# *FOUR AND FIVE PARTICLE PRODUCTION IN HIGH ENERGY COLLISIONS*

I. A. VERDIEV, A. M. POPOVA and K. A. TER-MARTIROSYAN

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The previously [3] derived asymptotic expressions for "truly inelastic" processes are used for the determination of the most likely configurations of momenta in reactions in which two particles are transformed into four or five particles at high energies. A general method for integrating over the momenta of the generated particles is proposed (in particular over the transverse momentum components). The general form of the energy distribution of the particles is found. By integrating this distribution, it is shown that for the total cross sections of these reactions values which are respectively proportional to $\xi^{-1}\ln^2\xi$ and $\xi^{-1}\ln^3\xi$ are obtained. These values are obtained by using asymptotic amplitudes which are the result of taking into account only one pole in the j-plane.

THE utilization of the previously obtained [1-3] asymptotic expressions for the amplitudes of inelastic processes has permitted [4] to carry out an investigation of the properties of the simplest process a + b → c + d + e. In the present paper a general method of integrating over the momenta of the generated particles and finding out the most important momentum configurations is illustrated on the example of more complicated reactions (a + b → 1 + 2 + 3 + 4 and a + b → 1 + 2 + 3 + 4 + 5).

## 1. THE ASYMPTOTIC BEHAVIOR OF AMPLITUDES AND DIFFERENTIAL CROSS SECTIONS

The asymptotic expressions of the amplitudes A(4 ← 2) and A(5 ← 2) for the production of four and five particles, respectively, at high energies and in the case of "truly inelastic" collisions[1)] are determined under certain conditions by the contributions of the diagrams represented in Fig. 1. These contributions have the form [3]:

$$A(4\leftarrow 2) = a_4(\varkappa_1, \varkappa_2, \varkappa_3) \times \left(\frac{s_{12}}{m^2}\right)^{j_0(t_{a1})} \left(\frac{s_{23}}{m^2}\right)^{j_0(t_{a12})} \left(\frac{s_{34}}{m^2}\right)^{j_0(t_{a123})}, \qquad (1)$$

$$A(5\leftarrow 2) = a_5(\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4) \times \left(\frac{s_{12}}{m^2}\right)^{j_0(t_{a1})} \left(\frac{s_{23}}{m^2}\right)^{j_0(t_{a12})} \left(\frac{s_{34}}{m^2}\right)^{j_0(t_{a123})} \left(\frac{s_{45}}{m^2}\right)^{j_0(t_{a1234})}. \qquad (2)$$

Here $\kappa_1$, $\kappa_2$, $\kappa_3$, $\kappa_4$ (and $\kappa_5$ for the case of Fig. 1 B) are the perpendicular components of the momenta of the produced particles, i.e., the projections of their momenta on a plane perpendicular to the direction $p_a = -p_b$. The squares of the energies of the produced particles in their c.m.s. are denoted by $s_{ik} = (p_i + p_k)^2$, and the squares of the momentum transfers, or of the "reggeon" momenta in Fig. 1 are denoted by $t_{a1} = (p_a - p_1)^2$, $t_{a12} = (p_a - p_1 - p_2)^2$, $t_{a123} = (p_a = p_1 - p_2 - p_3)^2$ etc. Here and everywhere below m is a quantity of the order of the mass of the particles, which has been introduced for the sake of convenience of notation.

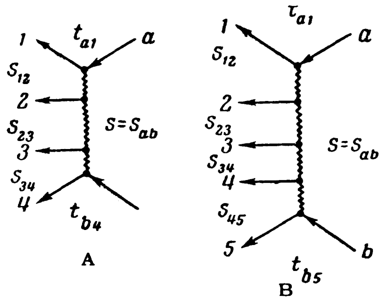


FIG. 1

According to Fig. 1, the coefficients $a_4$ and $a_5$ in (1) and (2) are of the form of a product of vertex parts and singular factors

$$a_4 = g(\varkappa_1)\, i\gamma_4(\varkappa_1, \varkappa_2')\, i\gamma_5(\varkappa_2', \varkappa_3')\, g(\varkappa_3')\, I(\varkappa_1)\, I(\varkappa_2') I(\varkappa_3'),$$

$$a_5 = g(\varkappa_1)\, i\gamma_4(\varkappa_1, \varkappa_2')\, i\gamma_5(\varkappa_2', \varkappa_3')\, i\gamma_6(\varkappa_3', \varkappa_4') \times g(\varkappa_4')\, I(\varkappa_1)\, I(\varkappa_2')\, I(\varkappa_3')\, I(\varkappa_4'), \qquad (3)$$

where $\kappa_2' = \kappa_2 + \kappa_1$, $\kappa_3' = \kappa_3 + \kappa_2 + \kappa_1$ etc. By momentum conservation ($\kappa_1 + \kappa_2 + \kappa_3 + \kappa_4 = 0$, for Fig. 1A and $\kappa_1 + \kappa_2 + \kappa_3 + \kappa_4 + \kappa_5 = 0$ for Fig. 1B), the quantity $a_4$ depends only on three two-dimensional vectors, i.e., on five variables, whereas the quantity $a_5$ depends on four vectors,

1)Collisions in which the energy of any pair of generated particles is large in their c.m.s.. The cases in which this energy is not large will not be considered, since these cases reduce [2] to the production of a smaller number of particles.

i.e., on seven variables. Besides, for Fig. 1A we have $t_{a123} = t_{b4}$, $t_{a12} = t_{b43}$ and $\kappa_3' = -\kappa_4$, $\kappa_2' = -(\kappa_3 + \kappa_4)$, and for Fig. 1B we have $t_{a1234} = t_{b5}$, $t_{a123} = t_{b54}$ etc. and $\kappa_4' = -\kappa_5$, $\kappa_3' = -(\kappa_4 + \kappa_5)$ etc.

