FERMION REGGE POLES IN PROCESSES INVOLVING VECTONS

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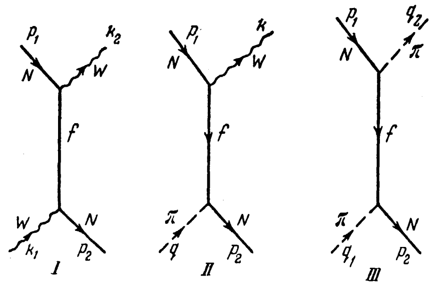
The asymptotic value of the cross section for production of vector mesons by π mesons and γ quanta on nucleons at angles near 180° is obtained. The analysis is based on the Regge pole hypothesis. Relations between the cross sections of various processes involving vector mesons are presented.

1. INTRODUCTION

DIFFERENT assumptions about the singularities of the partial amplitudes in the crossed channel lead in each case to a definite form of the asymptotic value of the cross section with respect to the energy. For given singularities in the partial amplitudes one can obtain a relation between the cross sections for different processes, using the isotopic relations, crossing symmetry, and the unitarity condition. These theoretical results can be tested directly by experiment.

In the present paper we obtain the asymptotic value and several relations for the cross sections of processes involving vector mesons (vectons, denoted by W in the following), assuming the existence of one or several Regge poles.



We consider three processes (see the figure): I—scattering of vectons on nucleons (if the mass of the initial vecton is $\lambda_1 = 0$: photoproduction of vectons; II—production of vectons by π mesons on nucleons; and III—scattering of π mesons on nucleons. In the analysis of the singularities of the partial amplitudes it is convenient to use amplitudes for transitions between states with definite parity. Such states are combinations of states with different helicities ν [1] and have the following form (in the parentheses we indicate the particles contained in the state; j is the total angular momentum):

$$(N, W) \quad |j;\ \nu;\ W;\ \pm\rangle = \frac{1}{\sqrt{2}}\left(\left|j;\ \nu;\ \frac{1}{2}\right\rangle \pm \left|j;\ -\nu;\ -\frac{1}{2}\right\rangle\right),$$

$$(N, \pi) \quad |j;\ \pi;\ \pm\rangle = \frac{1}{\sqrt{2}}\left(\left|j;\ \frac{1}{2}\right\rangle \pm \left|j;\ -\frac{1}{2}\right\rangle\right),$$

$$\nu = 1,\ 0,\ -1. \tag{1}$$

From these states one can construct the desired amplitudes for the processes I, II, and III, respectively:

$$\Phi_{\nu\nu'}^{j\pm} = \langle j;\ \nu;\ W;\ \pm | j;\ \nu';\ W;\ \pm\rangle, \tag{2a}$$

$$\varphi_{\nu}^{j\pm} = \langle j;\ \nu;\ W;\ \pm | j;\ \pi;\ \pm\rangle, \tag{2b}$$

$$\chi^{j\pm} = \langle j;\ \pi;\ \pm | j;\ \pi;\ \pm\rangle. \tag{2c}$$

The unitarity condition for the amplitudes (2) leads to the following relations: [2]

$$\Phi_{\nu\nu'}^{j\pm} - (\Phi_{\nu\nu'}^{\overset{*}{j}\pm})^{*} = \frac{2ik}{\omega}\varphi_{\nu}^{j\pm}(\varphi_{\nu'}^{\overset{*}{j}\pm})^{*},$$

$$\varphi_{\nu}^{j\pm} - (\varphi_{\nu}^{\overset{*}{j}\pm})^{*} = \frac{2ik}{\omega}\varphi_{\nu}^{j\pm}(\chi^{\overset{*}{j}\pm})^{*},$$

$$\chi^{j\pm} - (\chi^{\overset{*}{j}\pm})^{*} = \frac{2ik}{\omega}\chi^{j\pm}(\chi^{\overset{*}{j}\pm})^{*}. \tag{3}$$

