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FERROMAGNETISM IN SUPERCONDUCTING ALLOYS

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The possibility of coexistence of ferromagnetism and superconductivity in alloys containing paramagnetic impurities is demonstrated. Ordering of the impurity spins takes place because of exchange with the conduction electrons. The impurity "magnetization" s causes the appearance of a spatial electron spin density $\langle \sigma \rangle$. The exchange interaction as $\cdot \langle \sigma \rangle$ makes up for the loss in the kinetic energy of the electrons. The electron spin density is determined by the magnitude of the paramagnetic susceptibility. The latter differs from zero even for a superconductor at $T = 0$ owing to the effects of exchange scattering of electrons by paramagnetic impurities. Account of these effects leads to the appearance of a comparatively narrow mixed state region, separated from the pure superconducting phase by a line of transitions of the second kind, and separated from the ferromagnetic phase by a line of transitions of the first kind. The narrowness of the region of coexistence is associated with the smallness of the exchange interaction. Because of this effect the exchange scattering plays a smaller role in comparison with "sliding" of the Fermi surface, which hinders the formation of singlet Cooper pairs. The introduction of spin-orbit scattering by nonmagnetic impurities, whose concentration is usually larger, enables one to enlarge the region of coexistence and to explain the experimental results. The decrease of the effect of "sliding" of the Fermi surface is explained by nonconservation of the electron spin for spin-orbit interactions.

In recent years a considerable number of theoretical and experimental articles [1-6] have been devoted to the question of whether superconductivity and ferromagnetism can simultaneously exist in a given volume. This question was first considered on a phenomenological basis in the article by Ginzburg, [1] and subsequently by Zharkov,[2] who reached the conclusion that in typical ferromagnets the appearance of superconductivity is forbidden by the presence of large (in comparison with the critical fields of superconductors) internal fields in them. Up to the present time, superconductivity of ferromagnets has not been observed. Nevertheless, the appearance of ferromagnets with sufficiently low Curie temperature T_K does not exclude the possibility of the simultaneous coexistence of superconductivity and ferromagnetism.

At the same time, coexistence was observed earlier by Matthias et al. (see [4,5]) in alloys containing paramagnetic impurities. The ferromagnetic ordering of the ions in such a system comes about, apparently, owing to their indirect exchange interaction with the conduction electrons. In the normal state, T_K of the alloys is small together with the concentration of impurities, and for concentrations of order 1% the Curie temperature is comparable with T_c for superconductors.

Competition between both phenomena is possible in these systems. Actually, on the one hand, according to the BCS theory, [7] the electrons in a superconductor form pairs with zero resultant spin and a finite binding energy; at low temperatures this hampers their participation in an indirect interaction between ions; therefore the paramagnetic part of the electron susceptibility is diminished. On the other hand, the paramagnetic impurities strongly lower T_c of a superconductor. However, as will be shown below, the introduction of impurities leads, together with the decrease of T_c, to such modification of the pair wave function that the susceptibility of the electrons turns out to be finite at $T = 0$. The reason for this is that the spin of a pair is not conserved in the process of scattering by the spin of an impurity. The appearance of a finite electron susceptibility, in turn, facilitates ferromagnetic ordering in the superconducting phase at sufficiently low temperatures.

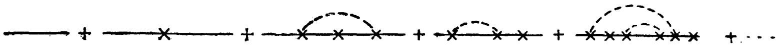

FIG. 1

1. FERROMAGNETISM OF THE IMPURITIES

In this section we shall discuss a model allowing us to satisfactorily investigate in turn the ferromagnetic ordering of the impurity spins both in the normal metal and in the superconductor. Basic to this model is the assumption that the impurity spins, being ordered, create a spatial electron spin density thanks to the exchange interaction with the conduction electrons. The loss in kinetic energy of the electrons is compensated by the average energy of the interaction of an impurity spin with the electron spin density, having the form $\mathbf{as} \cdot \langle \sigma \rangle$. Such a description of the impurity ferromagnetism was developed by Baltensperger,[3] and also in the article by Abrikosov and one of the authors;[8] it leads to a linear dependence of T_K on the impurity concentration in the region of small concentrations.

It was already noted above, however, that the lowering of the transition temperature and the finite magnitude of the paramagnetic susceptibility (at $T = 0°$) for superconducting alloys is due to the effects of scattering of electrons by impurities. In order to include these effects, the thermodynamical method with self-consistent field previously employed[8] is inadequate; it is necessary to take into account terms of next order in the magnitude of the exchange interaction. For this purpose we shall study the Green's function $\mathfrak{G}_{\alpha\beta}(x, x') = -\langle T(\psi_\alpha(x)\psi_\beta^+(x'))\rangle$ of the electrons in the presence of impurities. The Hamiltonian of the interaction with the impurities,

$$V = \sum_a \int \psi^+(x)\,(u_1(\mathbf{r} - \mathbf{r}_a) + u_2(\mathbf{r} - \mathbf{r}_a)\,\sigma \mathbf{S}_a)\,\psi(x)\,d^3\mathbf{x} \quad (1)$$

has exchange and nonexchange parts. (Here $\mathbf{S}_a$ is the spin of an impurity, the σ_α are the Pauli matrices: $\sigma_\alpha^2 = 1$.) Below we shall only need the average (over a random distribution of impurities) value of the Green's function $\mathfrak{G}_{\alpha\beta}(x - x')$.

The technique of averaging such quantities with the aid of diagrams is well-developed at the present time (see, for example, [9]). In the momentum representation the series for $\mathfrak{G}_{\alpha\beta}(x)$ is represented by Fig. 1, where the solid line corresponds to the Green's function $\mathfrak{G}^0_{\alpha\beta}(p, \omega_n) = \delta_{\alpha\beta}(i\omega_n - \xi(p))^{-1}$ of free electrons, the cross at the end of a dotted line corresponds to the Fourier component $\hat{v}(q) = u_1(q) + u_2(q)\sigma \cdot \mathbf{S}$ of the potential, and a single cross corresponds to the zero Fourier component $\hat{v}(0)$ (which corresponds to the Born approximation in a scattering calculation). As usual, integration is carried out over the momentum transfer $\mathbf{q}$. We have omitted diagrams containing intersecting dotted lines, since their contribution is small ($(p_0 l) \ll 1$, p_0 is the Fermi momentum, l is the mean free path).

In a normal metal the nonexchange part $nu_1(0)$ of a single cross is the usual renormalized chemical potential, and we shall omit it in what follows, but the exchange part $V^{-1}u_2(0)\sigma \cdot \sum_a \mathbf{S}_a$ leads to ferromagnetic "sliding" of the Fermi surfaces (assuming that $\sum_a \mathbf{S}_a$ is different from zero). The sum $\sum_a \mathbf{S}_a$, being proportional to the total number of impurities, is a classical vector which it is legitimate to replace by $nV\mathbf{s}$ ($\mathbf{s}$ is the average spin per impurity atom). Without going into the subsequent calculations, similar to those presented earlier,[10] we give the expression for the Fourier component of the Green's function with the spin directed along or opposite to the magnetization of the ions:

$$\mathfrak{G}_\pm(\mathbf{p}, \omega_n) = \Big[i\omega_n\Big(1 + \frac{1}{2|\omega_n|}(\tau_1^{-1} + \tau_2^{-1} \pm s\tau_{12}^{-1})\Big) - \xi \pm nu_2(0)\,s\Big]^{-1};$$

$$\tau_1^{-1} = \frac{nmp_0}{(2\pi)^2}\int |u_1(\theta)|^2\,do, \qquad \tau_2^{-1} = \frac{nmp_0 S^2}{(2\pi)^2}\int |u_2(\theta)|^2\,do,$$

$$\tau_{12}^{-1} = \frac{2nmp_0}{(2\pi)^2}\int u_1(\theta)\,u_2(\theta)\,do. \quad (2)$$

