QUANTUM THEORY OF THE PROPAGATION OF ELECTROMAGNETIC WAVES IN METALS IN A MAGNETIC FIELD

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Submitted to JETP editor November 24, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) 46, 1809-1819 (May, 1964)

The high-frequency conductivity of a metal in a quantized magnetic field is studied theoretically. It is shown that quantization of the electron states results in giant quantum oscillations of the dissipative part, due to Landau damping, of the conductivity tensor. The effect of these oscillations on the spectrum, damping, and polarization of a helicoidal electromagnetic wave in a metal with an arbitrary carrier dispersion law is considered. It is found that the damping and polarization of the helicoidal wave undergoes strong quantum oscillations upon variation of the magnetic field. The effect of electron scattering on the amplitude and waveform of the oscillations is investigated.

ELECTROMAGNETIC excitations of various types can exist in metals in the presence of a strong magnetic field. We have developed a theory for the propagation of electromagnetic waves under conditions of strong spatial dispersion, when the wavelength is much smaller than the mean free path of the conduction electrons[1,2].

In metals with unequal concentrations of the electrons and "holes" ($n_1 \neq n_2$), helicoidal electromagnetic fields exist with quadratic spectrum and elliptical polarization. The frequency ω of these excitations does not exceed the cyclotron frequency Ω of the conduction electron, and their wavelength is large compared with the dimensions of the electron orbits in the magnetic field. At sufficiently large carrier mean free path and not too small frequencies ω, the damping of the excitation is due to the spatial dispersion (Landau damping).

We have already considered[1,2] the classical limit $\hbar\Omega \ll T$ (T—temperature in energy units). In the present paper we investigate the quantum case $\hbar\Omega \gg T$. As is well known, the Landau damping is due to the electrons which move in phase with the wave. The quantization of the electron states causes such electrons to be located near the Fermi surface only for definite values of the magnetic field H. The Landau damping turns out to be in this case a periodic function of H. When ultrasound propagates through the metal, this leads to giant oscillations of the absorption coefficient as a function of the magnetic field[3,4]. Oscillations of a similar type should be observed also in the damping of the electromagnetic waves.

1. We consider a plane monochromatic electromagnetic wave with frequency ω and wave vector **k** propagating in an infinite metal at an angle φ to the direction of the constant magnetic field **H**. We choose a coordinate system such that the z axis is directed along **H** and the x axis is transverse to **k** and **H**.

The dispersion equation which determines the spectrum and the damping of the electromagnetic wave was obtained in[1,2] independently of the relation between $\hbar\Omega$ and T. Therefore in the quantum case $\hbar\Omega \gg T$ we can use the dispersion equation from[1,2] in which, however, the expression for the conductivity tensor $\sigma_{\alpha\beta}$ (**k**, ω, **H**) must take into account the quantization of the energy of the electrons in the magnetic field. The kinetic equation can in general not be used to calculate the tensor $\sigma_{\alpha\beta}$ in the limiting quantum case, and we must use the quantum equation for the electron density matrix. However, before we proceed to obtain a quantum formula for $\sigma_{\alpha\beta}$, we note the following. The spectrum of the electromagnetic wave is determined by the antihermitian part of the conductivity tensor. In the limiting cases when $kR \ll 1$ (R—characteristic Larmor radius of the electrons) all the electrons make the same contribution to this part of the tensor, the magnitude of which is not affected by the quantization. Therefore, if the hermitian part of the conductivity tensor $\sigma_{\alpha\beta}^{(h)}$ is small also in the quantum limit compared with the antihermitian part $\sigma_{\alpha\beta}^{(a)}$, then the spectrum of the electromagnetic excitations remains unchanged.

On the other hand, the damping of the wave, which as before is determined by the hermitian part $\sigma^{(h)}_{\alpha\beta}$, is essentially quantum.

In [2] we obtain a classical equation for the current density $\mathbf{j}(\mathbf{k}, \omega, \mathbf{H})$ in the case of a strong magnetic field and a strong spatial dispersion:

$$kR \ll 1 \ll |k_z l^*|, \qquad l^* = v/(\nu - i\omega), \tag{1.1}$$

where v—characteristic Fermi velocity and ν—frequency of collisions between the electrons and the scatterers.

In the case of a closed Fermi surface, the asymptotic expression for $\mathbf{j}$ is

$$\mathbf{j}(\mathbf{k}, \omega, \mathbf{H}) = \frac{Nec}{\mathrm{kH}}[\mathbf{kE}] + s_0\mathbf{h}(\mathbf{hE}) + \sum C\mathbf{w}(\mathbf{w}^*\mathbf{E}); \tag{1.2}*$$

here $\mathbf{E}$—electric field vector of the wave; $N = n_1 - n_2$; $\mathbf{h} = \mathbf{H}/H$.