The asymptotic expressions (1) and (2) are valid in the case when all squared momentum transfers (on which the exponents of the powers, $j_0(t) \approx 1 + j_0' t$, depend) are small. For large $s_{ik}$ they decrease rapidly as $|t_{ikl\ldots}|$ increase. In order that all $t_{ikl\ldots}$ be small, it is in particular necessary (but not sufficient) that all perpendicular components $\kappa_i$ of the momenta be small. Since for $s \to \infty$ the momenta of all particles are large[2)], this means that all produced particles can be divided into two groups and that the momenta of the particles in one of the groups are almost parallel to $p_a$ and those of the particles in the second group are almost parallel to $p_b$.

The asymptotic expressions (1) and (2) are written taking into account the contribution of only one pole (the vacuum pole) $j_0(t)$. Their form will be modified[3] if the singularity situated at the extreme right in the j-plane is more complicated. However this modification is inessential for the following.

The differential cross-section for the production of n particles (where n = 4, or n = 5) is related to the asymptotic amplitudes $A(n \leftarrow 2)$ through the usual expression:

$$d\sigma_n = \tfrac{1}{2} s^{-1} |A(n \leftarrow 2)|^2 d\tau_n, \qquad (4)$$

where

$$d\tau_n = \frac{1}{(2\pi)^{3n-4}} \frac{d\mathbf{p}_1}{2\varepsilon_1}\frac{d\mathbf{p}_2}{2\varepsilon_2}\cdots\frac{d\mathbf{p}_n}{2\varepsilon_n}\,\delta^4(P - \Sigma p_i) \qquad (5)$$

represents the statistical weight (phase space) of the final state. We will determine the momenta $\mathbf{p}_i$ of the generated particles by their longitudinal components $k_i$ and their transverse components $\kappa_i$: $\mathbf{p}_i = (k_i, \kappa_i)$, and $d\mathbf{p}_i = dk_i\, d\kappa_i$. In the ultra-relativistic limit (which is most important for $s \to \infty$), when $k_i \gg m$, one may use the following approximation for the energy of the particles:

$$\varepsilon_i = \sqrt{k_i^2 + m_i^2 + \varkappa_i^2} \approx k_i + (m_i^2 + \varkappa_i^2)/2k_i. \qquad (6)$$

After integrating in (5) over $\mathbf{p}_n$ and $k_1$ [the latter with account of Eq. (6)], we obtain

$$d\sigma_n = \frac{s}{4k_1k_n}\frac{2\pi}{(4\pi)^{2n-2}}\left|\frac{1}{s}A(n \leftarrow 2)\right|^2 \frac{d\varkappa_1}{\pi}\frac{d\varkappa_2}{\pi}\cdots\frac{d\varkappa_{n-1}}{\pi}\, d\xi_2 d\xi_3 \ldots d\xi_{n-1}, \qquad (7)$$

where $\xi_i = \ln(k_i/m)$. For all configurations of momenta which are essential for the following, the factor $s/4k_1k_n$ always equals one in the limit $s \to \infty$, and from the kinematics and Eqs. (1) and (2) it follows that the quantity $s^{-1}A(n \leftarrow 2)$ does not contain entire powers of the energy $s_{ik}$, and depends only on the logarithms $\xi_i$ of the momenta (see below) and on their transverse components $\kappa_i$.

If all particles are identical, then an additional factor $1/n!$ has to be introduced in (5). However the integration over the momenta in (7) produces n! identical configurations, which differ by permutations of the momenta and giving identical contributions to the cross-section. Therefore the factor n! cancels out, and in order to obtain the total cross section it is sufficient to take into account in the integration in (7) only essentially different configurations (this will in fact be done everywhere below).

## 2. FOUR PARTICLE PRODUCTION; MOMENTUM CONFIGURATIONS FOR "TRULY INELASTIC" PROCESSES

In the case of four particle production (Fig. 1A) there exist only three momentum configurations: one, of the type represented in Fig. 2A, and two symmetric ones, of the types represented in Fig. 2B, C, in which for $s \to \infty$ the energies of all particle pairs are large, and the quantities $t_{a1}$, $t_{ai2} = t_{b43}$ and $t_{a123} = t_{b4}$ are small. Figure 2A corresponds to the case when the generated particles split into two groups of two particles each (the case 2 + 2), so that particles 1 and 2 are emitted almost parallel to $p_a$ and the particles 3 and 4 almost parallel to $p_b$.

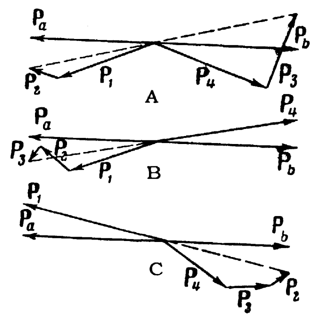


FIG. 2

In order that the energy $s_{12}$ be large, it is necessary[3)] that either $p_2 \gg p_1$ or $p_1 \gg p_2$. The first case is obviously useless, since it corresponds to small values of $p_1$. Indeed, for small $p_1$ and large

[2)]At least in the region which is essential for what follows.

[3)]Since otherwise after transforming to the c.m.s. of the particles 1 and 2 both momenta $p_1$ and $p_2$ will simultaneously be small. Hence the energy $(s_{12})^{1/2}$ will also be small.

values of the momentum $p_a \approx p_b \approx (s)^{1/2}/2$, the invariant $t_{a_1}$ will not be small. Therefore it is necessary that $p_1 \gg p_2$ (this is also indicated on Fig. 2A). Similarly, it is necessary that $p_4 \gg p_3$.