It follows from (3) that, if one of the amplitudes has a pole in j, all amplitudes must have it, too, where the residues at each pole are connected by the relations

$$r_{\nu\nu}^{\pm} r_{\nu'\nu'}^{\pm} = (r_{\nu\nu'}^{\pm})^2, \qquad r^{\pm} r_{\nu\nu}^{\pm} = (r_{\nu}^{\pm})^2, \tag{4}$$

$\nu = 1, 0, -1$, where $r^{\pm}_{\nu\nu'}$, $r^{\pm}_{\nu}$, and $r^{\pm}$ are the residues of the amplitudes $\Phi^{j\pm}_{\nu\nu'}$, $\varphi^{j\pm}_{\nu}$, and $\chi^{j\pm}$, respectively. Thus four residues of both parities are independent. For these we choose the residues of the processes II and III, i.e., $r^{\pm}_{\nu}$ and $r^{\pm}$.

One can, therefore, restrict the discussion to process II (process III has been considered by Gribov, Okun', and Pomeranchuk [3]); the amplitude and the cross section for process I can be obtained with the help of relation (4).

2. AMPLITUDE FOR THE PRODUCTION OF VECTONS IN THE u CHANNEL

The amplitude for process II in the u channel is determined by the following six helicity amplitudes:

$$f_1^u = \langle {}^1/_2 | {}^1/_2;\ 1\rangle, \qquad f_2^u = \langle -{}^1/_2 | {}^1/_2;\ 1\rangle,$$
$$f_3^u = \langle -{}^1/_2 | {}^1/_2;\ 1\rangle, \qquad f_4^u = \langle {}^1/_2 | -{}^1/_2;\ 1\rangle,$$
$$f_5^u = \langle {}^1/_2 | {}^1/_2;\ 0\rangle, \qquad f_6^u = \langle -{}^1/_2 | {}^1/_2;\ 0\rangle. \tag{5}$$

The amplitudes whose partial waves are given by the functions (2b) are connected with the functions (5) by the relations

$$\varphi_1^\pm = \frac{1}{\sqrt{2}}\left(-f_1^u/\sin\frac{\vartheta}{2} \pm f_2^u/\cos\frac{\vartheta}{2}\right),$$
$$\varphi_2^\pm = \frac{1}{\sqrt{2}}\left(f_3^u/\sin\frac{\vartheta}{2} \pm f_4^u/\cos\frac{\vartheta}{2}\right),$$
$$\varphi_3^\pm = \frac{\lambda}{2k_0}\left(f_5^u/\cos\frac{\vartheta}{2} \pm f_6^u/\sin\frac{\vartheta}{2}\right), \tag{6}$$

where λ is the mass of the vecton.

The asymptotic form of the amplitudes (6) for large $\cos\vartheta$ (large s) and finite u can be obtained in the usual way in terms of the residues of the Regge poles. To find the asymptotic cross section and polarization coefficients for process II one must only determine the connection between the functions (6) and the helicity amplitudes in the s channel. We obtain this connection via the invariant amplitudes, which also allows us to find a factorized expression for the total amplitude.

The invariant form of the amplitude II is [1)]

$$T_{II} = \bar{u}(p_2)\,\varepsilon_\mu J_\mu u(p_1), \quad \varepsilon_\mu k_\mu = 0, \quad (i\hat{p} + m)\,u(p) = 0, \tag{7}$$

$$J_\mu = i\gamma_5\{A_1\gamma_\mu\hat{k} + A_2 p_{1\mu} + A_3 p_{2\mu} - A_4 i\gamma_\mu - A_5 i\hat{k} p_{1\mu} - A_6 i\hat{k} p_{2\mu} + A_7 k_\mu - A_8 i\hat{k} k_\mu\}, \tag{7a}$$

where $\hat{k} \equiv \gamma_\mu k_\mu$, p_1 and p_2 are the momenta of the initial and final nucleons, $u(p)$ are the Dirac spinors, k and ϵ are the momentum and polarization of the vecton, and the A_i are scalar functions of the invariants $s = -(p_2 + k)^2$ and $u = -(p_1 - k)^2$, which have no kinematical singularities in u.