The electron spin density can be expressed in terms of the Green's function in the well-known manner:

$$\frac{1}{2}\langle\sigma\rangle = \frac{1}{2}T\sum_n \mathrm{Sp}\,\sigma \int \hat{\mathfrak{G}}(\mathbf{p}, \omega_n)\frac{d^3\mathbf{p}}{(2\pi)^3}. \quad (3)$$

Substituting here (2), we find

$$\frac{1}{2}\langle\sigma\rangle = nmp_0u_2(0)\,\mathbf{s}/2\pi^2 \quad (4)$$

(here the effects associated with the mean free path drop out completely). Finally, we shall evaluate the average values of the impurity spins in the homogeneous field $u_2(0)\langle\sigma\rangle$ of the magnetized electrons. For example,

$$\mathbf{s} = \frac{\mathrm{Sp}\,[\mathbf{S}\exp(-\beta u_2(0)\langle\sigma\rangle\mathbf{S})]}{\mathrm{Sp}\,[\exp(-\beta u_2(0)\langle\sigma\rangle\mathbf{S}].} \quad (5)$$

We note that in the previous formulas and everywhere farther on, we regard for simplicity the impurity's spin as a classical vector. From our point of view, further complication of the model would be an excessive refinement considering the semiphenomenological character of the model with self-consistent ("molecular") field. Therefore, strictly speaking, it is necessary to assume $S \gg 1$. In this approximation Eqs. (4) and (5) agree with those obtained earlier.[8] For the Curie temperature of the normal metal, we have

$$T_{K0} = nS^2 u_2(0)^2 mp_0/3\pi^2. \tag{6}$$

The scheme set forth can evidently be generalized to the case of a superconductor. In this connection, it is necessary to keep in mind that a superconductor is described by two Green's functions: the $\mathfrak{G}_{\alpha\beta}(x, x')$ already introduced and the function $\mathfrak{F}^+_{\alpha\beta}(x, x')$, which is defined as a thermodynamic mean of the form

$$\mathfrak{F}^+_{\alpha\beta}(x, x') = \langle T(\psi^+_\alpha(x)\psi^+_\beta(x'))\rangle.$$

Prior to averaging these functions satisfy the system of equations

$$\left(i\omega_n + \frac{1}{2m}\nabla^2 + \mu\right)\hat{\mathfrak{G}}_{\omega_n}(\mathbf{r}, \mathbf{r}') - \hat{V}\hat{\mathfrak{G}}_{\omega_n}(\mathbf{r}, \mathbf{r}') + \Delta(\mathbf{r})\hat{\mathfrak{F}}^+_{\omega_n}(\mathbf{r}, \mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}'),$$

$$\left(i\omega_n - \frac{1}{2m}\nabla^2 - \mu\right)\hat{\mathfrak{F}}^+_{\omega_n}(\mathbf{r}, \mathbf{r}') + \hat{V}^t\hat{\mathfrak{F}}^+_{\omega_n}(\mathbf{r}, \mathbf{r}') + \hat{\Delta}^+(\mathbf{r})\hat{\mathfrak{G}}_{\omega_n}(\mathbf{r}, \mathbf{r}') = 0, \tag{7}$$

where $V^t_{\alpha\beta} = V_{\beta\alpha}$, $\omega_n = \pi T(2n+1)$. The parameter $\hat{\Delta}^+(\mathbf{r})$ is defined by the condition

$$\hat{\Delta}^+_{\alpha\beta}(\mathbf{r}) = |\lambda| T \sum_{\omega_n} \mathfrak{F}^+_{\alpha\beta}(\mathbf{r}, \mathbf{r}'; \omega_n). \tag{8}$$

Similarly $\hat{\Delta}(\mathbf{r})$ can be expressed in terms of the function $\hat{\mathfrak{F}}_{\omega_n}(\mathbf{r}, \mathbf{r}')$ introduced below. $\hat{A}\hat{B}$ denotes everywhere the matrix product with respect to the spin variables.

In spite of the difference between a superconductor and a normal metal, which consists in the appearance of the $\mathfrak{F}$-function, one is able to reduce perturbation theory in both cases to formally identical form. For this purpose, we introduce two more functions:

$$\mathfrak{F}_{\alpha\beta}(x, x') = \langle T(\psi_\alpha(x)\psi_\beta(x'))\rangle,$$

$$\tilde{\mathfrak{G}}_{\alpha\beta}(x, x') = \mathfrak{G}_{\beta\alpha}(x', x),$$

which satisfy the system of equations [1)]

$$\left(i\omega_n + \frac{1}{2m}\nabla^2 + \mu\right)\hat{\mathfrak{F}}_{\omega_n}(\mathbf{r}, \mathbf{r}') - \hat{V}\hat{\mathfrak{F}}_{\omega_n}(\mathbf{r}, \mathbf{r}') - \hat{\Delta}(\mathbf{r})\hat{\tilde{\mathfrak{G}}}_{\omega_n}(\mathbf{r}, \mathbf{r}') = 0,$$

$$\left(-i\omega_n + \frac{1}{2m}\nabla^2 + \mu\right)\hat{\tilde{\mathfrak{G}}}_{\omega_n}(\mathbf{r}, \mathbf{r}') - \hat{V}^t\hat{\tilde{\mathfrak{G}}}_{\omega_n}(\mathbf{r}, \mathbf{r}') + \hat{\Delta}^+(\mathbf{r})\hat{\mathfrak{F}}_{\omega_n}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'). \tag{9}$$

It is possible to combine the four equations (7) and (9) into a single matrix equation

$$\begin{pmatrix} i\omega_n + \nabla^2/2m + \mu - \hat{V} & \hat{\Delta} \\ -\hat{\Delta}^{10} & -i\omega_n + \nabla^2/2m + \mu - \hat{V}^t \end{pmatrix} \tag{10}$$

$$\times \begin{pmatrix} \hat{\mathfrak{G}} & -\hat{\mathfrak{F}} \\ \hat{\mathfrak{F}}^+ & \hat{\tilde{\mathfrak{G}}} \end{pmatrix} = 1.$$

Expanding this equation in powers of the interaction and averaging over the positions of the impurities, we again obtain in the momentum representation the series shown in Fig. 1 for the matrix function of interest to us,

$$\begin{pmatrix} \hat{\mathfrak{G}} & -\hat{\mathfrak{F}} \\ \hat{\mathfrak{F}}^+ & \hat{\tilde{\mathfrak{G}}} \end{pmatrix}$$

Now the matrix

$$G^{(0)}(\mathbf{p}, \omega_n) = -(\omega_n^2 + \Delta^2 + \xi(\mathbf{p})^2)^{-1} \times \begin{pmatrix} i\omega_n + \xi & i\sigma_y\Delta \\ i\sigma_y\Delta & -i\omega_n + \xi \end{pmatrix}, \tag{10'}$$

plays the role of the zero-order Green's function, and a cross corresponds to the diagonal matrix

$$\begin{pmatrix} \hat{V}(\mathbf{p}, \mathbf{p}') & 0 \\ 0 & \hat{V}^t(\mathbf{p}, \mathbf{p}') \end{pmatrix}.$$

In conclusion of the present section, we consider for general orientation purposes an approximation in which the effects of scattering are negligible, i.e., we utilize only the first Born approximation in the exchange interaction. In this connection, only simple crosses are left in the diagrams for the Green's function of Fig. 1. Then, as is well-known,[10] at sufficiently high temperatures the paramagnetic impurities in this approximation do not have any effect on the superconductor properties. In particular, the temperature of the superconducting transition $T_c = T_{c0}$ remains unchanged; according to [10] this change is an effect of second order in the amplitude of the exchange interaction. The coexistence of ferromagnetism and superconductivity turns out to be impossible in such a system.

