LARGE DEFORMATIONS OF NUCLEI IN THE ANISOTROPIC OSCILLATOR MODEL

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The ground state energy, the chemical potential and the magnitude of the gap in the energy spectrum of the even-even nuclei Kr^{86}, Sr^{88}, Sr^{90}, Ba^{136}, and Ce^{140} are calculated for large deformations on the basis of the anisotropic oscillator model.

THE energy characteristics of the ground states and of single particle excited states of deformed nuclei have been obtained in a number of papers[1,2]. However, the discussion in these papers was restricted to deformations not exceeding $\delta = 0.3$ and which in fact refer only to the equilibrium deformations of nuclei. It is of interest, for example in the study of the process of fission, to obtain the dependence of the energy characteristics on the deformation for considerably larger values of the deformation under conditions when the deformation can certainly not be regarded as an equilibrium deformation.

The deformation process is accompanied by a change in the self-consistent field and by a rearrangement of the single particle levels in the nucleus. However, the times characterizing the deformations of the nucleus as a whole are considerably larger than the times of single particle relaxations, and, therefore, the deformation process can be regarded as being adiabatic.[3] In other words, in the process of deformation the nucleus passes only through those states which are energetically the most favorable for each given value of the deformation, and in the single particle model this corresponds to the nucleons filling the lowest energy levels.

Thus, in order to evaluate the dependence of the energy of the ground state of the nucleus on the deformation it is necessary to know the behavior of the single particle levels under a deformation of the self-consistent field. At the same time one should take into account the effects of pairing, and for this it is necessary to solve the system of equations for the gap and for the chemical potential utilizing a system of single particle levels.

In order to evaluate the system of single particle levels for large deformations one can utilize the anisotropic oscillator model with terms of the type l^2 and $\mathbf{s}\cdot\mathbf{l}$[1]. But in such an approach the condition of self-consistency of the problem is violated. Thus, for example, the quadrupole moment of the nucleus obtained as the sum of single particle quadrupole moments is considerably larger than the value of the quadrupole moment given by the liquid drop model. At the same time it is clear that the value of the quadrupole moment is correctly given by the drop model since, on the one hand, the approximation of constant density of nuclear material is sufficiently good, and, on the other hand, the quadrupole moment is determined only by the mass distribution. Therefore, in order for the problem to be self consistent a subsidiary condition must be imposed on the system of single particle levels. In the present article we have chosen for such a condition the agreement of the quadrupole moment of the nucleus computed in accordance with the single particle model, with the quadrupole moment obtained from the drop model.

The following program of calculation follows from what we have just said: 1) selection of the single particle Hamiltonian; 2) making the problem self-consistent; 3) taking the effects of pairing into account. In practical calculations it turns out to be more convenient to combine items 2) and 3).

We choose the single particle Hamiltonian in the form

$$H = H_{\text{osc}} + C\mathbf{ls} + Dl^2, \qquad (1)$$

where H_{osc} is the Hamiltonian for the anisotropic oscillator, $\mathbf{l}$ is the operator for the orbital angular momentum, $\mathbf{s}$ is the spin operator, C and D are constants. We restrict ourselves to a consideration of axially-symmetric deformations. For the deformation parameter we take the square of the eccentricity $\epsilon = 1 - (a/b)^2$ where a is the semiminor and b is the semimajor axis of the ellipsoid (we consider ellipsoidally-deformed nuclei). The Nilsson parameter δ is related to ϵ by the following equation:

$$\delta = [1 - (1 - \varepsilon)^{1/2}]/(1 - \varepsilon)^{1/3}.$$

Taking into account the constancy of the nuclear volume (cf.[1]) we shall take for the dependence on

the deformation of the longitudinal $\omega_{\parallel}$ and of the transverse $\omega_{\perp}$ frequencies of the oscillator the expressions

$$\omega_{\parallel} = \omega_0 (1 - \varepsilon)^{1/3}, \qquad \omega_{\perp} = \omega_0 (1 - \varepsilon)^{-1/3},$$

where ω_0 coincides with Nilsson's quantity ω_0^0. The constants appearing in the Hamiltonian must be chosen so as to obtain agreement with experiment. Since within the range of deformations considered by us there are no experimental data, we shall take the values of the constants C and D from Nilsson's article [1].

When pairing occurs the system of nucleons (protons or neutrons) is described by the Hamiltonian [2,4]

$$H = \sum_{\nu\sigma} \varepsilon_\nu^0 a_{\nu\sigma}^+ a_{\nu\sigma} - G \sum_{\nu\nu'} a_{\nu+}^+ a_{\nu-}^+ a_{\nu'-} a_{\nu'+}, \qquad (2)$$

where ϵ_ν^0 are single-particle energy levels. In order to take into account the conditions of the conservation of the number of particles N and of the consistency of the problem with respect to the value of the quadrupole moment Q we add to the Hamiltonian (2) the operators for the number of particles $\hat{N} = \sum_{\nu\sigma} a_{\nu\sigma}^+ a_{\nu\sigma}$ and for the quadrupole moment $\hat{Q}$ multiplied by appropriate Lagrangian multipliers λ and μ:

$$\tilde{H} = \sum_{\nu\sigma} (\varepsilon_\nu^0 - \lambda) a_{\nu\sigma}^+ a_{\nu\sigma} - G \sum_{\nu\nu'} a_{\nu+}^+ a_{\nu-}^+ a_{\nu'-} a_{\nu'+} - \mu \hat{Q}.$$

After this the pairing is taken into account in the usual manner.