Current conservation $J_\mu k_\mu = 0$ leads to the following relation between the functions A_i:

$$(s - m_2^2 - \lambda^2)A_3 = (u - m_1^2 - \lambda^2)A_2 - 2\lambda^2(A_1 + A_7),$$
$$(s - m_2^2 - \lambda^2)A_6 = 2A_4 + (u + m_1^2 - \lambda^2)A_5 - 2\lambda^2 A_8;$$
$$p_i^2 = -m_i^2, \qquad k^2 = -\lambda^2. \tag{7b}$$

It is seen from this, in particular, that the functions A_3 and A_6 are of order s^{-1} compared to the remaining functions for $s \to \infty$. The amplitudes A_7 and A_8 give no contribution to the total amplitude (7) on account of the condition $k\epsilon = 0$, but they play a definite role in the law of current conservation. Thus we have six independent amplitudes A_i. For $\lambda = 0$ the number of independent amplitudes reduces to four.

Going over to the center of mass system (c.m.s.) in the u channel and expanding (7) in helicity amplitudes (5), we obtain a relation between the functions $\varphi_i^\pm$ (6) and A_i:

$$\varphi_1^\pm = zF_1^\pm + F_{1\mp},$$
$$\varphi_2^\pm = 2F_2^\pm + \varphi_1^\pm, \quad \varphi_3^\pm = F_2^\pm + zF_1^\pm + F_3^\pm; \tag{8a}$$

$$F_1^\pm = -N_u^\pm(m_2 \mp E_{2u})\,[A_3 + (m_1 \mp \sqrt{u})A_6],$$
$$F_2^\pm = N_u^\mp\,[(m_1 \pm \sqrt{u})A_1 - A_4], \tag{8b}$$

$$F_3^\pm = \mp N_u^\mp(m_1 \pm E_{1u})\,(\mp k_{0u})^{-1}\,\{A_1(m_1 \mp \sqrt{u}) - A_4 \pm E_{2u}\,[A_3 + (m_1 \mp \sqrt{u})A_6] \pm \sqrt{u}\,[A_2 + (m_1 \mp \sqrt{u})A_5]\};$$
$$N_u^\pm = [(m_1 \pm E_{1u})(m_2 \pm E_2)]^{1/2}/8\pi u,$$
$$E_{1u} = \frac{u + m_1^2 - \lambda^2}{2\sqrt{u}},$$
$$E_{2u} = \frac{u + m_2^2 - \mu^2}{2\sqrt{u}},$$
$$k_{0u} = -\frac{u - m_1^2 + \lambda^2}{2\sqrt{u}},$$
$$z = \cos\vartheta_u = \frac{m_2^2 + \lambda^2 - s + 2k_{0u}E_{2u}}{2k_u q_u}. \tag{8c}$$

In these formulas m_1 and m_2 are the masses of the initial and final baryons, and μ and λ are the masses of the initial meson and the vecton.

As is seen from (8), all quantities with different parity differ by the sign in front of $\sqrt{u}$ (McDowell symmetry [4]). Therefore, we could have chosen as basic quantities, instead of the functions (6), the functions $F_i^\pm$, which are the coefficients in the expansion of the total amplitude (7) in the c.m.s. in terms of a set of pseudoscalar quantities containing Pauli matrices and enclosed between Pauli spinors χ_ν:

$$T_{II} = \chi_f^*\left\{\frac{(\sigma\mathbf{q})(\mathbf{q}\varepsilon)}{q^2}F_1^+ + \frac{(\sigma\mathbf{k})(\mathbf{q}\varepsilon)}{qk}F_1^- + \frac{(\sigma\mathbf{q})(\sigma\varepsilon)(\sigma\mathbf{k})}{qk}F_2^+ + (\sigma\varepsilon)F_2^- + \frac{(\sigma\mathbf{q})(\varepsilon\mathbf{k})}{qk}F_3^+ + \frac{(\sigma\mathbf{k})(\mathbf{k}\varepsilon)}{k^2}F_3^-\right\}\chi_i. \tag{8d}$$

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1)We consider the case where the combined intrinsic parities of the particles in each vertex of the graph II (cf. the Figure) are opposite. If the parities are the same, $i\gamma_5$ must be replaced by unity in all formulas.