The configuration represented in Fig. 2A (as are all the others of Fig. 2) is characterized by the fact that all momentum transfer variables $t_{a_1}$, $t_{a_{12}}$, $t_{a_{123}} = t_{b4}$ are determined by the transverse components only, of the momenta of the particles 1, 2, 3, and 4. Indeed, denoting, as before by $k_i$ the absolute values of the longitudinal components of the momenta and observing that the conservation laws of energy-momentum yield, when we take into account Eq. (6):

$$k_1 + k_2 = k_3 + k_4 \approx \frac{\sqrt{s}}{2} + O\left(\frac{m_i^2 + \varkappa_i^2}{k_i}\right),$$

we obtain

$$t_{a1} = (\varepsilon_a - \varepsilon_1)^2 - (k_a - k_1)^2 - \varkappa_1^2$$
$$\approx -\frac{2k_2}{\sqrt{s}}(m_1^2 - m_a^2) - \varkappa_1^2\left(1 - \frac{2k_2}{\sqrt{s}}\right).$$

Here it has been taken into account that $k_2 \ll k_1$, i.e., that $k_1 \approx (s)^{1/2}/2$. Neglecting terms of the order $m^2k_2/(s)^{1/2}$, we obtain

$$t_{a1} \approx -\varkappa_1^2. \tag{8}$$

In exactly the same manner[4)]

$$t_{a12} \approx -(\varkappa_1 + \varkappa_2)^2 = -\varkappa_2'^2,$$
$$t_{a123} \approx t_{b4} \approx -(\varkappa_1 + \varkappa_2 + \varkappa_3)^2 = -\varkappa_3'^2 = -\varkappa_4^2. \tag{9}$$

For the energy of the pairs of produced particles in the configuration represented in Fig. 2A, one obtains the values

$$s_{12} \approx 2(\varepsilon_1\varepsilon_2 - k_1k_2) \approx \sqrt{s}(m_2^2 + \varkappa_2^2)/2k_2,$$
$$s_{23} \approx 4k_2k_3, \qquad s_{34} \approx \sqrt{s}(m_3^2 + \varkappa_3^2)/2k_3; \tag{10}$$

these quantities satisfy the relation

$$s_{12}s_{23}s_{34} = s(m_2^2 + \varkappa_2^2)(m_3^2 + \varkappa_3^2). \tag{11}$$

It is not difficult to see that in all other configurations of the type 2 + 2 (i.e., when two pairs of particles are produced which are emitted in opposite directions), either the energies of the produced particles are small, or the momentum transfers are large. If, for instance, one permutes the indices 2 and 3 in Fig. 2A, i.e. one considers $p_3$, $p_1$, and $p_a$ as almost parallel (instead of $p_2$, $p_1$, and $p_a$, as in Fig. 2A), then one obtains for the momentum transfer $t_{a12}$, which according to Fig. 1A enters the asymptotic expression (1), the value

$$t_{a12} \approx -4k_2k_3 - (\varkappa_1 + \varkappa_2)^2 \approx -s_{23} - (\varkappa_1 + \varkappa_2)^2,$$

which increases in absolute value as the energy $s_{23}$ increases (in distinction from the arbitrarily small value obtained from (9) for Fig. 2A if the transverse components $\kappa_1$ and $\kappa_2$ are small). In this case not $t_{a12}$ will be small, but the invariant $t_{a13}$ which determines the asymptotic behavior of the diagram in Fig. 3, differing from the diagram in Fig. 1A by a permutation of the particles 2 and 3. Thus, after interchanging the particles 2 and 3 the diagram in Fig. 2A does not contribute to the asymptotic expression (1), corresponding to the diagram in Fig. 1A. Taking into account the contribution of the diagram in Fig. 3 corresponds to a trivial interchange of the particles 2 and 3, about which we talked at the end of the preceding section.

Figures 2B and 2C correspond to the case when three particles move in one direction, and the fourth in the opposite direction. In order that the invariants $t_{a1}$ and $t_{b4}$ be small, it is necessary that the momenta $p_a$ and $p_1$, and also $p_b$ and $p_4$ be almost parallel in all cases, and equal in magnitude. This means that in the case of Fig. 2B both momenta $p_2$ and $p_3$ be much smaller than $p_1$, and in the case of Fig. 2C, that momenta $p_3$ and $p_2$ be much smaller than $p_4$.

In order that the energy $s_{23}$ be large, it is necessary that either $p_2 \gg p_3$, or $p_3 \gg p_4$. It is easy to see that in the second case, the invariant $t_{a12}$ will not be small for the configuration of Fig. 2B. Therefore for the case of Fig. 2B

$$p_1 \gg p_2 \gg p_3.$$

On the contrary, in the case of Fig. 2C, $p_3 \gg p_2$ must hold, i.e.,

$$p_4 \gg p_3 \gg p_2.$$

In order that these inequalities be satisfied, the invariants $t_{a1}$, $t_{a12}$ and $t_{a123} = t_{b4}$ must have

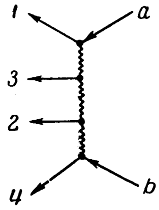

FIG. 3

4)In more detail:

$$t_{a12} = \left(k_a - k_1 - k_2 + \frac{m_a^2}{2k_a} - \frac{m_1^2 + \varkappa_1^2}{2k_1^2} - \frac{m_2^2 + \varkappa_2^2}{2k_2^2}\right)$$
$$-(k_a - k_1 - k_2)^2 - (\varkappa_1 + \varkappa_2)^2 \approx -(\varkappa_1 + \varkappa_2)^2 + O\left(\frac{m_i^2 + \varkappa_i^2}{k_i}\right),$$

since $k_a - k_1 - k_2$ is a quantity of the order of $(m_i^2 + \varkappa_i^2)/2k_i$.

the values (8) and (9), as is easily seen, and for the energies of the pairs of generated particles we obtain in the case of Fig. 2B

$$s_{12} = \sqrt{s}\,(m_2^2 + \varkappa_2^2)/2k_2, \quad s_{23} = k_2\,(m_3^2 + \varkappa_3^2)/k_3,$$
$$s_{34} = 4k_3k_4 = 2\sqrt{s}\,k_3 \tag{12}$$

and in the case of Fig. 2C

$$s_{12} = 4k_1k_2 = 2\sqrt{s}\,k_2, \quad s_{23} = k_3\,(m_2^2 + \varkappa_2^2)/k_2,$$
$$s_{34} = \sqrt{s}\,(m_3^2 + \varkappa_3^2)/2k_3. \tag{13}$$

Here it has been taken into account that if the indicated inequalities hold $k_1 = k_4 \approx p_a \approx p_b \approx (s)^{1/2}$ (up to terms of the order $k_\alpha/(s)^{1/2}$, with $\alpha = 2, 3$). These values of $s_{ik}$ obviously satisfy the relation (11) in all cases. In all other configurations of the type 3 + 1, except those given in Figs. 2B and C, either the energies of the pairs of generated particles are small, or the squared momentum transfers are large.