Following Belyaev [4], we obtain a system of equations from which we can find μ, λ, and Δ—the energy gap in the nucleon spectrum:

$$\frac{2}{G} = \sum_\nu \frac{1}{E_\nu},$$

$$\sum_\nu \left(1 - \frac{\varepsilon_\nu^0 - \mu q_\nu - \lambda}{E_\nu}\right) = N,$$

$$\sum_\nu q_\nu \left(1 - \frac{\varepsilon_\nu^0 - \mu q_\nu - \lambda}{E_\nu}\right) = Q_0;$$

$$E_\nu = [(\varepsilon_\nu^0 - \mu q_\nu - \lambda)^2 + \Delta^2]^{1/2}, \qquad (3)$$

where q_ν is the single-particle quadrupole moment, Q_0 is the quadrupole moment of the system of nucleons calculated in accordance with the drop model. The energy of the ground state of the Hamiltonian (2) is given in this case by the formula

$$E = \sum 2\varepsilon_\nu^0 V_\nu^2 - \Delta^2/G, \qquad 2V_\nu^2 = 1 - (\varepsilon_\nu^0 - \mu q_\nu - \lambda)/E_\nu.$$

The details of the subsequent steps in the calculations are as follows. We find the system of levels ϵ_ν^0 of the single-particle Hamiltonian (1). The levels are characterized by the quantum numbers $n_{\parallel}$ (the number of longitudinal oscillations), $n_{\perp}$ (the number of transverse oscillations), Λ (the component of the orbital angular momentum along the symmetry axis), Σ (the component of the spin along the symmetry axis). The terms $Cl \cdot s$, Dl^2 are taken into account in the first order perturbation theory approximation.

The solution of the system (3) was carried out separately for neutrons and for protons. The summation in (3) is taken over the 32 single-particle levels near the Fermi surface. The pair interaction constant G is selected by imposing the condition that Δ should coincide with the magnitude of the gap in the neutron and the proton spectra for zero deformations [5].

The single-particle quadrupole moment is calculated by means of the formula

$$q_\nu = 2\frac{\hbar}{m}\left(\frac{n_{\parallel} + 1/2}{\omega_{\parallel}} - \frac{n_{\perp} + 1}{2\omega_{\perp}}\right);$$

m is the nucleon mass. The quadrupole moment of the system of nucleons in the liquid drop model is given by the expression

$$Q_0 = \tfrac{2}{5} N R_0^2 \varepsilon / (1 - \varepsilon)^{1/3},$$

where R_0 is the radius of the undeformed nucleus. In our calculations we have assumed $R_0 = 1.2 \times 10^{-13} A^{1/3}$ cm. The quantity $\hbar\omega_0$ is set equal to $41A^{1/3}$ MeV.

The model we have chosen correctly describes

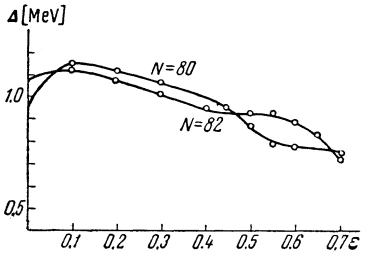

FIG. 1. Dependence of Δ on the deformation, for neutron systems of the nuclei Ba_{56}^{136}, Ce_{58}^{140} ($G = 0.018\ \hbar\omega_0$).

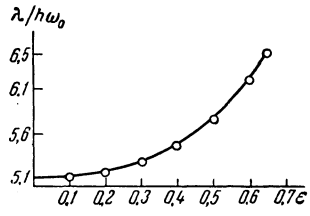

FIG. 2. Dependence on the deformation of the chemical potential of the neutron system of the nucleus Kr_{36}^{86}.

