SURFACE NUCLEON STRIPPING REACTIONS

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A simple expression has been obtained for the differential cross section for nucleon transfer reactions based on the model of peripheral collisions. Different angular distributions are obtained depending on the degree of "classicality" of the process. The available experimental data on transfer reactions involving heavy ions are in satisfactory agreement with the model of peripheral collisions. The formulas obtained can also be applied to other surface reactions.

IN reactions involving heavy ions it is usually assumed that nucleon transfer occurs in tangential collisions, since at smaller impact parameters fusion of nuclei occurs. This explains the existence of a sharp maximum in the angular distribution at the angle approximately corresponding to the Coulomb angle of scattering for a tangential trajectory. Experimental data show, however, that in addition to such a maximum there also exists an appreciable maximum at zero angle. In the case of transfer of many nucleons the "peripheral" maximum is often absent and the cross section falls off monotonically with increasing angle [1,2]. In connection with this a hypothesis is sometimes advanced that there exist at least two different mechanisms of the reaction [3], one of which is the "tunnel" mechanism of transfer in distant collisions discussed by Breit and Abel [4]. In actual fact the existence of two maxima, as well as the absence in certain cases of the "peripheral" maximum, is completely compatible with the "peripheral" mechanism of the reaction and, as is shown below, is a consequence of the wave properties of scattering. The classical picture arises as a result of the interference of a large number of waves, and, therefore, concepts on which the initial interpretation is based are valid only under the condition that the beam of tangential trajectories contains a sufficiently large number of orbital angular momenta. This is not always satisfied even in the case of heavy particles, and, therefore, appreciable deviations are observed from the trivial classical picture. Angular distributions of products of the transfer reaction have been evaluated in a number of papers [3,5], but these calculations are unnecessarily awkward and physically hard to see through. And yet a simple description of the transfer reaction is possible based essentially only on the assumption of the mechanism of "peripheral" nuclear collisions. Such an approach may also turn out to be useful in the case of other "surface" reactions.

For the sake of simplicity we assume that the spins of the initial and the final channels are small ($S < \Delta l$, cf., below). In this case the amplitude of the reaction can be written in the simple form

$$f(\vartheta) = \frac{1}{2ik}\sum (2l+1)\,\eta_l \exp(2i\delta_l)\,P_l(\cos\vartheta), \quad (1)$$

where η_l and δ_l are real. The quantity η_l determines the intensity of the reaction in the channel of orbital angular momentum l. Expression (1) is approximately valid also in the case when the direction of the orbital angular momentum (the plane of the reaction) is conserved in the reaction. In this case we must interpret l as the orbital angular momentum of the final pair of nuclei.

The hypothesis concerning the role played by peripheral conditions means that η_l has a maximum at a certain $l \approx l_0$ corresponding to the "tangential" trajectory. Near l_0 it is possible to write approximately

$$\eta_l = \eta_{l_0} q(l - l_0).$$

Below we consider the following two expressions for the function $q(x)$—the Gaussian form (G) and the exponential form (E):

$$q(x) = \exp(-\alpha^2 x^2) \qquad (G), \quad (2)$$

$$q(x) = \begin{cases} \exp(-\alpha x), & l > l_0 \\ 0, & l < l_0 \end{cases} \qquad (E). \quad (3)$$

The quantity $\Delta l = 1/\alpha$ defines the width of the packet with respect to l. In the significant region $l \approx l_0$ the phase δ_l may be represented in the form

$$\delta_l \approx \delta_l + {}^1/_2\theta(l - l_0), \quad (4)$$

where $\theta = 2(d\delta_l/dl)_{l=l_0}$ is the classical angle of deviation for the tangential trajectory [6] (not necessarily purely Coulomb) which, together with α, is regarded here as a parameter.