The first term in (1.2) is the non-dissipative current, analogous to the Hall current in the static case. The quantity

$$s_0 = \frac{2e^2}{k_z^2}\sum(\nu - i\omega)\left|\frac{dn}{d\zeta}\right| \tag{1.3}$$

characterizes the conductivity which is longitudinal relative to $\mathbf{H}$; here ζ—chemical potential, and the symbol Σ denotes summation of analogous expressions for different groups of carriers.

The value of s_0 is determined by the contribution of all the electrons on the Fermi surface. Therefore in the quasi classical approximation

$$T \ll \hbar\Omega \ll \zeta \tag{1.4}$$

the quantization of the electron states leads only to the appearance in the collision frequency ν of small terms which oscillate with variation of the magnetic field (the Shubnikov-deHaas effect). In the present paper we are not interested in this effect.

The situation is different with the last term in (1.2), the magnitude of which is due to the electrons moving in phase with the wave (the Landau damping). The complex vector $\mathbf{w}$ characterizing the average velocity of these electrons can be represented in the form [2]

$$w_x = i\frac{ck}{eH}\zeta_x, \qquad w_y = -i\frac{ck}{eH}\zeta_y, \qquad w_z = \frac{\omega}{k_z}, \tag{1.5}$$

where ζ_x and ζ_y are quantities of the order of the Fermi energy ζ, which depend on the form of the Fermi surface and on the orientation of the vectors $\mathbf{k}$ and $\mathbf{H}$ relative to the crystal axes. In the particular case when the vector $\mathbf{H}$ is parallel to a symmetry axis of high order, or when the spectrum of the carriers is isotropic, we have

$$\zeta_x = \sin\varphi\,(S/2\pi m)_{\substack{\varepsilon=\zeta\\ p_z=p_{z0}}}, \quad \zeta_y = 0; \tag{1.6}$$

*$[\mathbf{kE}] = \mathbf{k}\times\mathbf{E}$, $(\mathbf{hE}) = \mathbf{h}\cdot\mathbf{E}$.

here $S(\epsilon, p_z)$—area of the intersection between the surface $\epsilon(\mathbf{p}) = \epsilon$ and the plane p_z = const; $\mathbf{p}$—quasi-momentum; $m = (2\pi)^{-1}\,\partial S/\partial\epsilon$—electron effective mass; p_{z0} is a solution of the equation

$$\bar{v}_z(\zeta, p_z) \equiv -(2\pi m)^{-1}\frac{\partial S(\zeta, p_z)}{\partial p_z} = \frac{\omega}{k_z} \equiv w_z, \tag{1.7}$$

where $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$—electron velocity, and the bar denotes averaging over the positions of the electron on the orbit with specified values of ϵ and p_z. In the case of a convex singly-connected Fermi surface, Eq. (1.7) has only one solution.

In accord with the previously obtained data [2], C is given by

$$C_{\mathrm{cl}} = \frac{e^2}{2\pi\hbar^3}\int d\varepsilon\,\frac{\partial f}{\partial\zeta}\int dp_z\,|m|\,\delta(\omega - k_z\bar{v}_z). \tag{1.8}$$

Quantization causes the electrons with $\bar{v}_z = w_z$ to be on the Fermi surface only for different values of the magnetic field. For other values of H, such electrons are far from the Fermi surface. Consequently the quantity C, together with the Landau damping, decreases abruptly.

We note that in the calculation of the Landau damping in the classical limit $\hbar\Omega \ll T$ we can neglect the collisions between the electrons and the scatterers ($\nu \to 0$). In the quantum case (1.4) the scattering of the electrons leads to the smearing and smoothing of the quantum oscillations, and its role is appreciable [5].