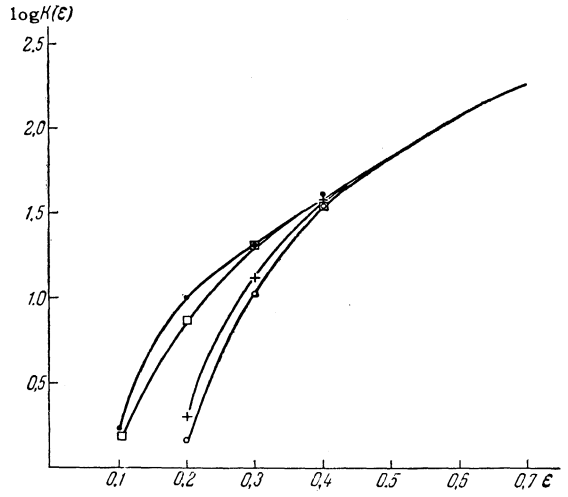

FIG. 3. Dependence of K(ϵ) on the deformation: ● – Kr_{36}^{86}; + – Ba_{56}^{136}; o – Ce_{58}^{140}; □ – Sr_{38}^{90}.

the general tendency in the variation of E, Δ, and λ with increasing deformation. Thus, for example, Δ decreases with increasing deformation, although for certain values of ϵ one observes an increase in Δ with increasing deformation, and this is a manifestation of the fact that the single-particle levels are crowded together near the Fermi surface for an appropriate value of the deformation.

The results of the calculations are presented in the table. It can be seen that the neutron system does not affect the proton system very much, and vice versa. It is characteristic that the dependence of Δ_n on the deformation is smoother in the case of closed shells (N = 50, N = 80) than for unfilled shells. A typical dependence of Δ on the deformation is shown in Fig. 1. The dependence on the deformation of the chemical potential λ for neutron and for proton svstems is of the same nature. Figure 2 shows this dependence for the neutron system of the nucleus Kr_{36}^{86}.

For $\epsilon \geq 0.4$ the energy of nuclear deformation can be written in the form

$$E(\varepsilon) = K(\varepsilon) S(\varepsilon) \sim K(\varepsilon)(1-\varepsilon)^{-1/6} A^{2/3},$$

where S(ϵ) is the surface of the deformed nucleus and K(ϵ) is the surface tension coefficient. For $\epsilon \geq 0.4$ the shell effects are unimportant, and the deformation energy is determined by the magnitude of the nuclear surface, with the surface tension K(ϵ) being independent of N and Z, and increasing with increasing deformation. This dependence is shown in Fig. 3.

The dependence of the effective energy levels ($\epsilon_{eff} = \epsilon_\nu^0 - \mu q_\nu$) on the deformation agrees well with Nilsson's system [1] of single-particle levels (for the range of deformations $\epsilon = 0 - 0.4$). For deformations corresponding to the fission barrier $\epsilon = 0.7$ ($\delta = 0.64$), the energy of nuclear deformation is of the order of hundreds of MeV, and therefore, in this range of deformations the model chosen cannot be regarded as satisfactory. Moreover, the anisotropic oscillator model in principle cannot yield a fission barrier.

The application of the results of these calculations to the determination of the magnitude of the deformation of the fission fragments immediately after the rupture of the neck yields $\epsilon \sim 0.25$. Calculations on the basis of the liquid drop model [3] lead to $\delta = 0.25$ ($\epsilon = 0.37$).

In conclusion the author expresses his gratitude

Nucleus	$G/\hbar\omega_0$	ε	0.00	0.10	0.20	0.30	0.40	0.50	0.55	0.60	0.65	0.70
Kr_{36}^{86}	0.020	Δ_n	1.24	1.23	1.21	1.19	1.16	1.12	1.08	1.07	1.04	0.95
	0.024	Δ_p	1.26	1.26	1.24	1.21	1.03	0.93	0.86	0.78	0.68	0.56
		E	0.0	3.5	20.0	41	82	152	200	270	340	430
Sr_{38}^{88}	0.020	Δ_n	1.23	1.21	1.19	1.16	1.13	1.07	1.03	0.98	0.91	0.83
	0.024	Δ_p	0.98	0.95	0.90	0.82	0.79	0.82	0.82	0.82	0.81	0.78
		E	0.0	3.1	15.3	40	81	150	209	260	340	440
Sr_{38}^{90}	0.020	Δ_n	1.37	1.36	1.35	1.32	1.28	1.24	1.21	1.18	0.98	0.87
	0.024	Δ_p	0.98	0.94	0.88	0.80	0.81	0.84	0.84	0.83	0.82	0.79
		E	0.0	3.8	17.0	43	86	158	208	270	350	460
Ba_{56}^{136}	0.018	Δ_n	0.96	1.15	1.12	1.07	1.00	0.87	0.79	0.78	0.77	0.75
	0.017	Δ_p	1.00	0.99	0.98	0.95	0.27	0.48	0.49	0.48	0.45	0.19
		E	0.0	−3.3	13.7	39	101	185	253	340	455	570
Ce_{58}^{140}	0.018	Δ_n	1.08	1.12	1.08	1.02	0.95	0.93	0.93	0.89	0.83	0.72
	0.017	Δ_p	0.98	0.97	0.95	0.91	0.63	0.63	0.61	0.58	0.52	0.25
		E	0.0	−0.5	15.6	51	98	182	253	345	470	590

For each nucleus the first line gives the neutron constant of the pair interaction G_n and the gap Δ_n [MeV] in the neutron spectrum, the second line gives the same quantities for the proton system, and the third line gives the energy of deformation of the nucleus E [MeV].

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