This circumstance is easiest of all to see at absolute zero temperature. Actually, in this case, because of the vanishing of the paramagnetic susceptibility of the superconductor, the "magneti-

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1)We remark that the symbol t implies also transposition with respect to spin and momentum variables, $V^t_{\alpha\beta}(\mathbf{p}, \mathbf{p}') = V_{\beta\alpha}(\mathbf{p}, \mathbf{p}')$.

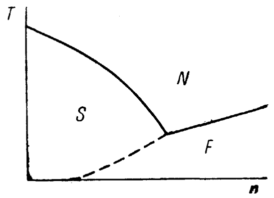

FIG. 2. Phase diagram (schematic) in which the effects of scattering in the ferromagnetic phase are neglected.

zation'' of the ions cannot lead to the appearance of an electron spin density. The phase diagram has the form shown in Fig. 2, where the dotted line denotes a first-order phase transition between the pure superconducting and the pure ferromagnetic phases. A picture of this kind was also obtained by Baltensperger, [3] from which he concluded that it is impossible to explain the coexistence of ferromagnetism and superconductivity on the basis of a model of the exchange interaction of electrons and impurity spins.

Meanwhile, as we have already indicated, in order to have ferromagnetic ordering in this model it is only necessary that the paramagnetic susceptibility of the superconductor remain finite at $T = 0$. The very fact that impurities are present is already sufficient for this purpose because, as we shall see, exchange scattering of electrons leads to a finite susceptibility of the order of the susceptibility of the normal metal for values of $\tau_2 T_{c0} \sim 1$. The spin-orbit interaction of electrons with nonmagnetic impurities [11] also leads to a finite susceptibility.

It is necessary to also note the formal necessity of calculation of the second Born approximation in the range of concentrations of interest to us. Actually, the region of possible coexistence of the phases is $T_K \sim T_{c0}$ or, according to Eq. (6), $nS^2u_2^2(0)mp_0/3\pi^2 \sim T_{c0}$. But $nS^2u_2^2(0)mp_0 \sim 1/\tau_2$, and therefore this is also the range of concentrations where the superconducting transition temperature T_c and the paramagnetic susceptibility χ_s change markedly in comparison with the corresponding values for a pure superconductor.

We formally obtained equations for the averaged Green's functions (Fig. 1) in the Born approximation, $nu_2(0)S\tau_2 \ll 1$. It should be assumed, however, that this limitation is unimportant for our results, and account of higher-order approximations reduces to the replacement of the Fourier components of the potential by the exact scattering amplitude.[2)] Actually, however, the exchange interaction is apparently several times ($\sim$3 to 5 times) weaker than the nonexchange interaction.

[2)]The correctness of this assertion was checked by Medvedev for nonmagnetic impurities (private communication).

Below we shall primarily be interested in the phase diagrams for the systems studied. For this purpose, we shall determine the lines in the (T, n) plane on which either the superconducting or the ferromagnetic characteristics vanish, i.e., the lines $\Delta = 0$ and $s = 0$. With an appropriate arrangement of the curves, these will be the lines of second order phase transitions. An incorrect arrangement indicates the presence of a first order phase transition.

2. T_K OF THE FERROMAGNETIC TRANSITION

In order to determine the Curie temperature T_K, it is sufficient to evaluate the spin density $\langle\sigma\rangle$ in the approximation linear in s. Substitution of the resulting expression for $\langle\sigma\rangle$ into Eq. (4) for s gives an equation for T_K in the approximation under consideration. The quantity $\langle\sigma\rangle$ is related by a simple relation to the linear-in-s correction $\hat{\mathfrak{G}}^{(1)}(p, \omega_n)$ to the averaged Green's function:

$$\langle\sigma\rangle = T\sum_{\omega_n}\int \mathrm{Sp}\,(\hat{\sigma}\hat{\mathfrak{G}}^{(1)}(\mathbf{p}, \omega_n))\frac{d^3\mathbf{p}}{(2\pi)^3}. \tag{11}$$

We recall that the function $\hat{\mathfrak{G}}(p, \omega_n)$ is the element standing in the upper left corner of the total matrix $\hat{G}$ [see Eq. (10)]. The linear-in-s correction to the latter is schematically represented by the series shown in Fig. 3. A point on this diagram denotes the matrix

$$nu_2(0)\,s\begin{pmatrix}\sigma & 0\\ 0 & \sigma^t\end{pmatrix}.$$

The impurity spin associated with a dotted line can obviously be regarded as free, i.e., $\overline{S_x^2} = \overline{S_y^2} = \overline{S_z^2} = 1/3$. The solid lines represent the already calculated Green's function of the superconductor in the paramagnetic state:[10]

$$\hat{G}^{(0)}(\mathbf{p}, \omega_n) = -(\tilde{\omega}_n^2 + \tilde{\Delta}_n^2 + \xi(\mathbf{p})^2)^{-1} \times \begin{pmatrix} i\tilde{\omega} + \xi(\mathbf{p}) & i\sigma_y\tilde{\Delta}_n \\ i\sigma_y\tilde{\Delta}_n & -i\tilde{\omega}_n + \xi(\mathbf{p})\end{pmatrix}, \tag{12}$$

where

$$\tilde{\omega}_n = \omega_n + \frac{1}{2}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)\frac{u_n}{\sqrt{u_n^2+1}},$$
$$\tilde{\Delta}_n = \Delta_n + \frac{1}{2}\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)\frac{1}{\sqrt{u_n^2+1}}, \tag{13}$$

$$\frac{\omega_n}{\Delta} = u_n\left(1 - \frac{1}{\tau_2\Delta}\frac{1}{\sqrt{u_n^2+1}}\right), \qquad u_n = \frac{\tilde{\omega}_n}{\tilde{\Delta}_n}. \tag{14}$$

It is convenient to carry out calculations for the vertex part $\hat{\Lambda}(p, \omega_n)$:

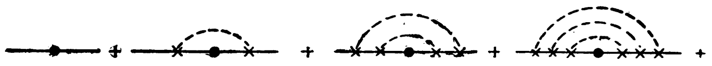

FIG. 3

$$\hat{G}^{(1)}(\mathbf{p}, \omega_n) = \hat{G}^{(0)}(\mathbf{p}, \omega_n)\,\hat{\Lambda}(\mathbf{p}, \omega_n)\,\hat{G}^{(0)}(\mathbf{p}, \omega_n). \quad (15)$$

Summing the series (Fig. 3) for $\hat{\Lambda}(\mathbf{p}, \omega_n)$, we obtain the following equation:

$$\hat{\Lambda}(\mathbf{p},\omega_n) = nu_2(0)\,\boldsymbol{\sigma}\mathbf{s} + \frac{n}{(2\pi)^3}\int \hat{V}(\mathbf{p}-\mathbf{p}')\hat{G}^{(0)}(\mathbf{p}', \omega_n)\hat{\Lambda}(\mathbf{p}', \omega_n)\,\hat{G}^{(0)}(\mathbf{p}',\omega_n) \times \hat{V}(\mathbf{p}'-\mathbf{p})\,d^3\mathbf{p}', \quad (16)$$

$$\hat{V}(\mathbf{p}-\mathbf{p}') = \begin{pmatrix} u_1(\mathbf{p}-\mathbf{p}') + u_2(\mathbf{p}-\mathbf{p}')\,\boldsymbol{\sigma}\mathbf{S} & 0 \\ 0 & u_1(\mathbf{p}'-\mathbf{p}) + \boldsymbol{\sigma}^t\,\mathbf{S}u_2(\mathbf{p}'-\mathbf{p}) \end{pmatrix}. \quad (17)$$