2. We now proceed to calculate the conductivity tensor when conditions (1.1) and (1.4) are satisfied. According to [6], in the absence of electron scattering $\sigma_{\alpha\beta}$ takes the form

$$\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H}) = \sum_{aa'}\frac{f_a - f_{a'}}{\hbar\omega_{aa'}}\int d^3r\,\exp(-i\mathbf{kr}) \times \frac{\langle a|j_\alpha(\mathbf{r})|a'\rangle\langle a'|j_\beta(0)|a\rangle}{\Delta + i(\omega_{a'a} - \omega)}. \tag{2.1}$$

Here $\langle a|j_\alpha(\mathbf{r})|a'\rangle$ is the matrix element of the operator of the α-component of the current density; a—complete set of quantum numbers characterizing the state of the electron in the magnetic field (magnetic quantum number n, quasi-momentum projection p_z and spin projection s_z on the direction of the magnetic field, and the coordinate X of the center of rotation); $\hbar\omega_{a'a} = \epsilon_{a'} - \epsilon_a$—energy difference; f_a—Fermi function of argument $(\epsilon_a - \zeta)/T$; the adiabatic parameter $\Delta \to 0$.

The energy eigenvalues ϵ_a depend on the quantum numbers n, p_z, and s_z, and are degenerate in X:

$$\varepsilon_{np_zs_z} = \varepsilon_{np_z} + s_z\mu H, \tag{2.2}$$

where $s_z = \pm 1$; $\mu = e\hbar/2m_0c$—Bohr magneton; m_0—mass of free electron; ϵ_{np_z} is determined in the quasi classical approximation by the equation[7]

$$S(\varepsilon, p_z) = 2\pi e\hbar H c^{-1} n. \tag{2.3}$$

It is easy to show that the expression in the numerator of (2.1) is hermitian. Therefore the hermitian part of the conductivity tensor, which is of interest to us, can be represented in the form

$$\sigma_{\alpha\beta}^{(h)} = \sum_{aa'} [f(\varepsilon_a) - f(\varepsilon_a + \hbar\omega)](\hbar\omega)^{-1}\pi\delta(\omega_{a'a} - \omega) \times \langle a | J_\alpha(\mathbf{k}) | a'\rangle \langle a' | j_\beta(0) | a\rangle, \tag{2.4}$$

where

$$\langle a | J_\alpha(\mathbf{k}) | a'\rangle = \int d^3r \langle a | j_\alpha(\mathbf{r}) | a'\rangle e^{-i\mathbf{kr}}. \tag{2.5}$$

To account for the scattering of the electrons in the relaxation time approximation, the δ-function with the energy conservation law in (2.4) should be "smeared out" by replacing the adiabatic parameter Δ with the collision frequency ν. Then[1])

$$\sigma_{\alpha\beta}^{(h)} = \sum_{aa'} \frac{f(\varepsilon_a) - f(\varepsilon_a + \hbar\omega)}{\hbar\omega} \frac{\nu}{\nu^2 + (\omega - \omega_{a'a})^2} \times \langle a | J_\alpha(\mathbf{k}) | a'\rangle \langle a' | j_\beta(0) | a\rangle. \tag{2.6}$$

In the case of isotropic electron scattering, and when conditions (1.1) and (1.4) are satisfied, formula (2.6) can be proved rigorously with the aid of a method developed previously[5].

Let us investigate formula (2.6). The different terms in the sums over n and n′ have different orders of magntiude. In the terms with $n' \neq n$ the energy difference $\omega_{aa'}$ is much larger than in the diagonal terms with $n' = n$. In fact, the matrix element $\langle a | J(k) | a'\rangle$ contains a factor $\delta_{p'_z, p_z + \hbar k_z}$, which expresses the conservation of the z-component of the momentum when the electron absorbs a quantum $\hbar\omega$ of the electromagnetic field. Therefore

$$\hbar\omega_{a'a} = \varepsilon_{n', p'_z, s_z} - \varepsilon_{n, p_z, s_z} \approx (n' - n)\hbar\Omega, \quad n' \neq n, \tag{2.7}$$

$$\hbar\omega_{a'a} = \varepsilon_{n, p_z + \hbar k_z, s_z} - \varepsilon_{n, p_z, s_z} \approx \hbar k_z v_{zn}, \quad n' = n;$$

$$v_{zn}(p_z) = \partial\varepsilon_{np_z s_z}/\partial p_z. \tag{2.8}$$

It follows hence that the terms with $n \neq n'$ for different n and n′ are quantities of the same order of magnitude. In other words, the sum of the terms with $n \neq n'$ is due to the contribution of all the electrons on the Fermi surface, and the corresponding part $\sigma_{\alpha\beta}^{(h)}$ does not contain any quantum effects (if we disregard the Shubnikov-deHaas oscillations). This part of $\sigma_{\alpha\beta}^{(h)}$ is small compared with the Landau damping and does not interest us here.