From the definition of a surface reaction mechanism it follows that

$$\Delta l / l_0 \ll 1. \tag{5}$$

We assume that $\Delta l \gg 1$, and write, taking (5) into account,

$$f(\vartheta) \approx \frac{l_0}{ik} \eta_{l_0} \exp[2i\delta_{l_0}] \int_{-\infty}^{+\infty} dl\, q(l - l_0) \times \exp[i\theta(l - l_0)]\, P_l(\cos\vartheta). \tag{6}$$

In the case $\vartheta \gg 1/l_0$ we can utilize the well known asymptotic representation for the Legendre polynomials. We then obtain from (6) the reaction amplitude in the form

$$(\vartheta) = -k^{-1}\sqrt{l_0/2\pi\sin\vartheta}\,\eta_{l_0} \times \exp[2i\delta_{l_0}]\{\exp[i(l_0 + {}^1\!/_2)\vartheta + i\pi/4]\, g(\theta + \vartheta) - \exp[-i(l_0 + {}^1\!/_2)\vartheta - i\pi/4]\, g(\theta - \vartheta)\}, \tag{7}$$

where g(t) is the Fourier component:

$$g(t) = \int_{-\infty}^{+\infty} q(x)\, e^{itx}\, dx = \begin{cases} \sqrt{\pi}\alpha^{-1}\exp(-t^2/4\alpha^2) & \text{(G)} \\ 1/(\alpha - it) & \text{(E)} \end{cases}. \tag{8}$$

The amplitude (7) oscillates strongly as a function of the angle ϑ and, therefore, in practice the cross section is defined by the square of the modulus of $f(\vartheta)$ averaged over the angle. In this case the oscillating term in $|f(\vartheta)|^2$ can be completely neglected, since on the basis of the inequality (5) its period (of the order of $\pi/2l_0$) is small compared to the range over which the function g(x) varies appreciably (of the order of α). As a result we obtain the average cross section in the form

$$\overline{\frac{d\sigma}{d\Omega}} = \overline{\sigma(\vartheta)} = \frac{l_0}{2k^2\alpha^2}\eta^2 \frac{1}{\sin\vartheta}\{|g(\theta - \vartheta)|^2 + |g(\theta + \vartheta)|^2\}. \tag{9}$$

Substituting (8) into formula (9) we obtain

$$\overline{\sigma(\vartheta)} = \frac{l_0}{2k^2\alpha^2\theta}\eta_{l_0}^2 \frac{\vartheta}{\sin\vartheta}\begin{cases} G_a(\vartheta/\theta) & \text{(G)} \\ \pi^{-1}E_a(\vartheta/\theta) & \text{(E)} \end{cases}, \tag{10}$$

where

$$G_a\left(\frac{\vartheta}{\theta}\right) = \frac{\theta}{\vartheta}\left\{\exp\left[-\frac{(\Delta l)^2}{2}(\theta - \vartheta)^2\right] + \exp\left[-\frac{(\Delta l)^2}{2}(\theta + \vartheta)^2\right]\right\} \tag{11}$$

$$E_a\left(\frac{\vartheta}{\theta}\right) = \frac{\theta}{\vartheta}\left\{\frac{1}{1 + (\Delta l)^2(\theta - \vartheta)^2} + \frac{1}{1 + (\Delta l)^2(\theta + \vartheta)^2}\right\}, \tag{12}$$

$$a = \theta\Delta l. \tag{13}$$

The expression (10) is not valid for small angles $\vartheta \lesssim 1/l_0$. The value for $\sigma(0°)$ is obtained directly from formula (6). For the variant G we obtain

$$\sigma(0°) = \frac{l_0^2}{k^2}\eta_{l_0}^2 \frac{\pi}{\alpha^2}\exp\left(-\frac{\theta^2}{2\alpha^2}\right), \tag{14}$$

while for the varient E we obtain

$$\sigma(0°) = \frac{l_0^2}{k^2}\eta_{l_0}^2 \frac{1}{\alpha^2 + \theta^2}. \tag{15}$$

Analytic expressions for the cross section for small angles $\vartheta \ll \theta$ can be obtained by utilizing the expression for $P_l(\cos\vartheta)$ in terms of the Bessel function which is valid under the condition $l \gg 1$, $\vartheta < 1$. For $l_0\vartheta \gg 1$ these expressions go over into (10). An essential characteristic feature of $\sigma(\vartheta)$ for $\vartheta < \theta$ is the existence of strong oscillations over intervals of the order of $1/l_0$. It is possible that for not very heavy particles these oscillations could be noticed.