The connection between the invariant amplitudes and the helicity amplitudes in the s channel is obtained directly from (8) and (6) by applying crossing to the amplitudes A_i (which implies the replacement $A_1 \rightleftarrows A_1$, $A_4 \rightleftarrows -A_4$, $A_2 \rightleftarrows A_3$, $A_5 \rightleftarrows A_6$) and making the replacement $m_1 \rightleftarrows m_2$, $u \rightleftarrows s$.

3. ASYMPTOTIC FORM OF THE AMPLITUDE FROM REGGE POLES

Evaluating in the usual way the contribution of the main Regge pole to the functions (6) for large z and finite u, we obtain[2)]

$$\varphi_i{}^{\pm} = \frac{r_i{}^{\pm}(\sqrt{u})\,(1+\sigma\exp(-i\pi l^{\pm}))}{\sin\pi l^{\pm}}\, s^{l^{\pm}}, \tag{9}$$

$$l = j - \tfrac{1}{2}, \qquad \sigma = \pm 1, \qquad s \cong -2q_u k_u z,$$

where σ is the signature quantum number. [5]

In virtue of the McDowell symmetry, the contributions from poles with different parities will differ by the sign in front of $\sqrt{u}$ in the residues $r(\sqrt{u})$ and the trajectories of the Regge poles $l(\sqrt{u})$. The asymptotic expressions for the invariant functions A_i and the total current J_μ can be written in the form of a sum of two terms with different parity differing in the sign in front of $\sqrt{u}$:

$$A_i = A_i{}^{+} + A_i{}^{-}, \qquad J_\mu = J_\mu{}^{+} + J_\mu{}^{-}. \tag{10}$$

In the following we shall, therefore, consider only terms with positive parity. The complex conjugate terms with negative parity in the physical region of the s channel ($u < 0$) will be taken into account in the calculation of the cross section.

The asymptotic form of the amplitudes A_i is obtained with the help of (9), using the connection formulas (8a) to (8d) and (7b):[3)]

$$A_4{}^{+} = A_1{}^{+}(m_1 - \sqrt{u}), \quad A_2{}^{+} = 2A_1{}^{+} - A_5{}^{+}(m_1 + \sqrt{u}),$$
$$A_3{}^{+} = -A_6{}^{+}(m_1 + \sqrt{u}), \quad A_7{}^{+} = -2A_1{}^{+} - A_8{}^{+}(m_1 + \sqrt{u}), \tag{11a}$$

$$sA_6{}^{+} = (m_1 - \sqrt{u})A_2{}^{+} - \lambda^2(A_5{}^{+} + 2A_8{}^{+});$$
$$A_1{}^{+} = (\bar{N}_u{}^{-})^{-1}(\varphi_2{}^{+} - \varphi_1{}^{+})/2,$$
$$sA_6{}^{+} = (N_u{}^{+}q_u k_u[(m_2 - \sqrt{u})^2 - \lambda^2])^{-1}\sqrt{u} + \varphi_1{}^{+},$$
$$\sqrt{u}A_5{}^{+} = (N_u{}^{-}[(m_1 + \sqrt{u})^2 - \lambda^2])^{-1}\{\sqrt{u}(m_1 + \sqrt{u})\varphi_2{}^{+}$$
$$-[m_1(m_1 + \sqrt{u}) - \lambda^2]\varphi_1{}^{+} + (m_1{}^2 - u - \lambda^2)\varphi_3{}^{+}\}. \tag{11b}$$

Substituting (11a) in (7a), we find

$$J_\mu^{+} = M_{II}^{+}\gamma_5(i\hat{f} - \sqrt{u})\Gamma_\mu{}^{+}, \quad f = p_1 - k, \qquad f^2 = -u, \tag{12}$$
$$\Gamma_\mu{}^{+} = \gamma_\mu + i\alpha^{+}(\gamma_\mu\hat{k} - \hat{k}\gamma_\mu) + i\beta^{+}k_\mu + is^{-1}\gamma^{+}p_{2\mu},$$
$$M_{II}{}^{+} = A_2{}^{+}/2, \qquad \alpha^{+} = -A_5{}^{+}/2A_2{}^{+}, \tag{12a}$$
$$\beta^{+} = (2A_8{}^{+} - A_5{}^{+})/A_2{}^{+}, \qquad \gamma^{+} = 2sA_6{}^{+}/A_2{}^{+}. \tag{12b}$$