On the other hand, the magnitude of the sum of the terms with $n' = n$ is determined by those electrons for which $v_{zn} \approx w_z \ll v$ (in particular, this can be only one term with a minimum value $|v_{zn} - w_z|$). This is the part of $\sigma_{\alpha\beta}^{(h)}$ which yields the Landau damping in the quantum case.

It is possible to show with the aid of rather laborious transformations that

$$M_{\alpha\beta}(np_z, n'p'_z) \equiv \sum_{X'} \langle np_z X | J_\alpha(\mathbf{k}) | n'p'_z X'\rangle \times \langle n'p'_z X' | j_\beta(0) | np_z X\rangle = \frac{1}{V}\sum_{X'} \langle np_z X | J_\alpha(\mathbf{k}) | n'p'_z X'\rangle \times \langle n'p'_z X' | J_\beta(-\mathbf{k}) | np_z X\rangle, \tag{2.9}$$

where V—volume of the crystal (we did not write out the spin quantum number s_z, since the operator j is diagonal in the electron spin).

In the quasi-classical case (1.4) the quantities $M_{\alpha\beta}$ (2.9) are smooth functions of n and p_z. On the other hand, the difference in the Fermi functions in (2.6) differs from zero only when $|\varepsilon_a - \zeta| \ll \hbar\Omega$. The function $[\nu^2 + (\omega - \omega_{a'a})^2]^{-1}$, however, has a sharp maximum at $v_{zn} \approx w_z$. Therefore in the case (1.4) the quantities $M_{\alpha\beta}$ in formula (2.6) can be replaced by their limiting classical values[9], i.e.,

$$M_{\alpha\beta}(np_z, np'_z) \to \frac{1}{V} w_\alpha w_\beta^* \delta_{p'_z, p_z + \hbar k_z}. \tag{2.10}$$

Thus, the conductivity current density $j(\mathbf{k}, \omega, \mathbf{H})$ can in this case be represented in the form (1.2), where

$$C = \frac{e^2}{2\pi\hbar^3} \frac{1}{2} \sum_{s_z} \frac{\hbar e H}{c} \sum_n \int dp_z [f(\varepsilon_{np_z s_z}) - f(\varepsilon_{np_z s_z} + \hbar\omega)] \times \frac{1}{\pi\hbar\omega} \frac{\nu}{\nu^2 + [\omega - k_z v_{zn}(p_z)]^2}. \tag{2.11}$$

In the case when $\hbar\Omega \ll T$, the summation over n can be replaced by integration, and (2.11) coincides with its classical limit (1.8).

3. Let us consider first the idealized case of absolute zero temperature and no electron scattering ($\nu \to 0$). Integration with respect to p_z in (2.11) using the δ-function yields

$$C = \frac{\Omega}{2\omega} \frac{e^2}{2\pi\hbar^3} \left|\frac{m}{k_z}\right| \sum_{ns_z} \left|\frac{\partial v_{zn}}{\partial p_{z0}}\right|^{-1} [\theta(\zeta - \varepsilon_{np_{z0}s_z}) - \theta(\zeta - \hbar\omega - \varepsilon_{np_{z0}s_z})], \tag{3.1}$$

where $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$.

Formula (3.1) shows that the function $C(\mathbf{k}, \omega, \mathbf{H})$ differs from zero if the magnetic field is such that for one of the n's the following inequalities are satisfied:

$$\zeta - \hbar\omega < \varepsilon_{np_{z0}s_z} < \zeta. \tag{3.2}$$

1)We note that (2.6) differs from the formula for the high frequency conductivity of the metal in a quantizing magnetic field, obtained by Azbel′[8] with the aid of the quantum kinetic equation.

The conditions (3.2) signify that in the interval $[\zeta - \hbar\omega, \zeta]$ there are near the Fermi surface electrons that move in phase with the wave. If one of the inequalities of (3.2) is not satisfied, then there are no electrons with $v_{zn} = w_z$ in this interval, and C vanishes.

Thus, in the case in question C, viewed as a function of H, is an aggregate of narrow and high rectangular maxima, which are periodic in the reciprocal of the magnetic field. The distance between maxima $\Delta H \sim H\hbar\Omega/\zeta$, and their width is $\delta H \sim H\hbar\omega/\zeta$. These quantum oscillations of the hermitian part of the high-frequency conductivity are analogous to the giant oscillations for the absorption of ultrasound by metals in the magnetic field[3].