In the significant region $\vartheta \lesssim 1$ we can replace in formula (10) $\sin\vartheta$ by ϑ, and then it can be seen from (10)–(12) that the form of the angular distribution for $\vartheta \approx 1$ is completely determined by the functions $G_a(\tau)$ or $E_a(\tau)$ (where $\tau = \vartheta/\theta$), which depend only on the one parameter a. For sufficiently large a the cross section (10) has a maximum at $\vartheta = \theta$. The shape of the maximum in the case of G and in the case of E is essentially different: for a Gaussian η_l the maximum is described by the Gaussian function, while in the exponential case it has the form of a power resonance function. The width of the maximum in both cases turns out to be of the order of $1/\Delta l$.

The graphs of the functions $G_a(\tau)$ and $E_a(\tau)$ are shown in Figs. 1 and 2 for different values of a. Both families of curves are similar to each other and to curves evaluated numerically in [3,5] on the basis of a specific model.

The maximum at $\vartheta \approx \theta$ ($\tau \approx 1$) disappears for certain $a = a_{cr}$, and in both cases $a_{cr} \approx 2$. Therefore, the condition for the existence of a maximum at $\vartheta \neq 0$ can be written as

$$\theta\Delta l > 2. \tag{16}$$

The inequality (16) reflects the uncertainty principle and is in fact the condition for the existence of a classical trajectory. If the condition (16) is not satisfied, then the averaged cross section for the reaction falls off monotonically from $\vartheta = 0$ at first as $1/\vartheta$, and for $\vartheta \gtrsim \theta$ like a Gaussian in the case of G, and approximately like $1/\vartheta^3$ in the case of E. The minimum in the angular distribution for $a > 2$ is associated with the factor $1/\sin\vartheta$. The latter depends relatively weakly on the angle, as

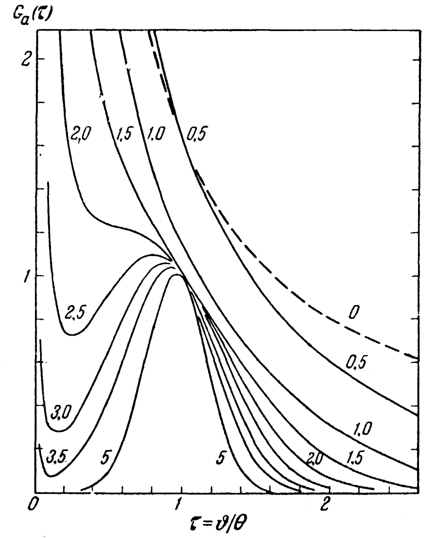

FIG. 1. Universal curves for the angular distribution of the products of a "peripheral" reaction in the case of the Gaussian distribution G. The numbers next to the curves are the values of the parameter a. The dotted curve is the limiting distribution (2τ) for a = 0, which corresponds to the distribution $1/\sin\vartheta$ for $\Delta l = 0$ (normalized to the point $\tau = 1$ for a = 0.5); in this case the contribution to the angular distribution is made by only one partial wave.

