilon_r/dx = \infty$.

2. When $\eta \gg \eta_{cr}$ the edge of the range (x_1, x_2) and the values of ϵ_r on both sides of the discontinuity for $\tilde{x} = x_1$ and $\tilde{x} = x_2$ are given by the expressions

$$\begin{aligned} &x_1/\delta l_0 = 3\,(\eta/4)^{1/3}, \quad x_2/\delta l_0 = \eta,\\ &\varepsilon_r\,(x \to x_1 - 0) = \delta\,(\eta/4)^{1/3},\\ &\varepsilon_r\,(x \to x_2 - 0) = -\,\delta/2\eta,\\ &\varepsilon_r\,(x \to x_1 + 0) = -\,\delta\,(2\eta)^{1/3},\\ &\varepsilon_r\,(x \to x_2 + 0) = -\,\delta\eta. \end{aligned} \tag{14}$$

As is evident from Fig. 1, the appearance of the discontinuity is accompanied by the formation of a weakly sloping part ("plateau") where the derivative $d\epsilon_r/dx$ is reduced appreciably. As a result when $\eta \gg \eta_{cr}$ (to which we refer everywhere below) the distribution $\epsilon_r(x)$ acquires a form which can be shown schematically as a step (Fig. 2a,b).

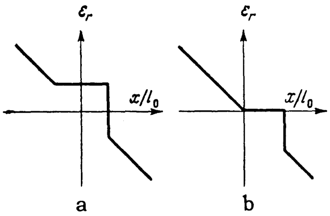

FIG. 2. Simplified schematic diagram showing the spatial distribution of the dielectric constant in a strong field ($\eta \gg 1$): a) field increasing in time ($\tilde{x} = x_1$); b) field decreasing in time ($\tilde{x} = x_2$).

On the scale length l_0 the width of the plateau Δx_p coincides with the height of the step $\Delta\epsilon_r$.

$$\Delta x_p/l_0 = \Delta\varepsilon_r \approx \begin{cases} 2\delta\eta^{1/3}, & \tilde{x} = x_1 \\ \delta\eta, & \tilde{x} = x_2 \end{cases}, \tag{15}$$

its height ϵ_{rp} above the level $\epsilon_r = 0$ is approximately $\Delta\epsilon_r/2$ and $\tilde{x} = x_1$ and is very close to zero $(\epsilon_{rp} \approx \delta)$ when $\tilde{x} = x_2$. It follows from Eq. (15) that for a given value of η the width of the plateau and the height of the step for $\tilde{x} = x_2$ are $\eta^{2/3}$ times greater than for $\tilde{x} = x_1$. However, it should be kept in mind that for a given $\eta = \eta_0$ the relation $\tilde{x} = x_2$ can be satisfied only after η increases to the value $\eta_{max} \geq \eta_0^3 \gg \eta_0$. A subsequent reduction in η is accompanied by a reduction in the dimensions of the step; this is first $(\eta_{max} \geq \eta \geq \eta_{max}^{1/3})$ due only to the reduction of the plateau level ϵ_{rp} and then $(\eta \leq \eta_{max}^{1/3})$ to the displacement of the coordinate of the discontinuity $\tilde{x} = x_2$ in the direction of decreasing x.

[3]Here and everywhere below we consider only slow changes of E_0 in time (the characteristic time for the change in E_0 being appreciably greater than the time required to achieve the equilibrium Boltzmann density distribution).

2. CHANGE IN JOULE LOSSES AND RESONANCE CHARACTERISTICS

The step approximation for the function $\epsilon_r(x)$ provides a simple way to estimate the change in power loss in the resonance region R ($|\epsilon| \ll 1$)

$$W = \frac{\omega}{8\pi}\,\delta E_0^2 \int\limits_{(R)} \frac{dx}{\varepsilon_r^2 + \delta^2} \tag{16}$$

as compared with the loss at small field amplitudes ($\eta \ll 1$)

$$W_0 = \frac{\omega}{8\pi}\,\delta E_0^2 \int\limits_{(R)} \frac{dx}{\varepsilon_0^2 + \delta^2} = \frac{\omega}{8} E_0^2 l_0. \tag{17}$$