We note that when the incident energy increases, the momenta $k_2$ and $k_3$ can always have such (ultrarelativistic) values, that for all three configurations the quantities $s_{12}$, $s_{23}$, and $s_{34}$ increase with the increase of s. If in this case $s \to \infty$

$$s_{12} = C_1\,(s/m^2)^{\alpha_1} \to \infty, \qquad s_{23} = C_2\,(s/m^2)^{\alpha_2} \to \infty,$$
$$s_{34} = C_3\,(s/m^2)^{\alpha_3} \to \infty, \tag{14}$$

then, according to (11) the positive numbers $\alpha_1$, $\alpha_2$, $\alpha_3$ and the constants $C_1$, $C_2$, and $C_3$ (which are independent of s) must satisfy the conditions

$$\alpha_1 + \alpha_2 + \alpha_3 = 1, \quad C_1C_2C_3 = (m_2^2 + \varkappa_2^2)\,(m_3^2 + \varkappa_3^2). \tag{15}$$

The contribution to the total cross section of the two configurations of the type represented in Figs. 2B and C is the same, and therefore, in the following we will consider only one of the two (e.g. Fig. 2B) and multiply the contribution by two.

## 3. THE CROSS SECTION FOR FOUR PARTICLE PRODUCTION

Taking into account what was said above, we write the differential cross-section of the four particle production process represented in Fig. 1, A in the form

$$d\sigma_4 = d\sigma_{4A} + 2d\sigma_{4B}, \tag{16}$$

where $d\sigma_{4A}$ refers to Fig. 2A, and $d\sigma_{4B}$ refers to Fig. 2B. Substituting in Eq. (7) the asymptotic expression (1) for the amplitude and taking into account (8), (9), and (11) (and also the fact that in all configurations of Fig. 2, $k_1 \approx k_4 \approx p_a \approx p_b \approx (s)^{1/2}/2$), we obtain

$$d\sigma_{4A} \approx \lambda_4\,(\varkappa_1, \varkappa_2', \varkappa_3')\exp\left[-2j_0'\,(\varkappa_1^2\xi_{12} + \varkappa_2'^2\xi_{23} + \varkappa_3'^2\xi_{34})\right]$$
$$\times \frac{d^2\varkappa_1}{\pi}\frac{d^2\varkappa_2'}{\pi}\frac{d^2\varkappa_3'}{\pi}\,d\xi_2 d\xi_3, \tag{17}$$

where, for the vacuum trajectory the following approximation has been used:

$$j_0\,(\varkappa_i^2) = 1 - j_0'\,\varkappa_i^2,$$

which is valid for small $\kappa_i^2$ which are essential in (17), and it has been taken into account, that $d^2\kappa_2 = d^2\kappa_2'$ for a given value of $\kappa_1$ and $d^2\kappa_3 = d^2\kappa_3'$ for given $\kappa_1$ and $\kappa_2$. The symbol $\lambda_4$ denotes the factor

$$\lambda_4 = 2\pi\,(4\pi)^{-6}\,|\,a_4\,(\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4)\,|^2\,m^{-6}$$
$$\times (m_2^2 + \varkappa_2^2)\,(m_3^2 + \varkappa_3^2),$$

which depends only on the transverse momentum components, and $\xi_{ik}$ denote the quantities $\ln(s_{ik}/m)$. According to Eq. (10),

$$\xi_{12} = \ln(s_{12}/m^2) = \xi - \xi_2 + \ln\left[(m_2^2 + \varkappa_2^2)/2m^2\right] \approx \xi - \xi_2,$$
$$\xi_{23} = \xi_2 + \xi_3, \qquad \xi_{34} = \xi - \xi_3, \tag{18}$$

where $\xi = \ln(s)^{1/2}/m)$ and $\xi_2 = \ln(k_2/m)$, and $\xi_3 = \ln(k_3/m)$ have been previously introduced. We assume that all quantities $\xi_{ik}$, $\xi$, and $\xi_i$ are large compared with unity.

For sufficiently large $\xi_{12}$, $\xi_{23}$, and $\xi_{34}$ the integration over $\kappa_1$, $\kappa_2$, and $\kappa_3$ in (17) are easily carried out, since in the integral only very small values of $\kappa_1^2$, $\kappa_2'^2$, and $\kappa_3'^2$ (of the order of $(2j_0'\xi_{ik})^{-1} \approx m^2/\xi_{ik}$) are essential, and the function $\lambda_4$ can be taken out of the integral, in the point $\kappa_1 \approx \kappa_2' \approx \kappa_3' \approx 0$ (i.e., for $t_{a1} \approx t_{a12} \approx t_{a123} = 0$). Taking into account that $d^2\kappa_i/\pi \equiv d\kappa_i^2$ and extending the integration with respect to $\kappa_1^2$, $\kappa_2'^2$, and $\kappa_3'^2$ up to infinity, we obtain

$$\frac{1}{\pi^3}\int \exp\left[-2j_0'\,(\varkappa_1^2\xi_{12} + \varkappa_2'^2\xi_{23} + \varkappa_3'^2\xi_{34})\right] d^2\varkappa_1 d^2\varkappa_2' d^2\varkappa_3'$$
$$= (2j_0')^{-3}\,[\xi_{12}\xi_{23}\xi_{34}]^{-1}.$$