It is easy to see, with the help of (7) to (7b) and the identity $(i\hat{f} - \sqrt{u})i\hat{f} = -(i\hat{f} - \sqrt{u})\sqrt{u}$, that the total current (12) satisfies the transversality condition $k_\mu J_\mu{}^{+} = 0$. With this condition one can add to the polarization vector of the vecton ϵ an arbitrary vector directed along k and remove the condition $\epsilon k = 0$. Then the last term in (12a) is always asymptotically smaller than the remaining terms and can be neglected. After removal of this term the quantity Γ_μ^{+} depends only on variables which enter in the vecton vertex of the graph II (cf. the figure) and can be regarded as the vertex part of the graph. The general expression (12) then takes the form of a pole term corresponding to the indicated graph with a "reggeon" as an intermediate particle.

Setting $\lambda = 0$ in (12) and reversing the time (changing the order of the Dirac matrices), we obtain an expression for the amplitude for the photoproduction of pions quoted earlier by us.[6] [4)]

It can be shown with the help of the unitarity relations (4), in exactly the same manner as the case $\lambda = 0$, [6-8] that the amplitudes for processes I and II can be expressed in the following factorized form [only the vertices in (12) must be replaced in correspondence with the graphs I and III (cf. the Figure)]:

$$T_I{}^{+} = \bar{u}(p_2)\,\varepsilon_{2\mu}\varepsilon_{1\nu}J_{\mu\nu}{}^{+}u(p_1), \quad J_{\mu\nu}{}^{+} = M_I{}^{+}\Gamma_{2\mu}{}^{+}(i\hat{f} - \sqrt{u})\Gamma_{1\mu}{}^{+}, \tag{13}$$

$$T_{III}{}^{+} = \bar{u}(p_2)J^{+}u(p_1), \quad J^{+} = M_{III}{}^{+}\gamma_5(i\hat{f} - \sqrt{u})\gamma_5; \tag{14}$$

the expressions for the $\Gamma_{i\mu}$ are given by formulas (12a) and (12b) with k replaced by k_i, and the factors M_I, M_{II}, and M_{III} are connected by the relation

$$M_I M_{III} = M_{II}{}^2. \tag{15}$$

An expression for the photoproduction of vectons can be obtained from (13) by setting $\lambda_1 = 0$ in the vertex $\Gamma_{1\mu}$ and M_I^{+}.

4. CROSS SECTION AND POLARIZATION COEFFICIENTS

For the calculation of the cross section and the polarization coefficients we could use directly the

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2)All factors which depend on $l(\sqrt{u})$ are included in the residues $r_i(l)$. In virtue of the definition (6), the residue r_3 contains a factor λ/k_0 and reduces to zero for $\lambda \to 0$.

3)To find the asymptotic A_i^{+} we may set all F_i^{-} in (8b) equal to zero.

4)Formulas (9) and (11) of[6] contain a misprint: the sign in front of $m/\sqrt{u}$ in the parentheses must be changed in the expressions for λ and α_2/α_1.