Since the integrand in Eq. (16) decreases rapidly on going away from the Fermi surface, one can regard $\hat{\Lambda}(\mathbf{p}, \omega_n)$ as a function $\hat{\Lambda}(\omega_n)$ that does not depend on the momentum. We shall seek a solution of Eq. (16) in the form (we assume that the direction of $\mathbf{s}$ is chosen as the z-axis)

$$\hat{\Lambda}(\omega_n) = \begin{pmatrix} \Lambda^{(1)}\sigma_z & \Lambda^{(2)}\sigma_x \\ -\Lambda^{(2)}\sigma_x & \Lambda^{(1)}\sigma_z \end{pmatrix}.$$

Isolating in (16) the terms with different spin dependence, we obtain the equations

$$\Lambda^{(1)}(\omega_n) = nu_2(0)\,s + \frac{1}{2}\left(\frac{1}{\tau_1} - \frac{1}{3\tau_2}\right)\frac{\Lambda^{(1)}(\omega_n) - iu_n\Lambda^{(2)}(\omega_n)}{\tilde{\Delta}_n(1+u_n^2)^{3/2}},$$

$$\Lambda^{(2)}(\omega_n) = \frac{1}{2}\left(\frac{1}{\tau_1} + \frac{1}{3\tau_2}\right)\frac{iu_n(\Lambda^{(1)}(\omega_n) - iu_n\Lambda^{(2)}(\omega_n))}{\tilde{\Delta}_n(1+u_n^2)^{3/2}}. \quad (18)$$

To evaluate of $\langle\sigma\rangle$ from (11), (15), and (18) it is first necessary, as usual, to carry out the summation over the frequency. It is more convenient, however, to add and subtract under the integral sign the corresponding expression for the normal metal. It is then legitimate to first integrate the resultant difference with respect to ξ, and the summation in the term which remains is then elementary. As a result, using (15) and the first equation of (18), we obtain a relation between $\langle\sigma\rangle$ and $\Lambda^{(1)}(\omega_n)$ in the following form:

$$\langle\sigma\rangle = -\frac{nmp_0u_2(0)}{\pi^2}\,s\left(1 - \pi T\sum_n(\Lambda^{(1)}(\omega_n) - nu_2(0)\,s) \times 2\left(\frac{1}{\tau_1} - \frac{1}{3\tau_2}\right)^{-1}\right).$$

Solving (18), we find the expression under the summation sign:

$$2\,(\Lambda^{(1)}(\omega_n) - nu_2(0)\,s)\left(\frac{1}{\tau_1} - \frac{1}{3\tau_2}\right)^{-1} = \left(\tilde{\Delta}_n(1+u_n^2)^{3/2} - \frac{1}{2}u_n^2\left(\frac{1}{\tau_1} + \frac{1}{3\tau_2}\right) - \frac{1}{2}\left(\frac{1}{\tau_1} - \frac{1}{3\tau_2}\right)\right)^{-1}.$$

Remembering the definition of $\tilde{\Delta}_n$, we finally obtain

$$\langle\sigma\rangle = -\frac{nmp_0u_2(0)}{\pi^2} \times s\left(1 - \pi T\sum_n\left(\Delta(1+u_n^2)^{3/2} - \frac{1}{3\tau_2}(1+2u_n^2)\right)^{-1}\right). \quad (19)$$

Here the expression inside the parentheses is obviously the relative susceptibility χ_s/χ_n of the electrons in a paramagnetic superconducting alloy. The nonexchange part of the scattering, as expected, drops out completely. The parameter $\Delta(T)$ appearing in this formula is determined from the equation

$$\ln T/T_{c0} = 2\pi T\sum_{n>0}\left(\frac{1}{\Delta\sqrt{u_n^2+1}} - \frac{1}{\omega_n}\right). \quad (20)$$

Substituting (19) into (4), it is easy to write an equation for T_K:

$$T_K = T_{K0}\left(1 - \pi T_K\sum_n\left(\Delta(T_K)\,(u_n^2(T_K)+1)^{3/2} - \frac{1}{3\tau_2}(2u_n^2(T_K)+1)\right)^{-1}\right). \quad (21)$$

For $\Delta = 0$ the sum vanishes and the right side of this equation changes into the Curie temperature T_{K0} of the normal alloy.

Further calculations can be carried out only in limiting cases. First let us turn to the case of small concentrations, $\tau_2\Delta \gg 1$. Expanding u_n and the right side of Eq. (21) in a series in powers of $(\tau_2\Delta)^{-1}$, we obtain in the linear approximation

$$T_K = T_{K0}\left(\frac{N_n(T_K)}{N} + \frac{\pi\Delta^2T_K}{3\tau_2}\sum_n\frac{7\omega_n^2 - \Delta^2}{(\omega_n^2+\Delta^2)^{7/2}}\right).$$

Since T_K is small together with the concentration of impurities ($T_K \ll \Delta$), one can neglect the first term in this formula with exponential accuracy [$\sim\exp(-\Delta/T_K)$], and in the second term one can replace the sum over ω_n by an integral

$$2\pi T\sum_n \to \int_{-\infty}^{+\infty}d\omega.$$

Then the expression for T_K takes the simple form:

$$T_K = \frac{\pi}{12}\frac{1}{\tau_2\Delta_{00}}T_{K0}. \quad (22)$$

In contrast to the normal metal, the dependence of T_K on the impurity concentration n is quadratic in this case. Thus, at sufficiently small concentrations and for $T < T_K$, ferromagnetic ordering in the superconducting phase becomes favored. The question of how far this mixed phase extends into the region of large concentrations can be solved, obviously, by finding the limit of existence of the superconducting state. In the case when the transition on this boundary is a second-order phase transition, it is sufficient for this purpose to find the line $\Delta = 0$.

3. DETERMINATION OF THE LINE $\Delta = 0$

In order to solve our problem it is sufficient to know the Green's function $\hat{G}(\mathbf{p}, \omega_n)$ in the approximation linear in Δ. It is convenient, however, to start from general expressions for the Green's functions, whose derivation is given in the Appendix. Using Eqs. (A4) and (A5), we obtain the equation for Δ:

$$\ln \frac{T}{T_{c0}} = \pi T \sum_n \left(\frac{1}{\sqrt{\Delta^2 + \eta_n^2}} - \frac{1}{|\omega_n|} \right), \tag{23}$$

where η_n is defined by Eq. (A7). In the limit $\Delta = 0$ these equations take a comparatively simple form:

$$\ln \frac{T}{T_{c0}} = \pi T \sum_n \left(\frac{\operatorname{sign} \operatorname{Re} \eta_n}{\eta_n} - \frac{1}{|\omega_n|} \right),$$

$$\omega_n + iI = \eta_n - \frac{\langle S_z^2 \rangle}{S^2} \frac{1}{\tau_2} \operatorname{sign} \operatorname{Re} \eta_n - \frac{\langle S_x^2 \rangle}{S^2} \frac{1}{\tau_2} \frac{2 \operatorname{Re} \eta_n}{\eta_n} \operatorname{sign} \operatorname{Re} \eta_n. \tag{24}$$