Outside the maxima, there is no Landau damping in this case (C = 0). In a metal with $N \neq 0$ there can propagate in this case an undamped helicoidal wave. Its spectrum and polarization remain the same as in the classical limit $\hbar\Omega \ll T$[1]:

$$\omega(\mathbf{k}) = ck\,|\mathbf{kH}|/4\pi\,|Ne|, \tag{3.3}$$

$$E_y = iE_x \sec\varphi, \qquad E_z = 0. \tag{3.4}$$

The electric field vector **E** in the helicoidal wave is elliptically polarized in the plane perpendicular to the constant magnetic field. The field component transverse to **k** revolves in a circle.

Comparison of (3.1) and (1.8) shows that the value of C at the maxima is $\Omega/2\omega \gg 1$ times larger than the limiting classical value C_{cl}. Therefore, if conditions (3.2) are satisfied, the hermitian part of the conductivity tensor becomes larger than the antihermitian part: $|\sigma^{(h)}_{\alpha\beta}| \gg |\sigma^{(a)}_{\alpha\beta}|$. Maxwell's equations for the spectrum, damping, and polarization of the wave take the form

$$\mathbf{E} - \mathbf{n}(\mathbf{nE}) = \frac{4\pi i\omega}{k^2c^2} \times \left\{\frac{Nec}{\mathbf{nH}}[\mathbf{nE}] + s_0\mathbf{h}(\mathbf{hE}) + C\mathbf{w}(\mathbf{w}^*\mathbf{E})\right\}; \quad n = \frac{\mathbf{k}}{k}. \tag{3.5}$$

In the right half of (3.5) we have left out the summation sign in the last term, since in the general case the positions of the maxima do not coincide for different groups of carriers (and for different p_{z0}.

To obtain the dispersion equation it is convenient to eliminate first the longitudinal component of the electric field $E_{\parallel}\mathbf{n} = \mathbf{E} - \mathbf{E}_{\perp}$ from the system (3.5). Taking the scalar product of (3.5) and **n**, we get

$$E_{\parallel} = -\frac{C(\mathbf{nw})(\mathbf{w}^*\mathbf{E}_{\perp}) + s_0(\mathbf{nh})(\mathbf{hE}_{\perp})}{C\,|\mathbf{nw}|^2 + s_0(\mathbf{nh})^2}. \tag{3.6}$$

After eliminating $E_{\parallel}$, the equation for $\mathbf{E}_{\perp}$ assumes the form

$$\mathbf{E}_{\perp} - i\frac{4\pi\omega Ne}{kc(\mathbf{kH})}[\mathbf{nE}_{\perp}] = i\frac{4\pi\omega s_0}{k^2c^2}C\mathbf{u}(\mathbf{u}^*\mathbf{E}_{\perp})[C\,|\mathbf{nw}|^2 + s_0n_z^2]^{-1};$$
$$\mathbf{u} = [\mathbf{n}[\mathbf{wh}]]. \tag{3.7}$$

Equating to zero the determinant of the system (3.7) we get the dispersion equation

$$\omega^2 = \left(\frac{kc\mathbf{kH}}{4\pi Ne}\right)^2\left\{1 - \frac{4\pi i\omega s_0}{k^2c^2}\frac{C\left(n_z^2|w_x|^2 + |w_y|^2\right)}{C\,|\mathbf{nw}|^2 + s_0n_z^2}\right\}. \tag{3.8}$$

It is easy to see that

$$|s_0/Cw_z^2| \sim kR \ll 1. \tag{3.9}$$

Therefore the relative damping of the wave is small. Neglecting $s_0m_z^2$ in the denominator of the second term in (3.8), we get

$$\omega^2 = \left(\frac{ck\mathbf{kH}}{4\pi Ne}\right)^2\left\{1 - \left(\frac{\omega\varkappa_D}{kck_z}\right)^2\frac{|w_x|^2n_z^2 + |w_y|^2}{|\mathbf{nw}|^2}\right\};$$
$$\varkappa_D^2 = 4\pi e^2\Sigma\,|dn/d\zeta| \tag{3.10}$$

which is the reciprocal of the square of the Debye-Hückel screening radius.

Let us investigate the influence of quantization on the propagation of the helicoidal wave in the general case of an anisotropic Fermi surface and arbitrary direction of the magnetic field, when $|w_x| \sim |w_y| \sim kRv$.