factorized form of the amplitude (12). However, it is more convenient to apply the recipe given at the end of the second section and to find the amplitudes in the s channel in terms of the residues of the Regge poles. Performing the indicated transformations and dividing the main contribution by s, we obtain

$$
\begin{aligned}
f_1^s &= \langle 1/2;\ 1 \mid 1/2\rangle = \tau_1^+ + \tau_1^-,\\
f_2^s &= \langle 1/2;\ 1 \mid -1/2\rangle = \tau_2^+ + \tau_2^-,\\
f_3^s &= \langle -1/2;\ 1 \mid -1/2\rangle = -i(\tau_2^+ - \tau_2^-),\\
f_4^s &= \langle -1/2;\ 1 \mid 1/2\rangle = -i(\tau_1^+ - \tau_1^-),\\
f_5^s &= \langle 1/2;\ 0 \mid 1/2\rangle = \tau_3^+ + \tau_3^-,\\
f_6^s &= \langle 1/2;\ 0 \mid -1/2\rangle = -i(\tau_3^+ - \tau_3^-);
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\tau_1^+ &= -\sqrt{2}uA_5^+/8\pi, \qquad \tau_2^+ = \sqrt{2}\sqrt{-u}(m_1A_5^+ + A_2^+)/8\pi,\\
\tau_3^+ &= \sqrt{-u}\{sA_6^+ - \lambda^2A_5^+ - (m_1 - \sqrt{u})A_2^+\}/8\pi\lambda\\
&= \lambda(A_5^+ + A_8^+)/4\pi.
\end{aligned}
\tag{17}
$$

In the case of several Regge poles the expression for τ_i and hence also for f_i^s has the form of a sum of terms each corresponding to a pole.

Using (9) and (11), we rewrite (17) in the form

$$\tau_i^\pm = \frac{\rho_i^\pm(1 + \sigma e^{-i\pi l^\pm})}{\sin \pi l^\pm} s^{l^\pm}. \tag{18}$$

We shall not write down the general form of the connection between the residues ρ_i and the residues r_i (9), which is easily derived from (11) and (17). We give this connection only in the limit $\lambda = 0$ for a comparison with the results of an earlier paper:[6]

$$
\begin{aligned}
\rho_1^+ &= \frac{\sqrt{u}}{m + \sqrt{u}}(mr_1^+ - \sqrt{u}\,r_2^+),\\
\rho_2^+ &= \frac{\sqrt{u}}{m + \sqrt{u}}(\sqrt{u}\,r_1^+ - mr_2^+), \quad \rho_3^+ = 0,\\
m_1 &= m_2 = m.
\end{aligned}
\tag{19a}
$$

The residues r and r′ introduced earlier by us[7] are related to (18) by

$$
\begin{aligned}
\rho_1^+ + \rho_2^+ &= -\sqrt{u}\left(\frac{m - \sqrt{u}}{m + \sqrt{u}}\right)^{1/2} r'^+,\\
\rho_1^+ - \rho_2^+ &= \sqrt{u}\left(\frac{m - \sqrt{u}}{m + \sqrt{u}}\right)^{1/2} r^+, \quad m_1 = m_2 = m.
\end{aligned}
\tag{19b}
$$

In the physical region of the s channel u is negative and hence the quantities with the + and − signs are complex conjugates. Setting

$$\rho_i^\pm = |\rho_i| e^{\pm i\varphi_i}, \qquad l^\pm = \mu \pm i\nu, \tag{20}$$

we separate in (18) the modulus and the phase:[3]

$$
\begin{aligned}
\tau_i^\pm &= R_i e^{\pm\pi\nu/2} e^{-ia} e^{\pm ib},\\
R_i &= \frac{\sqrt{2}\,|\rho_i|\,s^\mu}{(\operatorname{ch}\pi\nu - \cos 2a)^{1/2}},\\
a &= \frac{\pi}{2}\left(\mu - \frac{1-\sigma}{2}\right),\\
b &= \nu \ln s + \varphi + \chi, \qquad \chi = \operatorname{th}\frac{\pi}{2}\nu \operatorname{ctg} a,
\end{aligned}
\tag{21}*
$$

where δ is the signature quantum number.