Therefore

$$d\sigma_{4A}\,(\xi, \xi_2, \xi_3) = \sigma_4^0 \cdot 2d\xi_2 d\xi_3/\xi_{12}\xi_{23}\xi_{34}, \tag{19}$$

where [5], according to (3)

$$\sigma_4^0 = \sigma_0\beta_4\beta_5, \tag{20}$$

[5] In fact, this value is correct only in the case when all particles are identical (cf. above). If the particles are distinct, then in place of the expression $\beta_2\beta_3 d\xi_2 d\xi_3/\xi_{12}\xi_{23}\xi_{34}$ in (19) there will appear a sum of similar expressions over all possible permutations of the particles 1, 2, 3, and 4.

$$\sigma_0 = \frac{\pi}{m^2}\frac{1}{2j_0' m^2}\left(\frac{g_0^2}{4\pi}\right)^2,$$

$$\beta = \frac{1}{2j_0' m^2}\left(\frac{\gamma_i^0 m_i}{4\pi}\right)^2, \quad i = 2, 3, \tag{21}$$

with $g_0 = g(0)$ and $\gamma_i^0 = \gamma_i(0, 0)$ are the values of the vertex parts in (3) for vanishing $\kappa_i$, and m is the quantity introduced above, which is of the order of the particle mass.

Thus, according to (18) and (19), in the configuration of Fig. 2A the energy distribution of the generated particles (i.e., with respect to $\xi_2$ and $\xi_3$) is determined by

$$d\sigma_{4A}(\xi, \xi_2, \xi_3)$$
$$= \sigma_4^0 \cdot 2d\xi_2 d\xi_3/(\xi - \xi_2)(\xi + \xi_3)(\xi - \xi_3). \tag{22}$$

In obtaining this relation we have assumed that the energies (10) are large and that the momenta $k_2$ and $k_3$ are ultrarelativistic, in other words, that $k_i \ll (s)^{1/2}$ but $k_i \gg m$, with i = 2, 3. Both conditions are satisfied if

$$\sqrt{s}/L > k_i > Lm,$$

where $L \gg 1$, i.e., if

$$\xi - \lambda \geqslant \xi_i \geqslant \lambda, \quad i = 2, 3, \tag{23}$$

where $\lambda = \ln L$. From (18) it can be seen directly that the conditions $\xi_{ik} > 1$ are satisfied if $\lambda > 1$.

Thus, the distribution (22) is certainly true in the region (23); one can show that for $\xi > 1$ the contribution of this region determines the main contribution to the cross section $d\sigma_{4A}$. [When going beyond the limits (23), we reach, for $\xi - \lambda < \xi_i$ the region of "almost elastic" collisions, and for $\xi_i < \lambda$ we reach the region of nonrelativistic values of the momenta of particles 2 or 3. The contribution of both regions can be investigated separately, and for $\xi \to \infty$ turns out to vanishingly small compared to the contribution of the region (23).]

The cross section $d\sigma_{4B}$ corresponding to the configuration of Fig. 2B is determined by an equation of the same form as (17), the integration over the transverse components is carried out in the same manner as above and yields the result (19)—(21). However, in this case, the quantities $\xi_{ik}$ have, according to (12), the values

$$\xi_{12} = \xi - \xi_2, \qquad \xi_{23} \approx \xi_2 - \xi_3, \qquad \xi_{34} = \xi + \xi_3.$$

Therefore

$$d\sigma_{4B}(\xi, \xi_2, \xi_3)$$
$$= \sigma_1^0 \cdot 2d\xi_2 d\xi_3/(\xi - \xi_2)(\xi_2 - \xi_3)(\xi + \xi_3), \tag{24}$$

and the conditions $\xi_{ik} > 1$ lead in this case to

$$\xi - \lambda \geqslant \xi_2 \geqslant 2\lambda, \qquad \xi_2 - 2\lambda \geqslant \xi_3 \geqslant \lambda \tag{25}$$

in place of (23).

## 4. FIVE-PARTICLE PRODUCTION

In the case of five particle production there are only the four configurations represented in Fig. 4, in which all energies $s_{ik}$ are large, and the momentum transfers are small (and depend only on the transverse components of the momenta of the five produced particles). In all other configurations, except those in fig. 4, these conditions are not satisfied.

Comparing Fig. 4 and Fig. 1B (as well as Fig. 2 and Fig. 1A), one can note a simple rule for finding such configurations. Namely, the four configurations in Fig. 4 can be obtained from the diagram in Fig. 1B, dividing the latter by horizontal lines in all possible ways into two parts, intersecting every time the propagator of one of the four reggeons. In order to obtain each configuration one must consider that all particles for which the lines are situated above the separation line are emitted almost parallel into one direction, and the particles with lines below the separation line, are emitted in the opposite direction. Besides, those particles for which the lines in diagrams of the type of Fig. 1B are furthest out must have the maximal momentum in each group, and each consecutive particle (with their lines lying closer to the separation line) must have considerably smaller momenta.

The four configurations represented in Fig. 4 are pairwise symmetric: the first two (as well as the last two) contribute, obviously, identically to the total cross-section. Therefore it is sufficient to consider only the first of them (Fig. 4A) and the third (Fig. 4C). The squares of the momentum transfers have the same values in all four cases[6]:

[6]We give as an example the computation of $t_{a12}$ for the configuration in Fig. 4 A, in which $k_3 \ll k_2 \ll k$, $k_a - k_1 - k_2 \approx k_3$ (for this configuration the conservation laws imply $k_1 + k_2 + k_3 = k_4 + k_5 \approx k_a \approx k_b \approx (s)^{1/2}/2$):

$$t_{a12} = (p_a - p_1 - p_2)^2$$
$$= \left(k_a - k_1 - k_2 + \frac{m_a^2}{2k_a} - \frac{m_1^2 + \varkappa_1^2}{2k_1} - \frac{m_2^2 + \varkappa_2^2}{2k_2}\right)^2$$
$$- (k_a - k_1 - k_2)^2 - (\varkappa_1 + \varkappa_2)^2$$
$$\approx -(\varkappa_1 + \varkappa_2)^2 - \frac{k_3}{k_2}(m_2^2 + \varkappa_2^2) \approx -(\varkappa_1 + \varkappa_2)^2,$$

since $k_3/k_2 \ll 1$.