In the region of not too strong magnetic fields, satisfying the condition

$$H^2 \ll 4\pi n\zeta, \qquad H \ll 10^6\ \text{Oe}, \tag{3.11}$$

$w_z^2 \ll |w_y|^2$, and the spectrum of the wave is of the form

$$\omega = \frac{ck}{4\pi}\left|\frac{\mathbf{kH}}{Ne}\right|\left\{1 - \frac{H^2}{8\pi N^2\sin^2\varphi}\left(1 + \frac{\zeta_x^2}{\zeta_y^2}\right)\sum\left|\frac{dn}{d\zeta}\right|\right\}. \tag{3.12}$$

Thus, in spite of the large value of the Landau damping at the maxima of the giant oscillations of conductivity, there exists in the metal a weakly damped helicoidal electromagnetic wave. This wave can propagate because the dissipative current $C\mathbf{w}(\mathbf{w}\cdot\mathbf{E})$ connected with the Landau damping is missing if the vector of the electric field of the wave **E** is orthogonal to **w**. Therefore in the case (3.11) the helicoidal wave is elliptically polarized in the plane perpendicular to the vector with components $(w_x, w_y, 0)$. The transverse part of $\mathbf{E}_{\perp}$ again revolves in a circle. Consequently, giant quantum oscillations of the hermitian part of the conductivity lead to sharp periodic changes in the longitudinal component of the field $E_{\parallel}$ (i.e., polarization of the wave), and also to relatively small changes in the spectrum [second term in the right half of (3.12)].

We note that if the positions of the maxima of the giant oscillations $\sigma^{(h)}_{\alpha\beta}$ for different groups of carriers coincide, then the energy dissipation of the helicoidal wave

$$\mathbf{j}\mathbf{E}^* = \sum C \mid \mathbf{w}\mathbf{E} \mid^2$$

vanishes only when $E_x = E_y = 0$, and consequently there is no weakly damped helicoidal wave.

In the region of strong magnetic fields

$$H^2 \gg 4\pi n\zeta, \qquad H > 10^6 \text{ Oe}, \tag{3.13}$$

the quantity $|w_y^2|$ can be neglected compared with w_z^2 and

$$\omega^2 = \left(\frac{kc\,\mathbf{kH}}{4\pi Ne}\right)^2 \left\{1 - \frac{4\pi}{H^2}(\zeta_x^2 + \zeta_y^2) \sum \left|\frac{dn}{d\zeta}\right|\right\}. \tag{3.14}$$

In this case the electric field $\mathbf{E}$ is polarized in the same way as outside the maxima of the function C (3.4).

Let us discuss also the special case when $n_y w_y = 0$. This can occur if the spectrum of the electrons is isotropic or if the vector $\mathbf{H}$ is directed along a symmetry axis of order higher than 2 ($w_y = 0$), and also for $\mathbf{k} \parallel \mathbf{H}$ ($n_y = 0$). The spectrum of the electromagnetic wave is determined in this case by formula (3.14), independently of the value of the second term in (3.14). An undamped helicoidal wave exists in this case in the region of strong fields (3.13). In the region of weaker fields (3.11) this wave should have at the maxima of C an imaginary wave vector and will attenuate within a wavelength. The quantum effect consists in the latter case in the vanishing of the undamped wave (3.3) at the maxima of giant oscillations of the conductivity.

We note that if both vectors $\mathbf{k}$ and $\mathbf{H}$ are parallel to a high-order symmetry axis, then $\zeta_x = \zeta_y = 0$ and there is no Landau damping.

4. We consider further the case of finite temperatures T and assume that

$$\hbar\omega \ll T \ll \hbar\Omega. \tag{4.1}$$

Then

$$f(\varepsilon_a) - f(\varepsilon_a + \hbar\omega) \approx \hbar\omega\, \partial f(\varepsilon_a - \zeta)/\partial\zeta,$$

and for $\nu \to 0$ formula (2.11) leads to the following expression for C:

$$C = \frac{e^2}{2\pi\hbar^3} \sum_{s_z} \frac{\hbar eH}{8cT} \frac{\mu_0}{|k_z|} \operatorname{ch}^{-2}\left(\frac{\zeta - \varepsilon_{n_0 p_{z0} s_z}}{2T}\right); \tag{4.2*}$$

Here

$$\mu_0^{-1} = |\partial v_{zn_0}(p_{z0}) / \partial p_{z0}|,$$

n_0—magnetic quantum number n for which $|\zeta - \epsilon_{np_{z0}s_z}|$ is minimal.