Using (21), we obtain an expression for the differential cross section:

$$
\begin{aligned}
I_0 &= \sum_{i=1}^{6} |f_i^s|^2 = 2\sum_{i=1}^{3}(|\tau_i^+| + |\tau_i^-|)^2\\
&= (R_1^2 + R_2^2 + R_3^2)\operatorname{ch}\pi\nu.
\end{aligned}
\tag{22}
$$

We give only the polarization coefficient corresponding to a transverse polarization of the final nucleon (the notation is taken over from the work of Moravcsik[9]):

$$I_0P_m = (R_1^2 + R_2^2 + R_3^2)\operatorname{sh}\pi\nu, \qquad P_m = \operatorname{th}\pi\nu. \tag{23}†$$

In the case of several Regge poles the expression (21) must be replaced by a sum over the corresponding poles. Then the cross section and the polarization coefficient of the transverse polarization of the nucleon take the form (m is the number of the pole)

$$
\begin{aligned}
I_0 = \sum_{k=1}^{3}\Big\{\sum_m (R_k^m)^2 \operatorname{ch}\pi\nu_m + 4\sum_{m>m'} R_k^m R_k^{m'}\big[e^{\pi(\nu_m+\nu_{m'})/2}\\
\times \cos(a_m - a_{m'} - b_m + b_{m'})\\
+ e^{-\pi(\nu_m+\nu_{m'})/2}\cos(a_m - a_{m'} + b_m - b_{m'})\big]\Big\},
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
I_0P_m = \sum_{k=1}^{3}\Big\{\sum_m (R_k^m)^2 \operatorname{sh}\pi\nu_m + 4\sum_{m>m'} R_k^m R_k^{m'}\big[e^{\pi(\nu_m+\nu_{m'})/2}\\
\times \cos(a_m - a_{m'} - b_m + b_{m'})\\
- e^{-\pi(\nu_m+\nu_{m'})/2}\cos(a_m - a_{m'} + b_m - b_{m'})\big]\Big\}.
\end{aligned}
\tag{25}
$$

We note that in the case of one pair of poles with different parity the cross section and the polarization coefficients do not oscillate with energy. Only the coefficients determining the polarization correlations oscillate. In the case of several pairs of poles all quantities oscillate.

5. RELATIONS BETWEEN THE CROSS SECTIONS OF DIFFERENT PROCESSES

The reactions of the form II (cf. the Figure) for the production of vector mesons on stable targets can be of the following types:

$$
\begin{aligned}
&1)\ \pi + N \to N + W, \qquad 2)\ \pi + N \to B + K^*,\\
&3)\ \bar{K} + N \to B + W, \qquad 4)\ \bar{K} + N \to N + \bar{K}^*,
\end{aligned}
\tag{26}
$$

where $W = \omega, \rho, \varphi$; $B = \Lambda, \Sigma$. The reactions for the photoproduction and scattering of vectons on nucleons have the form

*ch = cosh, th = tanh, ctg = cot.

†sh = sinh.

$$1)\ W+N\to W+N,\quad 2)\ \bar{K}^*+N\to\bar{K}^*+N,$$
$$3)\ \gamma+N\to B+K^*,\quad 4)\ \gamma+N\to W+N. \qquad (27)$$

In virtue of the unitarity condition, which leads to the factorization of the asymptotic forms of the amplitudes for the indicated processes (12), (13), and (14), there exist, in the case of a single pair (even and odd) of Regge poles, relations between the cross sections of processes (26) and (27) which, in analogy to the reactions with a photon,[6,8] have the following form:[5)]

$$\left(\frac{d\sigma}{d\Omega}\right)_{VV'}\left(\frac{d\sigma}{d\Omega}\right)_{MM'}=\left(\frac{d\sigma}{d\Omega}\right)_{VM'}\left(\frac{d\sigma}{d\Omega}\right)_{MV'}, \qquad (28)$$

where V, V′ = W, $\bar{K}^*$, γ; M = π, $\bar{K}$, η. The indices in (28) denote the bosons participating in the reaction. Analogous relations hold between the corresponding polarization coefficients.

There still exist two types of relations which have been considered in detail in the work of Gribov, Okun', and Pomeranchuk.[3] These are the relation between the direct and crossed reactions and the isotopic relations which arise from the fact that the Regge poles have a definite isotopic spin. The relations of the first type have been given in exhaustive fullness in [3] for the case of scalar (pseudoscalar) mesons. Replacing these by vector mesons, we can obtain from these relations the corresponding relations for our case. In the quoted paper of Gribov et al.[3] only several illustrations of the isotopic relations are presented. Therefore, we consider these relations in more detail.