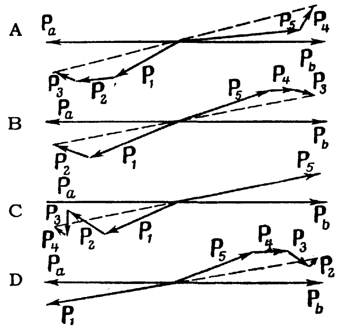


FIG. 4.

$$t_{a1} = -\varkappa_1^2, \qquad t_{a12} = -\varkappa_2'^2, \qquad t_{a123} \equiv t_{b45} = -\varkappa_3'^2,$$

$$t_{a1234} = t_{b5} = -\varkappa_4'^2,$$

where $\kappa_2' = \kappa_2 + \kappa_1$, $\kappa_3' = \kappa_3 + \kappa_2 + \kappa_1$, $\kappa_4' = \kappa_4 + \kappa_3' = -\kappa_5$, and for the energies we obtain: in the case of Fig. 4A

$$s_{12} = \sqrt{s}\,(m_2^2 + \varkappa_2^2)/2k_2, \qquad s_{23} = k_2\,(m_3^2 + \varkappa_3^2)/k_3,$$

$$s_{34} = 4k_3k_4, \quad s_{45} = \sqrt{s}\,(m_4^2 + \varkappa_4^2)/2k_4 \tag{26}$$

and in the case of Fig. 4B

$$s_{12} = \sqrt{s}\,(m_2^2 + \varkappa_2^2)/2k_2, \qquad s_{23} = k_2\,(m_3^2 + \varkappa_3^2)/k_3,$$

$$s_{34} = k_3\,(m_4^2 + \varkappa_4^2)/k_4, \qquad s_{45} = 2\sqrt{s}\,k_4. \tag{27}$$

In all cases a relation analogous to (11) holds:

$$s_{12}s_{23}s_{34}s_{45} = s\,(m_2^2 + \varkappa_2^2)\,(m_3^2 + \varkappa_3^2)\,(m_4^2 + \varkappa_4^2).$$

Substituting (2) into (7), we obtain, taking into account these values of the energies and momentum transfers (and also of the fact that for all configurations of Fig. 4 $k_1 \approx k_5 \approx p_a \approx p_b \approx (s)^{1/2}/2$) the following value for the contribution to the cross section from the configuration of Fig. 4A:

$$d\sigma_{5A} = \lambda_5\,(\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4)$$
$$\times \exp\,[-2j_0'\,(\varkappa_1^2\,\xi_{12} + \varkappa_2'^2\xi_{23} + \varkappa_3'^2\xi_{34} + \varkappa_4'^2\xi_{45})]$$
$$\times \frac{d^2\varkappa_1}{\pi}\,\frac{d^2\varkappa_2'}{\pi}\,\frac{d^2\varkappa_3'}{\pi}\,\frac{d^2\varkappa_4'}{\pi}\,d\xi_2 d\xi_3 d\xi_4, \tag{28}$$

After integrating over the transverse momentum components $\kappa_i$, we obtain, completely analogous to (19)

$$d\sigma_{5A}\,(\xi, \xi_2, \xi_3, \xi_4) = \sigma_5^0\cdot 2d\xi_2 d\xi_3 d\xi_4/\xi_{12}\xi_{23}\xi_{34}\xi_{45}, \tag{29}$$

where, according to (3),

$$\sigma_5^0 = \sigma_0\beta_2\beta_3\beta_4,$$

and $\sigma_0$ and $\beta_i$ (i = 2, 3, 4) have the values (21).

Taking into account that in the case of Fig. 4A the quantities have, according to (26), the values

$$\xi_{12} = \xi - \xi_2, \qquad \xi_{23} = \xi_2 - \xi_3,$$

$$\xi_{34} = \xi_3 + \xi_4, \qquad \xi_{45} = \xi - \xi_4,$$

we obtain for the configuration in Fig. 4A the following energy distribution for the produced particles:

$$d\sigma_{5A}\,(\xi, \xi_2, \xi_3, \xi_4)$$
$$= \sigma_5^0\,\frac{2d\xi_2 d\xi_3 d\xi_4}{(\xi - \xi_2)\,(\xi_2 - \xi_3)\,(\xi_3 + \xi_4)\,(\xi - \xi_4)}, \tag{30}$$

which is applicable in the region

$$\xi - \lambda \geqslant \xi_2 \geqslant 2\lambda, \qquad \xi_2 - \lambda \geqslant \xi_3 \geqslant \lambda,$$
$$\xi - \lambda \geqslant \xi_4 \geqslant \lambda. \tag{31}$$

For the part of the cross section corresponding to the configuration of Fig. 4C, we obtain in exactly the same manner the expressions (28) and (29), in which

$$\xi_{12} = \xi - \xi_2, \qquad \xi_{23} = \xi_2 - \xi_3,$$

$$\xi_{34} = \xi_3 - \xi_4, \qquad \xi_{45} = \xi + \xi_4,$$

i.e.,

$$d\sigma_{5C}(\xi, \xi_2, \xi_3, \xi_4) = \sigma_5^0\,\frac{2d\xi_2 d\xi_3 d\xi_4}{(\xi - \xi_2)\,(\xi_2 - \xi_3)\,(\xi_3 - \xi_4)\,(\xi + \xi_4)}\,. \tag{32}$$

This distribution is applicable in the region in which all denominators are large and the particles are ultrarelativistic, i.e.,

$$\xi - \lambda \leqslant \xi_2 \leqslant 3\lambda, \qquad \xi_2 - \lambda \leqslant \xi_3 \leqslant 2\lambda,$$
$$\xi_3 - \lambda \leqslant \xi_4 \leqslant \lambda. \tag{33}$$

In calculating the total cross section for $\xi \to \infty$, the regions (30) and (32) give the main contribution.