From the last equation it is more convenient to immediately determine the quantity of interest to us

$$\eta_n^{-1} = \frac{\left[\omega_n + \frac{\langle S_z^2 \rangle}{S^2} \frac{1}{\tau_2} + iI \right]}{\left[\left(\omega_n + \frac{1}{\tau_2} \right) \left(\omega_n + \frac{\langle S_z^2 \rangle}{S^2} \frac{1}{\tau_2} \right) + I^2 \right]}. \tag{25}$$

We chose the solution with sign Re η_n = sign ω_n in accordance with the limiting case $I = 0$ (see [10]). With the aid of (25), we finally obtain

$$\ln \frac{T_c}{T_{c0}} = 2\pi T_c \times \sum_{n>0} \left(\frac{\omega_n + \langle S_z^2 \rangle / S^2 \tau_2}{(\omega_n + 1/\tau_2)(\omega_n + \langle S_z^2 \rangle / S^2 \tau_2) + I^2} - \frac{1}{\omega_n} \right), \tag{26}$$

where $\omega_n = \pi T_c (2n + 1)$. Let us denote the temperature on the line $\Delta = 0$ by T_c. In the paramagnetic phase ($I = 0$) this equation reduces to one found earlier [see [10], formula (22)]. In this case T_c decreases monotonically with increase of concentration from the value T_{c0} of a pure superconductor to the intersection with the curve of T_K for a normal ferromagnet.

Let us denote the coordinates of the point of intersection of the curves T_c and T_K by T^x and n^x. We determine the behavior of these curves in the neighborhood of (T^x, n^x) for $T < T^x$. Simultaneously solving Eqs. (21) and (26), we arrive at the following equation for the determination of T^x:

$$\ln \frac{T_{c0}}{T^x} = 6\chi \sum_{n=0}^{\infty} \frac{1}{(2n+1)(2n+1+3\chi)}, \tag{27}$$

where n^x is obtained from the relation

$$T^x = n^x m p_0 S^2 u_2^2(0) / 3\pi^2, \text{ and } \chi = \overline{|u_2(\theta)|^2} / u_2^2(0)$$

characterizes the anisotropy of the scattering.

The results of a numerical solution of Eq. (27) are shown in Fig. 4. For isotropic scattering ($\chi = 1$), $T^x/T_{c0} = 1/4e$. We confine our attention below, to an investigation of precisely this case. s and Δ are small near the intersection point, and one can carry out a power series expansion with respect to them. We shall not dwell on the simple but tedious calculations, and we present only the results:

$$\frac{T_K - T^x}{T^x} \approx 13 \frac{n - n^x}{n^x}, \qquad \frac{T_c - T^x}{T^x} \approx (1 + 2.2\zeta) \frac{n - n^x}{n^x}.$$

(Here ζ is the Born parameter: $\zeta = (n u_2(0) S \tau_2)^{-1}$.)

It is obvious hence that the curves T_K and T_c have positive slopes, with the T_K curve almost vertical (independent of ζ), and with the curve T_c (for $\zeta \ll 1$) close to the continuation of T_{K0}. Such an arrangement of the curves indicates that there is no region of coexistence of ferromagnetism and superconductivity near the intersection point. The transition between them is accomplished by a first-order phase transition. We note that failure of the Born approximation ($\zeta \sim 1$) brings the T_c and T_K curves together,

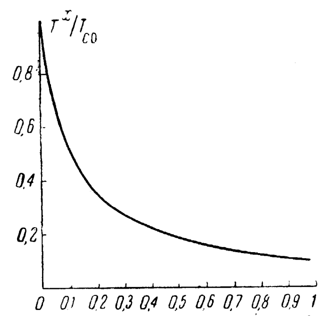

FIG. 4. The dependence of the temperature T^x, the point of intersection of the T_{K0} and T_{c0} curves, on the parameter χ which characterizes the anisotropy of the scattering.

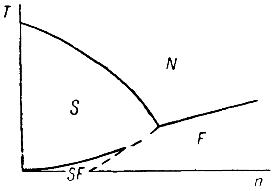

FIG. 5. Phase diagram (schematic) with account of exchange scattering.

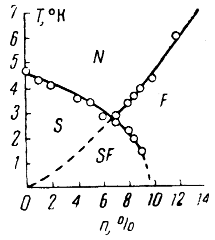

FIG. 6. Phase diagram for $Y_{1-x}Gd_xO_2$ obtained by Matthias et al.[4]

but reversal of their relative position ($T_K > T_c$) is hardly possible (for this the strong inequality $\zeta \gg 1$ must be fulfilled).

A similar investigation in the region of large concentrations at low temperatures indicates that the condition $\zeta > 1.5$ is necessary in order for the boundary of the ferromagnetic and mixed superconducting and ferromagnetic regions to be a line of second-order phase transitions. Therefore, as follows from the analysis presented, the most probable type of phase diagrams must turn out to be the diagram shown in Fig. 5. Here, the region of coexistence is strongly shifted into the region of small concentrations and temperatures, and its dimensions are rather sensitive to the nature of the interaction of an electron with a paramagnetic impurity.

A phase diagram of a different type (see Fig. 6) was observed in the experiments of Matthias and coworkers [4] with impurities of $GdOs_2$ in Y. It is necessary, of course, to mention that, owing to the Meissner effect, Matthias et al. [4] were not able to register the Curie temperature, T_K, in the superconducting phase ($T_K < T_c$) with the aid of magnetic measurements. Nevertheless, it is clear from the diagram mentioned that the T_c curve does not undergo any significant changes upon transition from the paramagnetic phase to the ferromagnetic phase. The same, apparently, also applies to the T_K curve for transition from the normal to the superconducting phase. The reason one is not able to describe such behavior of the T_c and T_K curves within the framework of the model considered above is as follows. Upon transition from the paramagnetic phase to the ferromagnetic phase, a term $\sim \mathbf{I}\cdot\boldsymbol{\sigma}$ is included in the Hamiltonian and this term, leading to "sliding" of the electron Fermi surfaces, almost completely suppresses the superconductivity ($\zeta \ll 1$) in the region of concentrations $n \sim n^x$. On the other hand, scattering processes, which induce transitions between the Fermi surfaces and, by the same token, hinder their "sliding," are usually several times weaker and cannot lead to restoration of superconductivity in the system.

4. THE ROLE OF THE SPIN-ORBIT INTERACTION

Now let us consider in more detail the role of spin-orbit scattering in the phenomena under investigation. We have already mentioned that the spin-orbit interaction, according to [11], increases the paramagnetic susceptibility of the electrons. In the limit of large concentrations, $l_{s.o} \ll \xi_0$, the electron susceptibility in a superconductor becomes equal to the susceptibility of the normal metal, i.e., in this case T_K must not undergo changes on going through the point (T^x, n^x). In the paramagnetic phase, the spin-orbit interaction does not have any effect on the thermodynamics of a superconductor; in particular, there is no effect on T_{c0}. In the ferromagnetic phase, the situation is different. Since in spin-orbit scattering, as in exchange scattering, the electron spin is no longer conserved, "sliding" of the Fermi surfaces does not turn out to exert such a destructive action on the Cooper effect. Therefore, one can anticipate that, for $l_{s.o} \ll \xi_0$, the dependence of T_c on the concentration of paramagnetic impurities will remain the same as in the paramagnetic phase.