Table I

Reaction cross section	$N(T=1/2)$	$\Delta(T=3/2)$
$\sigma(pK^-\to\rho^-\Sigma^+)=\sigma(n\bar{K}^0\to\rho^+\Sigma^-)$	0	9
$\sigma(pK^-\to\rho^0\Sigma^0)=\sigma(n\bar{K}^0\to\rho^0\Sigma^0)$	1	4
$\sigma(pK^0\to\rho^0\Sigma^+)=\sigma(p\bar{K}^0\to\rho^+\Sigma^0)$	2	2
$\sigma(pK^-\to\rho^+\Sigma^-)=\sigma(n\bar{K}^0\to\rho^-\Sigma^+)$	4	1

The isotopic relations can be classified according to the isotopic spin of the intermediate state ("reggeon") in the u channel. For a "reggeon" with the isotopic spin T = ½ (nucleon trajectory) we have the following relations:

$$\frac{\sigma(n\bar{K}^0\to\Sigma^0\omega)}{\sigma(nK^-\to\Sigma^-\omega)}=\frac{\sigma(pK^-\to\Sigma^0\omega)}{\sigma(p\bar{K}^0\to\Sigma^+\omega)}=\frac{1}{2}. \qquad (29)$$

The relations between the cross sections of processes which can go through two intermediate states are given in Tables I and II. Each column contains the ratios between the cross sections of processes going through an intermediate state with the isotopic spin indicated. This intermediate state is denoted by the particle having the same quantum numbers as the corresponding Regge pole. Besides the quoted relations, there are relations for the production of ρ mesons by π mesons and of K* mesons by K mesons, which can be obtained from Tables I and II of the paper of Gribov et al.[3] by changing π and K in the final state to ρ and K*. Comparing with the experimental ratio of the reaction cross sections for large s and finite u, one can determine which of the Regge poles is the main one.

Table II

Reaction cross section		$\Lambda(T=0)$	$\Sigma(T=1)$
$\sigma(p\pi^-\to K^{*0}\Sigma^0)=\sigma(n\pi^+\to K^{*+}\Sigma^0)$	$\sigma(\gamma n\to K^{*+}\Sigma^-)=\sigma(\gamma p\to K^{*0}\Sigma^+)$	0	2
$\sigma(p\pi^+\to K^{*+}\Sigma^+)=\sigma(n\pi^+\to K^{*0}\Sigma^+)=\sigma(n\pi^-\to K^{*0}\Sigma^-)=\sigma(p\pi^-\to K^{*+}\Sigma^-)$	$\sigma(\gamma p\to K^{*+}\Sigma^0)=\sigma(\gamma n\to K^{*0}\Sigma^0)$	1	1

In reactions involving the photon the isotopic spin conservation law is satisfied only in one of the vertices. The relations between the cross sections for this case have the following form (we indicate in parentheses the particle with the quantum numbers of the Regge pole):

$$\begin{aligned}\sigma(\gamma p\to\eta p)&=\sigma(\gamma n\to\eta n)\ (N),\\ \sigma(\gamma p\to\pi^+n)&=\sigma(\gamma n\to\pi^-p)\ (N,\Delta),\\ \sigma(\gamma p\to K^+\Lambda)&=\sigma(\gamma n\to K^0\Lambda)\ (\Lambda,\Sigma).\end{aligned} \qquad (30)$$

In the second column of Table II we give the ratios between cross sections having a different form depending on the isotopic spin of the "reggeon."

5)We note that the relations in[6,8] are actually written down for the probabilities, i.e., cross sections divided by the statistical weight. In (28) the initial and final states are identical on both sides of the equation, so that additional statistical weight factors are not required.

We note that the relations (28) hold only if there is a single pair (even and odd) of Regge poles. If there are several pairs of Regge poles, the relation (28) is no longer valid. The isotopic spin relations are preserved also in the case of several pairs of poles (or even cuts), if only they all have a definite isotopic spin. In this sense these relations are more fundamental.

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