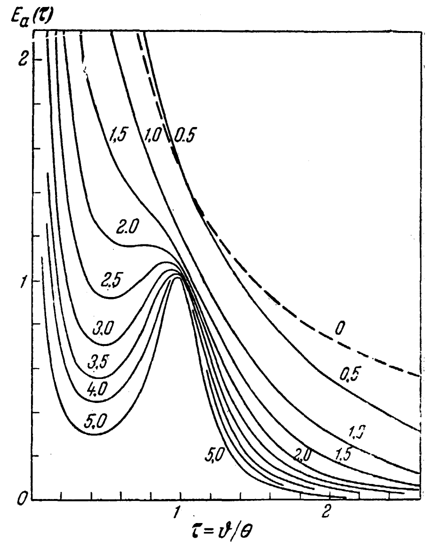

FIG. 2. The same as in Fig. 1, but for an asymmetric exponential distribution E. The values of the function $E_a(\tau)$ for a = 0 are normalized to the curve a = 0.5 at the point $\tau = 1$.

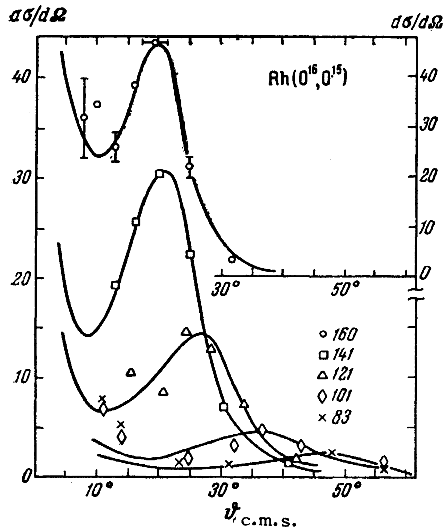

FIG. 3. Differential cross sections for the reaction Rh(O^{15}, O^{16}) (in mb/sr) at different energies (given in the diagram in MeV) in accordance with reference[1]. For the energy of 160 MeV the scale is given on the right. The curves are those calculated in the case of the variant E.

a result of which in the case of G when the shape of the angular distribution is fundamentally determined by the exponential factor the angle $\vartheta^{(\min)}$ corresponding to the minimum is very small. For $a \gg 1$ we have

$$\tau^{(min)} = \vartheta^{(min)} / \theta \to 1 / a^2$$

and in practice it turns out that $\vartheta^{(\min)} < \theta/4$ for $\sigma(\vartheta^{(\min)})/\sigma(\vartheta^{(\max)}) \lesssim 1/2$ (cf., Fig. 1). In contrast to this in the exponential case of E the quantity $\tau^{(\min)}$ is always greater than $1/3$. This difference between the cases of G and E is quite significant, and in making comparisons with experiment (cf. below) forces us to prefer the variant E, which is also more likely physically.

In those cases when the "peripheral" maximum is sufficiently pronounced in making comparisons with experimental data the parameters θ and Δl are, in fact, determined independently from the position of the center of the maximum and from its width. The normalization of the absolute value of the cross section determines $\eta_{l_0}^2$. Figure 3 shows experimental data on the cross section of the reaction Rh(O^{16}, O^{15}). The energies in the center of mass system are respectively equal to 138, 122, 105, 87, and 72 MeV. The theoretical curves evaluated for the case of the variant E are in satisfactory agreement with the experimental data. There is some discrepancy at small angles, but the accuracy of the experimental points in this range is not great[1], and it is not clear whether this disagreement is significant. The value of the parameter a turns out to be approximately the same in all cases ($a^2 \approx 16$), the angle θ has values of 18; 22; 27; 38 and 48°, from which the values of Δl are respectively equal to 13; 10.5; 8.5; 6.0 and 5.0. In the case of the variant G the minimum is obtained at $\vartheta \lesssim 5°$.

As should have been expected, $\Delta l/l_0$ turns out to be of the order of the ratio of the thickness of the diffuse layer to the sum of the nuclear radii.