The importance of accounting for these effects is due to the fact that paramagnetic alloys usually contain a considerable number of nonmagnetic impurities (inhomogeneities or boundaries may also play the role of impurities); the spin-orbit interaction, which in heavy metals is comparable with the exchange interaction, may be very important, since the effect of the impurities is proportional to their concentration. Such a situation apparently also occurs in the experiments of Matthias et al.[4] Below we shall study the effect of the spin-orbit interaction on T_c of a ferromagnetic superconductor.

The amplitude of the impurity scattering associated with the spin-orbit interaction has the form*

$$V_{so}(\mathbf{p}, \mathbf{p}') = iu_{so}(\mathbf{p}, \mathbf{p}')\,[\mathbf{n}\mathbf{n}']\,\boldsymbol{\sigma}, \qquad \mathbf{n} = \mathbf{p}/|\mathbf{p}|.$$

*$[\mathbf{n}\mathbf{n}'] = \mathbf{n}\times\mathbf{n}'$.

Repeating the derivation of the general equations for this case, we arrive at (see footnote 1) the previous system of equations (A3) in which, however, quantities marked with a bar are defined differently, namely:

$$\bar{\varphi}(\mathbf{p}) = n_{so}\int (u_1(\mathbf{p}-\mathbf{p}') + i\,[\mathbf{n}\mathbf{n}']\,\boldsymbol{\sigma}\,u_{so}(\mathbf{p}-\mathbf{p}')) \times \varphi(\mathbf{p}')\,(u_1(\mathbf{p}'-\mathbf{p}) + i\,[\mathbf{n}'\mathbf{n}]\,\boldsymbol{\sigma}u_{so}(\mathbf{p}'-\mathbf{p}))\frac{d^3p'}{(2\pi^3)}. \qquad (28)$$

Since we are interested in the qualitative aspect of the effect, we keep the exchange interaction only in terms of the form $\mathbf{I} = nu_2(0)\mathbf{s}$, having taken advantage of its relative smallness in comparison with the nonexchange interaction in real metals ($\zeta \sim 1/5$). In order to determine T_c it is sufficient to know the $\mathfrak{F}$-function in the linear (with respect to Δ) approximation, which can be immediately determined from Eq. (A3):

$$\mathfrak{F} = \mathfrak{G}'(\Delta + \bar{\mathfrak{F}})\,\tilde{\mathfrak{G}}', \qquad (29)$$

where $\tilde{\mathfrak{G}}'$ is the Green's function, averaged over impurities, of the normal metal

$$\mathfrak{G}' = (i\omega_n - \xi(\mathbf{p}) - I\sigma_z - \bar{\mathfrak{G}})^{-1},$$

and $\tilde{\mathfrak{G}}'$ is obtained from $\mathfrak{G}'$ by change of sign of the frequency ω_n and of the field I.

It is easy to determine $\bar{\mathfrak{G}}'$ with the aid of Eq. (28), and the final expression for the $\mathfrak{G}'$-function has the form

$$\mathfrak{G}' = (i\omega_n\eta_n - \xi(\mathbf{p}) - I\sigma_z)^{-1}, \qquad (30)$$

$$\eta = 1 + \frac{1}{2|\omega_n|}\left(\frac{1}{\tau_1} + \frac{1}{\tau_{so}}\right),$$

$$\frac{1}{\tau_{so}} = \frac{n_{so}mp_0}{(2\pi)^2}\int |u_{so}(\theta)|^2 \sin^2\theta\, do. \qquad (31)$$

We shall solve Eq. (29) for small values of I, which corresponds to the neighborhood of the intersection point (T^x, n^x). Correct to terms of second order in I, one can write the following expansion for $\bar{\mathfrak{F}}$:

$$\bar{\mathfrak{F}} = \Lambda_0 + \Lambda_0'(\mathbf{In})^2 + \Lambda_\sigma(\boldsymbol{\sigma}\mathbf{I}) + \Lambda_n(\boldsymbol{\sigma}\mathbf{n})(\mathbf{In}). \qquad (32)$$

Substituting this expression into (29) and, in addition, expanding $\mathfrak{G}'$ and $\tilde{\mathfrak{G}}'$ in powers of I, we obtain the following expression for $\mathfrak{F}$ to the approximation used:

$$\mathfrak{F} = \mathfrak{G}'\tilde{\mathfrak{G}}'(1 + (\mathfrak{G}'^2 + \tilde{\mathfrak{G}}'^2 - \mathfrak{G}'^2\tilde{\mathfrak{G}}'^2)I^2)(\Delta + \Lambda_0) + \Lambda_0'(\overline{\mathbf{In}})^2 + \Lambda_\sigma(\boldsymbol{\sigma}\mathbf{I}) + \Lambda_n(\overline{\boldsymbol{\sigma}\mathbf{n}})(\mathbf{In}) + \mathfrak{G}'\tilde{\mathfrak{G}}'(\mathfrak{G}' - \tilde{\mathfrak{G}}')\{((\Delta + \Lambda_0)\mathbf{I}\boldsymbol{\sigma} + \Lambda_\sigma I^2 + \Lambda_n(\mathbf{In})^2) + \text{terms odd in } \xi. \qquad (33)$$

We did not begin to write out the terms odd in ξ because we are actually not interested in the $\mathfrak{F}$-function itself, but in its integrated value (with respect to momentum). Namely

$$\int_{-\infty}^{+\infty}\mathfrak{F}(\mathbf{p})\frac{d\xi}{\pi} = \frac{1}{|\omega_n|\eta_n}(\Delta + \Lambda_0 + \Lambda_0'(\mathbf{In})^2 + \Lambda_\sigma(\boldsymbol{\sigma}\mathbf{I}) + \Lambda_n(\boldsymbol{\sigma}\mathbf{n})(\mathbf{In})) - \frac{1}{|\omega_n|^3\eta_n^3}(\Delta + \Lambda_0)I^2 + \frac{i\,\mathrm{sign}\,\omega_n}{(\omega_n\eta_n)^2}((\Delta + \Lambda_0)\,\boldsymbol{\sigma}\mathbf{I} + \Lambda_\sigma I^2 + \Lambda_n(\mathbf{In})^2). \qquad (34)$$

It is convenient to introduce a new system of notation:

$$I(\omega\eta)^{-2} \to I, \quad \Lambda_0'(\omega\eta)^2 \to \Lambda_0', \quad \Lambda_n(\omega\eta) \to \Lambda_n, \quad \Lambda_\sigma(\omega\eta) \to \Lambda_\sigma; \qquad (35)$$

then in terms of the new notation

$$\int_{-\infty}^{+\infty}\mathfrak{F}(p)\frac{d\xi}{\pi} = \frac{1}{|\omega_n|\eta}((\Delta + \Lambda_0) - (\Delta + \Lambda_0 - i\Lambda_\sigma) \times I^2 + (\Lambda_0' + i\Lambda_n)(\mathbf{In})^2 + i(\Delta + \Lambda_0 - i\Lambda_\sigma)\,\boldsymbol{\sigma}\mathbf{I} + \Lambda_n(\boldsymbol{\sigma}\mathbf{n})(\mathbf{In})). \qquad (36)$$