## 5. TOTAL CROSS SECTIONS; THE CASE $\gamma_i(0, 0) = 0$

According to (16) and (22)–(25) the total cross section for the production of four particles has the expression

$$\sigma_4\,(\xi) = \sigma_{4A}\,(\xi) + 2\sigma_{4B}(\xi);$$

$$\sigma_{4A}\,(\xi) = \sigma_4^0\cdot 2\int_\lambda^{\xi-\lambda} d\xi_2 \int_\lambda^{\xi-\lambda} \frac{d\xi_3}{(\xi - \xi_2)\,(\xi_2 + \xi_3)\,(\xi - \xi_3)}\,,$$

$$\sigma_{4B}\,(\xi) = 2\sigma_4^0 \int_{2\lambda}^{\xi-\lambda} d\xi_2 \int_\lambda^{\xi_2-\lambda} \frac{d\xi_3}{(\xi - \xi_2)\,(\xi_2 - \xi_3)\,(\xi + \xi_3)}\,.$$

Similarly, for the cross section of five particle production we obtain from (30)–(33)

$$\sigma_5(\xi) = 2\sigma_{5A}(\xi) + 2\sigma_{5C}(\xi);$$

$$\sigma_{5A}(\xi) = 2\sigma_5^0 \int_{2\lambda}^{\xi-\lambda} d\xi_2 \int_{\lambda}^{\xi_2-\lambda} d\xi_3 \int_{\lambda}^{\xi_3-\lambda} \frac{d\xi_4}{(\xi-\xi_2)(\xi_2-\xi_3)(\xi_3+\xi_4)(\xi-\xi_4)},$$

$$\sigma_{5C}(\xi) = 2\sigma_5^0 \int_{3\lambda}^{\xi-\lambda} d\xi_2 \int_{2\lambda}^{\xi_2-\lambda} d\xi_3 \int_{\lambda}^{\xi_3-\lambda} \frac{d\xi_4}{(\xi-\xi_2)(\xi_2-\xi_3)(\xi_3-\xi_4)(\xi+\xi_4)}.$$

For $\xi \to \infty$, in particular for $\xi \gg \lambda$ these integrals go beyond the simple asymptotic expression which will be computed in the following paper [5]:

$$\sigma_{4A} \approx \sigma_{4B} \approx (\ln\xi)^2/\xi, \qquad \sigma_{5A} \approx \sigma_{5C} \approx (\ln\xi)^3/\xi.$$

Therefore

$$\sigma_4(\xi) \approx 3\sigma_4^0 (\ln\xi)^2/\xi, \qquad \sigma_5(\xi) = 4\sigma_5^0(\ln\xi)^3/\xi,$$

where $2\xi = \ln(s/m^2)$.

Adding the cross sections for the production of an arbitrary number of particles which have been obtained in this manner, one can show [5] that for the total cross section for the interaction of the colliding particles a and b, there results a value which increases for $\xi \to \infty$. However, from the unitarity condition for the elastic scattering amplitude under zero angle, it follows, that for $\xi \to \infty$ the total cross section must have a constant value.

This contradiction can be removed, assuming that the vertex parts $\gamma_i(\kappa_i', \kappa_{i+1}')$ corresponding to the emission of a particle by a reggeon, vanishes for $\kappa_i' \to 0$ and $\kappa_{i+1}' \to 0$, e.g., linearly with respect to $\kappa_i'^2\kappa_{i+1}'^2$:

$$\gamma_i \approx \varkappa_i'^2 \varkappa_{i+1}'^2 \gamma_{i0}', \tag{34}$$

where $\gamma_{i0}'$ is a constant [7]. This gives rise to completely different formulas for the energy distribution of the particles and for the total cross sections.

Carrying out the integration over the transverse components of the momenta, exactly as was done above, but taking into account the form (34) of the vertex parts, we obtain the following expressions for the differential cross sections for four particle production

$$d\sigma_{4A} = \sigma_4'^0 \frac{d\xi_2 d\xi_3}{(\xi-\xi_2)^3(\xi_2+\xi_3)^5(\xi-\xi_3)^3}, \tag{35}$$

$$d\sigma_{4B} = \sigma_4'^0 \frac{d\xi_2 d\xi_3}{(\xi-\xi_2)^3(\xi_2-\xi_3)^5(\xi+\xi_3)^3}, \tag{36}$$

where

$$\sigma_4'^0 = \frac{\sigma_0}{6}\beta_2'\beta_3', \qquad \beta_i' = \frac{4!}{2j_0'm^2}\left(\frac{\gamma_{i0}'m_i}{4\pi(2j_0')^2}\right)^2, \qquad i = 2, 3, \tag{37}$$

and $\sigma_0$ has the expression (21). These equations replace the expressions (22), (24) and (21), which were obtained under the assumption that $\gamma_i(0,0) \neq 0$.

Similarly, one obtains for the cross section of five particle production in place of (30) and (32) the expressions

$$d\sigma_{5A} = \sigma_5'^0 \frac{d\xi_2 d\xi_3 d\xi_4}{(\xi-\xi_2)^3(\xi_2-\xi_3)^5(\xi_3+\xi_4)^5(\xi-\xi_4)^3}, \tag{38}$$

$$d\sigma_{5C} = \sigma_5'^0 \frac{d\xi_2 d\xi_3 d\xi_4}{(\xi-\xi_2)^3(\xi_2-\xi_3)^5(\xi_3-\xi_4)^5(\xi+\xi_4)^3}, \tag{39}$$

where $\sigma_5'^0 = \frac{1}{6}\sigma_0\beta_2'\beta_3'\beta_4'$.