Using (1.11) for C_{cl}, we can readily represent (4.2) in the form

$$C = C_{cl} M, \quad M = \frac{\hbar\Omega}{8T} \sum_{s_z} \operatorname{ch}^{-2}\left\{\frac{\varepsilon_{n_0 p_{z0} s_z} - \zeta}{2T}\right\}. \tag{4.3}$$

When H varies, the argument of the hyperbolic cosine in (4.3) $|\zeta - \epsilon_{n_0 p_{z0} s_z}|/2T$ oscillates from zero to a value $\hbar\Omega/4T$. In this case the hermitian part of the conductivity tensor

$$\sigma^{(h)}_{\alpha\beta} = \sum C w_\alpha w^*_\beta$$

experiences giant quantum oscillations, which are fully analogous to the oscillations of ultrasound absorption[3].

If the conductivity $C w_\alpha w^*_\beta$ connected with the Landau damping is the largest at the maxima of the giant oscillations, then the qualitative character of the results of the preceding section remains the same. We shall stop to discuss the case when this quantum part of the conductivity is much smaller than the classical part and can exert an influence only on the damping and polarization of a wave whose spectrum is determined by (3.3).

In the case of relatively strong magnetic fields, satisfying the condition

$$\sum \left(\frac{ck_{\parallel}\zeta_y}{eH}\right)^2 |k_z l^*| M \ll 1, \tag{4.4}$$

the quantity $|s_0|$ is large compared with the term $\Sigma C\, |\mathbf{n} \cdot \mathbf{w}|^2$, which can be neglected in (3.8). The damping of the helicoidal wave ω'' obtained directly from (3.8) is of the form

$$\frac{\omega''}{\omega} = \frac{k}{4\pi\hbar^3 N} \sum \frac{\mu_0}{\Omega}(\zeta_x^2 + \zeta_y^2) M(H) \sim \sum kRM(H), \tag{4.5}$$

and the polarization of the electric field $\mathbf{E}$ coincides with (3.4). Consequently, in the case of (4.4) the quantization of the electrons does not influence the spectrum or the polarization of the wave, and its damping is experienced by the quantum oscillations described by the function M(H) (4.3).

We note that in the special cases when $m_y w_y = 0$ (the vector $\mathbf{H}$ is directed along a high-order symmetry axis or along the vector $\mathbf{k}$), there is no need to satisfy the condition (4.4) in order for formula (4.5) to be valid.

*ch = cosh.

In the case when

$$\sum\left(\frac{ck_y\zeta_y}{eH}\right)^2|k_zl^*|M\gg 1, \qquad (4.6)$$

which is the inverse of (4.4), it is necessary to neglect in Maxwell's equations (3.5) the term with s_0, and to sum in the last term the analogous expressions for the different groups of carriers. The spectrum of the helicoidal wave does not change in this case, and the damping is determined by the formula

$$\frac{\omega''}{\omega}=\frac{k}{4\pi\hbar^3N}\left\{\sum\frac{\mu_0}{\Omega}\zeta_x^2M-\left(\sum\frac{\mu_0}{\Omega}\zeta_x\zeta_yM\right)^2\left(\sum\frac{\mu_0}{\Omega}\zeta_y^2M\right)^{-1}\right\}. \qquad (4.7)$$

The expression (4.7) differs from (4.5) in that a second term appears in the right side. This difference is quite important. Analysis shows that the relative damping of the wave is always of the order of kRM_{min}, i.e., the relative amplitude of the damping oscillations is of the order of unity, whereas the quantity $M(H)$ itself experiences giant oscillations. Here

$$M_{min}\sim\frac{\hbar\Omega}{2T}\exp\left(-\frac{\hbar\Omega}{2T}\right). \qquad (4.8)$$

This decrease in the amplitude of the quantum oscillations ω'' compared with (4.5) is due to the fact that in the maxima of the giant oscillations the main terms of (4.7), which contain M_{max}, cancel out. It must be emphasized that in these cases, when $n_yw_y = 0$, the damping of the wave ω'' is determined by formula (4.5) independently of the relation between $|s_0|$ and $C|\mathbf{n}\cdot\mathbf{w}|^2$. Therefore the anisotropy of the Fermi surface exerts an essential influence on the maximum value of the damping and the amplitude of its oscillations.

The transverse part of the field $\mathbf{E}_\perp$ is circularly polarized, while the longitudinal component $E_\parallel$ is determined from the condition of the orthogonality of $\mathbf{E}$ to the dissipative Landau current

$$\sum C\,(\mathbf{Ew})\,(\mathbf{w}^*\mathbf{n})=0. \qquad (4.9)$$

The quantity $E_\parallel$ experiences giant oscillations.

Thus, in the case of (4.6), the quantization of the electrons leads to an appreciable change not only in the damping but also in the polarization of the helicoidal wave.