For a given field amplitude E_0 that is sufficiently greater than E_0^{cr} ($\eta^{1/3} \gg 1$) the quantity W falls off strongly or increases sharply depending on the value of ϵ_r in the region of the plateau (i.e., on the direction of the previous change in amplitude

$$W \approx \begin{cases} \frac{\omega}{2\pi} E_0^2 l_0 \eta^{-1/3}, & \text{increasing field}, \quad \eta \leqslant \eta_{max} \\ \frac{\omega}{8\pi} E_0^2 l_0 \eta, & \text{decreasing field}, \quad \eta \leqslant \eta_{max}^{1/3} \end{cases}. \tag{18}$$

Before investigating the resonance properties of a layer we should point out that the results given above are not fundamentally limited to the one-dimensional case; in systems with more complicated field and density distributions (two or three-dimensional objects, plane layer irradiated by a wave at oblique incidence) the component of **E** parallel to $\nabla\epsilon$ in the region of the point $\epsilon_r = 0$ is, as a rule, determined by the same equation (2). In general, the important feature of inhomogeneous systems is the dependence of the constant E_0 on the nature of the distribution of ϵ and this dependence can generally be taken into account within the framework of a one-dimensional problem. We shall assume that the layer is located inside a plane condenser to the plates of which is applied an ac voltage with amplitude V_0. Then, as before the field in the plasma is described by Eq. (2) while the constant E_0, whose magnitude determines the nature of the distribution $\epsilon_r(x)$, can in turn be a functional of this distribution:

$$E_0 = V_0 \Bigg/ \left[\int\limits_{L_1}^{L_2} \frac{\varepsilon_r}{\varepsilon_r^2 + \delta^2} dx + i\delta \int\limits_{L_1}^{L_2} \frac{dx}{\varepsilon_r^2 + \delta^2}\right], \tag{19}$$

where L_1 and L_2 are the coordinates of the condenser plates; the plate separation $\Delta L = L_2 - L_1$ is assumed to be fairly large in the estimates we make below ($\Delta L \gtrsim l_0$). The impedance of the system is equal to the denominator of Eq. (19) multiplied by $4\pi/i\omega$. Setting the imaginary part equal to zero (i.e., the real part of the denominator) we obtain the resonance condition

$$\int\limits_{L_1}^{L_2} \frac{\varepsilon_r}{\varepsilon_r^2 + \delta^2} dx = 0. \tag{20}$$

It then follows in particular, that resonance is possible only for a function $\epsilon_r(x)$ that changes sign and that the number of resonances can be greater than unity. The resonance amplitude E_0^{res} ($\omega = \omega_{res}$), the width of the line ν and the quality factor are determined by the magnitude of the integral

$$K = \int\limits_{L_1}^{L_2} \frac{dx}{\varepsilon_r^2 + \delta^2}. \tag{21}$$

It is clear from Eq. (16) that the power loss for a fixed E_0^2 is proportional to the same integral ($W \sim KE_0^2$). Hence, all that has been indicated above concerning the increase and decrease in W/E_0^2 also applies completely to the width of the resonance line ν: when $\eta^{1/3} \gg 1$ the resonance in an increasing field ($\tilde{x} = x_1$) is much more pronounced ($\nu/\nu_0 \approx \eta^{-1/3}$) and is much weaker ($\nu/\nu_0 \approx \eta$) in a decreasing field ($\tilde{x} = x_2$) than in the absence of nonlinear effects ($\nu = \nu_0 \approx \omega_{res} l_0/\Delta L$). In the first case the power loss W, is always small far from resonance as in any system with a high Q ($W/W_0 \approx \eta^{1/3}$) and increases sharply at resonance ($E_0^{res} \sim 1/K \sim \eta^{1/3}$, $W/W_0 \approx \eta^{2/3}$). In the second case the maximum value that E_0 can assume falls off sharply ($E_0^{res} \sim \eta^{-1}$) and W always remains small[4)] ($W/W_0 \approx \eta^{-1}$).