With the aid of Eqs. (29) and (36), we obtain, after rather lengthy calculations, the following system of equations:

$$|\omega|\eta\Lambda_0 = \frac{1}{2}\left(\frac{1}{\tau_1} + \frac{1}{\tau_{so}}\right)(\Delta + \Lambda_0 - (\Delta + \Lambda_0 - i\Lambda_\sigma)I^2 + (\Lambda_0' + i\Lambda_n)I^2),$$

$$|\omega|\eta\Lambda_0' = \frac{1}{2}\left(\left(\frac{1}{\tau_1} + \frac{1}{\tau_{so}}\right) - 3\left(\frac{1}{\tau_1'} + \frac{1}{\tau_{so}'}\right)\right)(\Lambda_0' + i\Lambda_n),$$

$$|\omega|\eta\Lambda_\sigma = \frac{1}{2}\left(\frac{1}{\tau_1'} - \frac{1}{\tau_{so}'}\right)\Lambda_n + \frac{i}{2\tau_1}(\Delta + \Lambda_0 - i\Lambda_\sigma),$$

$$|\omega|\eta\Lambda_n = \frac{1}{2}\left(\left(\frac{1}{\tau_1} - \frac{1}{\tau_{so}}\right) - 3\left(\frac{1}{\tau_1'} - \frac{1}{\tau_{so}'}\right)\right)\Lambda_n - \frac{i}{2\tau_{so}}(\Delta + \Lambda_0 - i\Lambda_\sigma), \qquad (37)$$

where

$$\frac{1}{\tau_1'} = \frac{nmp_0}{8\pi^2}\int |u_1(\theta)|^2\sin^2\theta\, do,$$

$$\frac{1}{\tau_{so}'} = \frac{n_{so}mp_0}{8\pi^2}\int |u_{so}(\theta)|^2\sin^4\theta\, do.$$

In order to solve this system of equations, we shall use the fact that the spin-orbit interaction is always much smaller than the ordinary interaction ($\tau_1 \ll \tau_{s.0}$). In addition, since one can expect a significant effect in the region of concentrations $\tau_{s.0}T_{c0} \lesssim 1$, in order to solve the last three equations we shall assume that the condition $\tau_1\omega_n \ll 1$

is fulfilled (values of $n \sim 1$ are essential in the terms associated with I). In the first nonvanishing approximation with respect to τ_1/τ_{s0}, we have

$$|\omega|(\Delta + \Lambda_0) = |\omega|\eta\Delta$$
$$-\frac{1}{2\tau_1} I^2 (\Delta + \Lambda_0 - i\Lambda_\sigma - (\Lambda_0' + i\Lambda_n)),$$
$$-\frac{3}{2}\frac{1}{\tau_1'}(\Lambda_0' + i\Lambda_n) + i|\omega|\eta\Lambda_n = 0,$$
$$i\left(|\omega| + \frac{1}{2\tau_{so}}\right)(\Delta + \Lambda_0 - i\Lambda_\sigma) = i(\Delta + \Lambda_0)|\omega|\eta + \frac{1}{2\tau_1'}\Lambda_n,$$
$$-\frac{3}{2}\frac{1}{\tau_1'}\Lambda_n = \frac{i}{2\tau_{so}}(\Delta + \Lambda_0 - i\Lambda_\sigma); \quad |\omega|\eta = 1/2\tau_1. \tag{38}$$

First let us solve the last two equations:

$$\Delta + \Lambda_0 - i\Lambda_\sigma = \frac{(\Delta + \Lambda_0)|\omega|\eta}{|\omega| + 2/3\tau_{so}},$$
$$\Lambda_n = -\frac{1}{3}\frac{\tau_1'}{\tau_{so}}\frac{i(\Delta + \Lambda_0)|\omega|\eta}{|\omega| + 2/3\tau_{so}}. \tag{39}$$

From the second, we now find

$$\Lambda_0' + i\Lambda_n = \frac{2}{9}\frac{(\tau_1')^2}{\tau_{so}}\frac{(\Delta + \Lambda_0)(\omega\eta)^2}{|\omega| + 2/3\tau_{so}}. \tag{40}$$

Substituting (39) and (40) into the first equation, we finally determine $\Delta + \Lambda_0$:

$$\Delta + \Lambda_0 = \Delta\eta\left(1 - \frac{(\omega\eta)^2}{|\omega|(|\omega| + 2/3\tau_{so})} I^2\right). \tag{41}$$

On the right sides of Eqs. (34)–(40), one can put $\Delta + \Lambda_0 = \Delta\eta$, since these quantities themselves appear in $\mathfrak{F}$ with the coefficient I:

$$\Delta + \Lambda_0 - i\Lambda_\sigma = \frac{|\omega|\eta}{|\omega| + 2/3\tau_{so}}\Delta\eta,$$
$$\Lambda_n = -\frac{1}{3}\frac{\tau_1'}{\tau_{so}}\frac{i|\omega|\eta}{|\omega| + 2/3\tau_{so}}\Delta\eta,$$
$$\Lambda_0' + i\Lambda_n = \frac{2}{9}\frac{(\tau_1')^2}{\tau_{so}}\frac{(\omega\eta)^2}{|\omega| + 2/3\tau_{so}}\Delta\eta. \tag{42}$$

Now, making the substitution $I \to I(\omega\eta)^2$ [see (35)] and substituting (41), (42) into (36), we obtain

$$\int_{-\infty}^{+\infty}\mathfrak{F}(p)\frac{d\xi}{\pi} = \frac{\Delta}{|\omega_n|}\left(1 - \frac{I^2}{|\omega_n|(|\omega_n| + 2/3\tau_{so})}\right). \tag{43}$$

The contribution from $\Delta + \Lambda_0 - i\Lambda_\sigma$ and Λ_n need not be written out because the first gives zero upon summation over ω_n, and the second gives zero upon integration over the angles. The I-dependent contribution of $\Delta + \Lambda_0$ to $\tau_{s.0}/\tau_1$ is many times larger than the contribution from $\Lambda_0' + i\Lambda_n$.

Thus, the equation for T_c has the form

$$\ln\frac{T_c}{T_{c0}} = -2\pi T_c\sum_{n=0}^{\infty}\frac{1}{\omega_n^2(\omega_n + 2/3\tau_{so})} I^2 \tag{44}$$

or

$$\frac{1}{2}\frac{T_c - T_{c0}}{T_{c0}} = -\left(\frac{I}{\pi T_{c0}}\right)^2\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2(2n+1+2/3\pi\tau_{so}T_{c0})}.$$

From this we see that the coefficient in front of I^2 decreases with increase of the impurity concentration, and in the limit $\tau_{s.0}T_{c0} \to 0$:

$$\frac{T_c - T_{c0}}{T_{c0}} = -\frac{3\pi}{8}\tau_{so}T_{c0}\left(\frac{I}{T_{c0}}\right)^2 \to 0. \tag{45}$$

Therefore, in the presence of a sufficiently large number of spin-orbit impurities ($\tau_{s.0}T_{c0} \ll 1$), the appearance of ferromagnetism is almost not reflected in the behavior of T_c. In this limiting case, the region of simultaneous coexistence of ferromagnetism and superconductivity is limited from above by the curves of T_{c0} and T_{K0}, and from below—by the axis of abscissas, in accordance with the results of Matthias et al. [4,6].