If we assume that the thickness of the layer in which the reaction occurs is a certain quantity characteristic of a given reaction and depends only weakly on the energy of the incident particle, then Δl must be approximately proportional to the relative velocity of the particles at the moment of collision:

$$\Delta l \sim \sqrt{1 - B/E}, \qquad (17)$$

where B is the energy of the Coulomb barrier, while E is the energy in the center of mass system. Indeed, the width of the "peripheral" maximum in reactions involving the transfer of one nucleon increases with decreasing energy of the particle [1] and, as can be seen from the values given above, the variation of Δl with energy is in approximate agreement with (17).

Figure 4 shows similar data for other reactions involving the transfer of one nucleon. The theoretical curves are calculated for the variant E for the following values of the parameters: for the reaction $F^{19} \to F^{18} - \theta = 18°$, $a^2 = 12$ ($\Delta l = 11$); for the reaction $N^{14} \to N^{13} - \theta = 20°$, $a^2 = 16$ ($\Delta l = 11.5$); for the reaction $O^{16} \to O^{15} - \theta = 18°$, $a^2 = 16$ ($\Delta l = 13$); for the reaction $C^{12} \to C^{11} - \theta = 20°$, $a^2 = 5$ ($\Delta l = 6.4$). It can be seen that the "peripheral" maximum is absent in the reaction $C^{12} \to C^{11}$ which is characterized by the smallest value of Δl. There exist also other experimental data in which no rise towards $\vartheta = 0°$ has been observed [7]. This would provide evidence in favor of an expression of the Gaussian type for η_l.

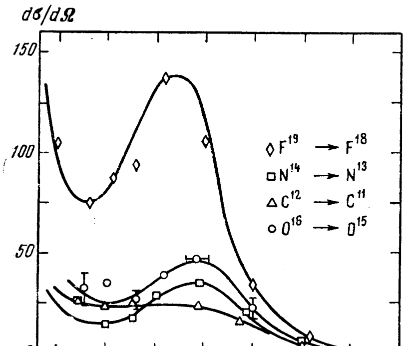

FIG. 4. Differential cross sections for reactions involving the transfer of one nucleon to the Rh nucleus (in mb/sr) for incident ion energy of 10 MeV per nucleon[1]. The curves are calculated using variant E.

In reactions involving the transfer of a large number of nucleons the "peripheral" maximum, as a rule, is absent [1], although this is not the general case [2]. The absence of a "peripheral" maximum can be explained by a relatively small value of a. Figure 5 shows experimental data for the reaction $Rh(O^{16}, F^{18})$, and also theoretical curves calculated using the variant E. In this case the variant G gives results which are closely the same, and for values of the parameters which differ little from those obtained above. The best agreement with experiment is obtained for $\theta = 10° - 12°$ and $\Delta l = 4-5$. For $\theta = 0$ agreement with experimental points shown in Fig. 1 becomes worse, but, apparently, the difference does not exceed experimental error.

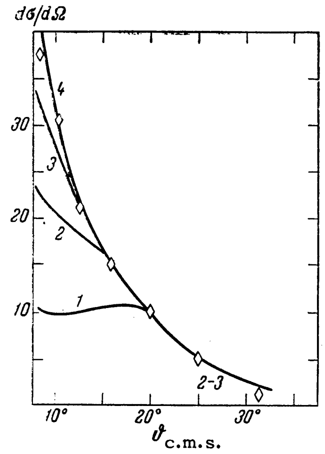

FIG. 5. Differential cross section (in mb/sr) for the reaction $Rh(O^{16}, F^{18})$ at an energy of 160 MeV in accordance with[1] (the degree of accuracy is not indicated in[1]). The theoretical curves (E) are normalized to the experimental value for $\vartheta = 0$, the parameter a was chosen from the condition of best agreement for $\vartheta > \theta$. Curve 1 – $\theta = 20°$, $a^2 = 4$ ($\Delta l = 6$); curve 2 – $\theta = 15°$, $a^2 = 2.5$ ($\Delta l = 6$); curve 3 – $\theta = 12.5°$, $a^2 = 1.0$ ($\Delta l = 5$); curve 4 – $\theta = 10°$, $a^2 = 0.5$ ($\Delta l = 4$).