5. Let us investigate the influence of scattering of electrons on the quantum oscillations of the helicoidal wave in the case (4.1). In the limit when $\nu\ll\omega\ll|k_zv|$, the main contribution to the Landau damping is made by electrons with $p_z\approx p_{z0}$. Therefore in formula (2.11) the energy ϵ_{np_z} and the velocity $v_{zn}(p_z)$ can be expanded in powers of $p_z - p_{z0}$ and only the linear terms of the expansion retained. As a result we can represent the expression for C in the form

$$C=C_{\text{cl}}\sum_{ns_z}M(n,s_z); \qquad (5.1)$$

$$M(n,s_z)=\frac{\hbar\Omega}{8\pi T}\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}\text{ch}^{-2}\left[\Delta(n,s_z)-\frac{x}{a}\right], \qquad (5.2)$$

$$\Delta(n,s_z)=(\zeta-\varepsilon_{np_{z0}s_z})/2T,\quad a=\frac{2Tk_z^2}{\mu_0\omega\nu},$$

$$x=\frac{(p_z-p_{z0})|k_z|}{\nu\mu_0}. \qquad (5.3)$$

We consider first the term with $n = n_0$, for which $|\Delta|$ is minimal. When H varies, the quantity $\Delta(n_0s_z)$ oscillates within the limits from $-\hbar\Omega/4T$ to $\hbar\Omega/4T$. If the amplitude of the oscillations Δ is much larger than $1/a$, then $M(n_0, s_z)$ experiences giant oscillations. The maximum of $M(n_0, s_z)$ is reached when $\Delta = 0$. In the case when $a \gg 1$

$$M_{max}=\hbar\Omega/8T. \qquad (5.4)$$

The minimum value of the function $M(n_0, s_z)$ is realized when $|\Delta| = \Delta_{max} \equiv \hbar\Omega/4T$:

$$M_{min}\approx\hbar\Omega/4\pi Ta\Delta_{max}^2=2\omega\nu\mu_0/\pi\hbar\Omega k_z^2. \qquad (5.5)$$

If the condition

$$a\Delta_{max}\approx k_z^2l|l^*|\hbar\Omega/\zeta\gg 1 \qquad (5.6)$$

is satisfied, it is sufficient to retain in the sum of (5.1) only the one term with $n = n_0$, and C oscillates together with $M(n_0, s_z)$.

At low frequencies $(\omega\ll\nu)$ we can set p_{z0} equal to zero; the expansion of ϵ_{np_z} contains then only the even powers of p_z and the expression for $M(n, s_z)$ differs from (5.2) in the fact that in the argument of the hyperbolic cosine x/a must be replaced by $(x/b)^2$, where $b^2 = 4T\,k_z^2/\mu_0\nu^2$. It can be shown that the inequality (5.6) leads to giant oscillations of the quantity C in this case, too.

Using the spectrum of the helicoidal wave (3.3) and expressing k in terms of ω and H, we can represent the condition (5.6) and the inequality $kR \ll 1$ in the form

$$\hbar\omega_0^2/mc^2\nu>1+\nu/\omega,\qquad \omega\lll\Omega^3(c/v\omega_0)^2, \qquad (5.7)$$

where $\omega_0 = (4\pi Ne^2/m)^{1/2}$ coincides in order of magnitude with the plasma frequency of the metal.

The inequalities (5.7) together with $\hbar\Omega \gg T$ determine the region of applicability of the obtained results. We note that for pure metals the first of the inequalities (5.7) is well satisfied when $\omega > 0.01\,\nu$.

In the case of low frequencies, when the condition (5.6) is replaced by its inverse, many n's are significant in the sum of (5.1). Simple calculations using the Poisson summation formula and the saddle point method lead to a result analogous to that obtained previously[5]:

$$C = C_{\text{cl}}\left\{1 + \frac{2\,|\,k_z\,|}{\pi\nu}\left(\frac{\hbar\Omega}{\mu_0}\right)^{1/2}\sum_{s=1}^{\infty}\frac{A_s}{\sqrt{s}}\cos\left(\frac{scS}{e\hbar H} - \pi s - \frac{\pi}{4}\right)\right\},$$

$$A_s = \left(\frac{2\pi^2 sT}{\hbar\Omega}\right)\operatorname{sh}^{-1}\left(\frac{2\pi^2 sT}{\hbar\Omega}\right). \quad (5.8)$$

In this case the relative amplitude of the quantum oscillations of the damping of the helicoidal wave is of the order of $(\omega_0/\nu)(\hbar\omega/mc^2)^{1/2} \ll 1$.

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Translated by J. G. Adashko
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