The deformation of the plasma structure not only changes the line width ν but also shifts the resonance frequency ω_{res} given by Eq. (20). In the absence of nonlinearity this condition is given by

$$\int\limits_{L_1}^{L_2} \frac{\varepsilon_0}{\varepsilon_0^2 + \delta^2} dx = 0, \tag{22}$$

while when $\eta^{1/3} \gg 1$ and $\tilde{x} = x_1$, as can be easily shown using the step approximation,

$$\int\limits_{L_1}^{L_2} \frac{\varepsilon_0}{\varepsilon_0^2 + \delta^2} dx = -2l_0 + 0(\eta^{-1}). \tag{23}$$

Comparing the two expressions we see that the entire distribution $\epsilon_0(x)$ must be shifted toward negative values so that the resonance frequency is reduced. As η increases this relative change ap-

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[4)]Actually this is the manifestation of the screening of the field by the layer with zero dielectric constant[11] (the plateau characterized by $\epsilon_r \approx \delta$ plays this role).

proaches a field-independent limit:

$$\frac{\Delta\omega}{\omega_{\text{res}}} \approx -l_0/\Delta L + 0(\eta^{-1}). \tag{24}$$

Thus, the resonance characteristics of a layer are sensitive to the magnitude and direction of the field change. As a result, amplitude modulation of the voltage on the condenser plates will cause the impedance to change over wide limits and lead to marked nonlinear distortion of the current envelope over a modulation period. As applied to an inhomogeneous system this means that at some critical field amplitude the envelope of a signal reflected by a plasma should exhibit strong nonlinear distortion.

CONCLUSION

The results that have been obtained can be summarized as follows: In an inhomogeneous plasma interacting with a time-increasing high-frequency field the transition through the point $\epsilon_r = 0$ occurs discontinuously; if the field is decreasing the discontinuity is shifted toward negative values of ϵ_r and the plasma "surrounds" itself by a screening layer with dielectric constant equal to zero.

If there is a density discontinuity at the point $\epsilon_r = 0$ the resonance effects which are weak at the boundary in a plasma with a smooth distribution of ϵ_r can be intensified greatly. In particular, it becomes completely possible to realize resonance interactions of the field with higher multipole moments of a plasma structure [1,2,12] (in the absence of nonlinear effects the multipole resonances of higher order are especially weak [1,2] because of the diffuseness of the boundary). In a plasma screened by a layer of zero dielectric constant the quasistatic resonances (which can be treated only on the basis of a particular model) become impossible.

Here we have neglected completely the effect of the deformation of the spatial distribution of plasma on the plasma resonances associated with the excitation of longitudinal (plasma) oscillations. The quality factor of these resonances is increased markedly, as indicated earlier, [3] because of a different nonlinear effect—the "equalization" of the electron velocity distribution function in the region in which the electrons are synchronized with the travelling plasma wave. [13]

The possibility of controlling the resonance properties of plasma objects by changing the signal amplitude is an extremely interesting one in a number of applications in which resonance effects in a bounded plasma are used (resonance acceleration of plasmoids, diagnostic methods, production of controlled transmission filters, and so on). It is also noteworthy that the nonlinear effects treated here are not limited to laboratory experiments but can also manifest themselves in reflection of radio waves from the ionosphere; in this case, under certain conditions, the vertical component of the electric field has a sharp maximum at the point $\epsilon_r = 0$.[14] We show below that the nonlinearity becomes appreciable starting at relatively low levels of high-frequency power. For example, let $\nu = 10^8$ sec^{-1}, $\omega = 10^{10}$ sec^{-1}, $T = 10^4$ °K (parameters that are typical for laboratory conditions). Then the critical field value $E_0^{cr} = (12.3\ mkT\nu^3/e^2\omega)^{1/2} \approx 1$ V/cm. To produce such fields in wave guides with transverse cross-section ~ 10 cm^2 requires a power $\sim 10^{-2}$ W. Under ionospheric conditions the role of the constant E_0 at small angles of incidence is played by $1.2\,(c/2\pi\omega l_0)^{1/2} E_i$ [14] where c is the velocity of light and E_i is the amplitude of the incident wave on entry into the layer. In the F-layer ($\nu \approx 3 \times 10^3$ sec^{-1}, $T \approx 10^3$ °K, $l_0 \approx 10^7$ cm) the critical value of the amplitude is $E_i^{cr} = 7\,(l_0 mkT\nu^3/e^2c)^{1/2} \approx 10^{-4}$ V/cm. At the height of the F-layer (approximately 300 km) a field of intensity of this kind can be realized with an isotropic radiator with a power of approximately $\sim 10^5$ W. If two waves at approximately the same frequency are reflected from the ionosphere the resulting change in electron density gradient should lead to cross-modulation which would be especially strong if the point of reflection of one wave were close to the resonance point of the other.

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320