In order to obtain the total cross sections these expressions have to be integrated over $\xi_i$ in the intervals which were given above. The principal contribution to the integrals comes from the integration over all $\xi_i$ [both in the case (35) and (36) and in the case (38) and (39)] near the upper limit, where the differences $\xi_i - \xi_{i+1}$ in the denominators of the integrands have values of the order of $\lambda$, and the sums $\xi_i + \xi_{i+1}$ are of the order of $2\xi$. Therefore the $\xi_i$-integrals of the cross sections (36) and (39) will be proportional to $(2\xi)^{-3}$ and for (35) and (38) they will be small, since they are proportional to $(2\xi)^{-5}$. Thus, in the case (34), the major contribution comes from configurations of the type in Figs. 2B, C or Figs. 4C, D, corresponding to the case when one of the emitted particles moves in one direction, and all the others move in the opposite direction.

Taking into account the fact, that [8]

$$\int_{2\lambda}^{\xi-\lambda} d\xi_2 \int_{\lambda}^{\xi_2-\lambda} \frac{d\xi_3}{(\xi-\xi_3)^3(\xi_2-\xi_3)^5(\xi+\xi_3)^3} = \frac{1}{(2\lambda^2)^3(2\xi)^3}\left[1 + O\left(\frac{\lambda}{\xi}\right)\right],$$

$$\int_{3\lambda}^{\xi-\lambda} d\xi_2 \int_{2\lambda}^{\xi_2-\lambda} d\xi_3 \int_{\lambda}^{\xi_3-\lambda} \frac{d\xi_4}{(\xi-\xi_2)^3(\xi_2-\xi_3)^5(\xi_3-\xi_4)^5(\xi+\xi_4)^3} = \frac{1}{(2\lambda^2)^5(2\xi)^3}\left[1 + O\left(\frac{\lambda}{\xi}\right)\right],$$

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[7] It was shown before [3] that for $\varkappa_i' \to 0$ and $\varkappa_{i+1}' \to 0$ the function $\varkappa_i$ can in general have the form $(\varkappa_i'^2\varkappa_{i+1}'^2)^k$ where k is a positive integer. We consider the case when k = 1; the calculations are completely analogous in the case k = 2, 3, ...

[8] The integrals are calculated extremely simply by substituting for the factors $(\xi+\xi_3)^3$ or $(\xi+\xi_4)^3$ in the denominators of the integrands their values at the upper integration limit.

we obtain for the total cross sections for four- and five-particle production, the expressions

$$\sigma_4(\xi) \approx 2\sigma_4'^0/(2\lambda^2)^3(2\xi)^3, \qquad \sigma_5(\xi) = 2\sigma_5'^0/(2\lambda^2)^5(2\xi)^3, \quad (40)$$

which decreases inversely proportional to the cube of the quantity $2\xi = \ln(s/m^2)$.

In exactly the same manner, taking into account the form (34) for the vertex part will modify the results obtained previously[4] for the three particle production processes. Under the assumption $\gamma_i(0,0) \neq 0$ the following expression had been obtained for the differential cross section $d\sigma_3$ of this process (cf. Eqs. (21) and (23) in[4])

$$d\sigma_3 = 2\sigma_3^0(\xi^2 - \xi_2^2)\,d\xi_2, \qquad \sigma_3^0 = \sigma_0\beta_2,$$

where $\sigma_0$ and $\beta_2$ are defined in Eq. (21) of the present paper. Taking into account the form (34) for the vertex part, we obtain instead

$$d\sigma_3 = 2\sigma_3'^0\frac{d\xi_2}{(\xi^2 - \xi_2^2)^3}, \qquad \sigma_3'^0 = \frac{\sigma_0}{6}\beta_2', \quad (41)$$

where $\beta_2'$ was defined above in Eq. (37).

For the total cross section of three particle production we obtain the expression

$$\sigma_3 = 2\sigma_3'^0\int\limits_{\lambda}^{\xi-\lambda}\frac{d\xi_2}{(\xi^2 - \xi_2^2)^3} = \frac{2\sigma'^0_3}{2\lambda^2(2\xi)^3}\left[1 + O\left(\frac{\lambda}{\xi}\right)\right],$$

which decreases as the third power of the logarithm of the energy, in the same manner as Eq. (40).

It is easy to note that if the vertex part behaves like $\gamma_i \approx (\kappa_i'^2\kappa_{i+1}'^2)^k\gamma_{i0}'$ for $\kappa_i \to 0$ and $\kappa_{i+1} \to 0$[3], where $k = 2, 3, \ldots$, then all cross sections of inelastic processes will decrease even faster, as $1/(2\xi)^{2k+1}$.

## CONCLUSION

All the results that have been derived above refer not only to the production of four or five (or three) particles in "truly inelastic" collisions, but also to the case[3] where the same number of groups of particles is produced with low energies

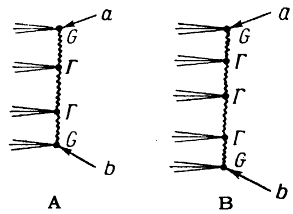


FIG. 5

of each particle in the c.m.s. of each group. The asymptotic expression for the corresponding amplitudes is determined by the contribution of diagrams such as those represented in Fig. 5. The main physical conclusion from what was said above is that if four or five groups of such particles are produced then, in the c.m.s. of the reaction, these particles will be emitted inside a narrow cone around the initial direction, and so that the total momenta of the particles within the different groups differ significantly in magnitude.

This conclusion seems not to be modified if one replaces the "pole asymptotic expressions" (1) and (2) corresponding to the contribution of one isolated pole in the j-plane by the asymptotic expressions[3] corresponding to more complicated singularities in the j-plane.

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