APPENDIX

Equations for the Green's Functions in the Mixed Phase

Summing the series on the right side (Fig. 1), we obtain for $\hat{G}(p, \omega_n)$ the equation represented in Fig. 7 or, multiplying both sides of this equation from the left by $1/\hat{G}^{(0)}(p, \omega_n)$ (see (10′)), we have in explicit form

$$\begin{pmatrix} i\omega_n - \xi - I\sigma_z - \overline{\mathfrak{G}} & i(\Delta + \overline{\mathfrak{F}})\sigma_y \\ i\sigma_y(\Delta + \overline{\mathfrak{F}}^+) & \sigma_y(-i\omega_n - \xi + I\sigma_z - \overline{\overline{\mathfrak{G}}})\sigma_y \end{pmatrix} \times \begin{pmatrix} \mathfrak{G} & -i\mathfrak{F}\sigma_y \\ -i\sigma_y\mathfrak{F}^+ & \sigma_y\widetilde{\mathfrak{G}}\sigma_y \end{pmatrix} \tag{A.1}$$

(the indices p and ω_n are temporarily omitted). We used the following representation of the Green's functions:

$$\hat{\mathfrak{F}}^+ = i\mathfrak{F}\sigma_y, \qquad \hat{\mathfrak{F}}^+ = -i\sigma_y\mathfrak{F}^+, \qquad \hat{\widetilde{\mathfrak{G}}} = \sigma_y\widetilde{\mathfrak{G}}\sigma_y$$

and similarly for the quantities with bars. The bar denotes an average of the type

$$\begin{pmatrix} \overline{\mathfrak{G}} & -\overline{\mathfrak{F}} \\ \overline{\mathfrak{F}}^+ & \overline{\widetilde{\mathfrak{G}}} \end{pmatrix} = n\int\left\langle\begin{pmatrix} u_1(\mathbf{q}) + u_2(\mathbf{q})\,\boldsymbol{\sigma}\mathbf{S} & 0 \\ 0 & u_1(\mathbf{q}) - u_2(\mathbf{q})\,\boldsymbol{\sigma}\mathbf{S} \end{pmatrix} \times \begin{pmatrix} \mathfrak{G}(\mathbf{p}') & -\mathfrak{F}(\mathbf{p}') \\ \mathfrak{F}^+(\mathbf{p}') & \widetilde{\mathfrak{G}}(\mathbf{p}') \end{pmatrix} \times \begin{pmatrix} u_1(\mathbf{q}) + u_2(\mathbf{q})\,\boldsymbol{\sigma}\mathbf{S} & 0 \\ 0 & u_1(\mathbf{q}) - u_2(\mathbf{q})\,\boldsymbol{\sigma}\mathbf{S} \end{pmatrix}\right\rangle_S \frac{d^3\mathbf{p}'}{(2\pi)^3}, \tag{A.2}$$

and $\langle\ldots\rangle_S$ is the average over spin directions in the field $u_2(0)\langle\boldsymbol{\sigma}\rangle\cdot\mathbf{S}$. We used the relation $\sigma_y\sigma_\alpha = -\sigma_\alpha^t\sigma_y$ for the derivation of this formula.

It is clear from general considerations that all

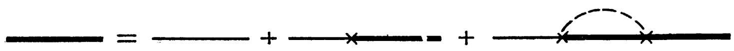

FIG. 7

of the functions entering into Eq. (A1) are diagonal spin matrices in the case of exchange scattering. Multiplying from the left by $\begin{pmatrix}1 & 0\\ 0 & i\sigma_y\end{pmatrix}$, and from the right by $\begin{pmatrix}1 & 0\\ 0 & -i\sigma_y\end{pmatrix}$, it is possible to recast Eq. (A1) in the form

$$\begin{pmatrix} i\omega_n - \xi - I\sigma_z - \mathfrak{G} & \Delta + \overline{\mathfrak{F}} \\ -(\Delta + \overline{\mathfrak{F}}^+) & -i\omega_n - \xi + I\sigma_z - \overline{\overline{\mathfrak{G}}} \end{pmatrix} \times \begin{pmatrix} \mathfrak{G} & -\mathfrak{F} \\ \mathfrak{F}^+ & \overline{\mathfrak{G}} \end{pmatrix} = 1. \qquad (A.3)$$

From this it is easy to obtain the solution

$$\begin{pmatrix} \mathfrak{G} & -\mathfrak{F} \\ \mathfrak{F}^+ & \overline{\mathfrak{G}} \end{pmatrix} = -(-(i\omega_n - \xi - I\sigma_z - \mathfrak{G}) \times (i\omega_n + \xi - I\sigma_z + \overline{\overline{\mathfrak{G}}}) + (\Delta + \overline{\mathfrak{F}})(\Delta + \overline{\mathfrak{F}}^+))^{-1} \times \begin{pmatrix} i\omega_n + \xi - I\sigma_z + \overline{\overline{\mathfrak{G}}} & \Delta + \overline{\mathfrak{F}} \\ -(\Delta + \overline{\mathfrak{F}}^+) & -i\omega_n + \xi + I\sigma_z + \mathfrak{G} \end{pmatrix}.$$

Substituting the solution obtained into Eq. (A.2), one can show that $\overline{\mathfrak{G}} = -\overline{\mathfrak{G}}$ and $\overline{\mathfrak{F}} = \overline{\mathfrak{F}}^+$. Then, in terms of the symbols

$$i\omega_n - I\sigma_z - \mathfrak{G} = i\tilde{\omega}_n - \tilde{I}_n\sigma_z, \qquad \Delta + \overline{\mathfrak{F}} = \tilde{\Delta}_n$$

($\tilde{\omega}_n$ and $\tilde{I}_n$ are real) we can write the solution in the form

$$\begin{pmatrix} \mathfrak{G} & -\mathfrak{F} \\ \mathfrak{F}^+ & \overline{\mathfrak{G}} \end{pmatrix} = -(-(i\tilde{\omega}_n - \tilde{I}_n\sigma_z)^2 + \tilde{\Delta}_n^2 + \xi^2)^{-1} \times \begin{pmatrix} i\tilde{\omega}_n - \tilde{I}_n\sigma_z + \xi & \tilde{\Delta}_n \\ -\tilde{\Delta}_n & -i\tilde{\omega}_n + \tilde{I}_n\sigma_z + \xi \end{pmatrix}. \qquad (A.4)$$

Here the quantities with tildes are given by the following relations:

$$\tilde{\omega}_n + i\tilde{I}_n\sigma_z = \omega_n + iI\sigma_z + \frac{1}{2}\left(\frac{1}{\tau_1} + \frac{\langle S_z^2\rangle}{S^2}\frac{1}{\tau_2}\right) \times \frac{\eta_n}{\sqrt{\Delta^2 + \eta_n^2}} + \frac{\langle S_x^2\rangle}{S^2}\frac{1}{\tau_2}\frac{\eta_n^*}{\sqrt{\Delta^2 + (\eta_n^*)^2}},$$

$$\tilde{\Delta}_n = \Delta\left(1 + \frac{1}{2}\left(\frac{1}{\tau_1} - \frac{\langle S_z^2\rangle}{S^2}\frac{1}{\tau_2}\right)\frac{1}{\sqrt{\Delta^2 + \eta_n^2}} - \frac{\langle S_x^2\rangle}{S^2}\frac{1}{\tau_2}\frac{1}{\sqrt{\Delta^2 + (\eta_n^*)^2}}\right), \qquad (A.5)$$

where η_n, by definition equal to

$$\eta_n = \frac{\tilde{\omega}_n + i\tilde{I}_n\sigma_z}{\tilde{\Delta}_n}\Delta, \qquad (A.6)$$

satisfies the equation

$$\omega_n + iI\sigma_z = \eta_n\left(1 - \frac{\langle S_z^2\rangle}{S^2}\frac{1}{\tau_2}\frac{1}{\sqrt{\Delta^2 + \eta_n^2}}\right) - \frac{\langle S_x^2\rangle}{S^2}\frac{1}{\tau_2}\frac{2\,\mathrm{Re}\,\eta_n}{\sqrt{\Delta^2 + (\eta_n^*)^2}}. \qquad (A.7)$$

The sign chosen for the radical is the one for which the real part is positive.

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