The angle θ is always smaller than the Rutherford angle of scattering for a tangential trajectory. This may be the result of nuclear interaction at the instant of collision [3], and, therefore, the experimental determination of this quantity is of considerable interest. Here we must bear in mind that the cross section (10) does not depend on the sign of θ and a change in the scattering angle θ_{Ruth} by an amount Δ produces the same effect as a change by the amount $2\theta_{\text{Ruth}} - \Delta$. Therefore, for example, the value $\theta \lesssim 10°$ given above for the reaction $Rh(O^{16}, F^{18})$ at an energy of 160 MeV means essentially that the angle of deviation may vary from 25° to 45° ($\theta_{\text{Ruth}} = 35°$).

Formula (10) also allows us to determine the magnitude of the cross section for angles $\vartheta \sim 180°$. For the reaction $Rh(O^{16}, F^{18})$ we have $(d\sigma/d\Omega)_{20°} \approx 15$ mb/sr. On setting in formulas (10), (12) in the case E the angle $\theta \approx 10°$ we find that for Δl

≈ 4 the ratio $\sigma(160°)/\sigma(20°) = 1/60$ $(\approx(20°/60°)^2)$ i.e., $\sigma(160°) \approx 0.25$ mb/sr. This value agrees well with the estimate given in [1]. It is interesting to note that beginning with the angle 160° the cross section for the transfer reactions shows an appreciable rise towards 180°[1], as should be expected in accordance with formula (10) due to the presence in it of the factor $1/\sin\vartheta$.

From the analysis given above it can be seen that the model of peripheral collisions agrees with the available experimental data on angular distributions of products of transfer reactions involving heavy ions and that there is no basis for speaking of other reaction mechanisms.

The finite width of the "peripheral" maximum is not related to a scatter in the classical trajectories of the particles, but is rather due to quantum fluctuations for a single trajectory. The partial waves forming a classical trajectory are coherent, and the angular distribution is the result of interference of all the waves in an interval of the order of Δl around l_0. The scattering angle is not uniquely related to the impact parameter, and, therefore, it is not possible to recalculate the angular distribution to give the distribution of impact parameters [1,7] and in this way to separate out the "tunnel" transfers by fixing attention on particles scattered through definite angles. This, probably, explains the apparent disagreement with the "tunnel" theory for particles above the barrier, while for particles below the barrier, when the reaction indeed takes place at large impact parameters, good agreement with experiment is obtained [8]. The "tunnel" mechanism of nucleon transfer for large impact parameters is a limiting case of the mechanism of peripheral collisions and must go over into the latter continuously. For particles above the barrier the "tunnel" mechanism could be utilized for estimating the behavior of η_l for large values of l $(l > l_0 + \Delta l)$. From this point of view it is natural to expect that the exponential tail of the probability of the transfer reaction for large values of l (i.e., the value of Δl) will be smaller in the case of transfer of strongly bound particles or of several particles. Apparently, this is in agreement with available experimental data.

Since in the above discussion we have essentially not used any assumptions about the specific nature of the reaction, formula (10) is also applicable to other surface reactions, i.e., under the condition that the inequality (5) is satisfied. Such an approximation can, in particular, be useful for describing the reactions of stripping and of capture of deuterons, tritons, He^3 and α-particles by heavy nuclei. Experimental data for the reaction Bi(d, T) at $E_d = 20$ MeV [9] show that the angular distributions of the tritium nuclei are indeed similar to the curves shown in Figs. 1 and 2, and they could be made to agree with formula (10) for certain values of the parameters θ and Δl. This would indicate the "peripheral" nature of the stripping reactions in heavy nuclei. However, we must take into account that the value of l_0 in this case is appreciably smaller than in the reactions with heavy ions, and, possibly, it is necessary also to take into account the scatter in l and θ associated with the transfer of angular momentum.

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