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ON THE POSSIBILITY OF A GEOMETRIC INTERPRETATION OF THE WEAK INTERACTIONS OF LEPTONS

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The possibility of interpreting the weak interactions of leptons as an effect of space-time curvature over distances of the order of $l \sim (G/\hbar c)^{1/2} \sim 6 \times 10^{-17}$ cm is being considered. It is assumed that the leptons are described by a unique spinor in an n-dimensional space in which the physical four-dimensional space represents a surface. The equations of this surface are constructed in a self-consistent manner from the values of the spinor on that surface and taking into account the condition of being Euclidean at infinity. A covariant zero-mass equation is postulated for the values of the spinor on the surface, which allows one to introduce electromagnetic interactions, and in first approximation yields a description of the weak interactions. The conditions which lead to the V − A type weak interactions are discussed.

1. INTRODUCTION

IN recent times the opinion has been expressed that the weak interactions of elementary particles are somehow connected with a change in the structure of space-time over small distances, namely over distances of the order of the "fundamental length" [1,2]:

$$l \sim \sqrt{G/\hbar c} \sim 6 \cdot 10^{-17} \text{ cm}, \qquad (1.1)$$

where G is the weak interaction coupling constant[1)] One can cite several properties of the weak interactions which are evidence in favor of this hypothesis. The weak interactions are universal, i.e., all presently known particles, with the possible exception of the photon, participate in it. The weak interactions have simple and beautiful symmetry properties, and it would be interesting to connect these with the properties of space. Finally, the weak interactions contain a ready-made standard of length (1.1) connected with the constant G, which is determined from particle decay experiments.

However, to date, there does not exist a single example showing intuitively how a modification of the structure of space at small distances can effectively lead to weak interactions. In this paper we discuss a model in which, under certain assumptions, we succeed in demonstrating the connection of the weak interactions with the properties of space.

The basic physical requirements imposed on the model are the following. Limiting ourselves to the consideration of weak interactions only we will not include in our model any strongly interacting particles, and will require that the model describe the four leptons only: the electron, the muon and the two neutrinos. Since among these particles two are charged and two are neutral, we require that the model admit the possibility of introducing electromagnetic interactions. Besides, the model must yield an effective four-fermion interaction, which should correspond to the interaction leading to the decay of the muon.

We go over to a consideration of the fundamental assumptions on which our model is based. We assume that the metric of the space is in general not Euclidean, but has the general form:

$$ds^2 = g_{ik} dx^i dx^k; \qquad g_{ik} = g_{ki}. \qquad (1.2)$$

We will consider a one-particle problem, described by a wave function $\psi(x)$. In this case the preceding assumption can be made concrete by assuming that there is a deviation of the metric from the Euclidean metric in the neighborhood of a four-dimensional point x_0, over distances $|x^k - x_0^k| \lesssim l$ and that at large distances, i.e., $|x^k - x_0^k| \gg l$, the space should become flat. Intuitively one may imagine that the space becomes curved only near the particle, remaining Euclidean at a distance from the particle.

In order to obtain a description of the leptons, one must introduce into this space quantities of a spinor nature. In the proposed model the transformation functions will also depend on these quantities, and due to the essential nonlinearity the definition of the spinors becomes a complicated

1)In the following we assume everywhere $\hbar = c = 1$.

problem. We will try to avoid this difficulty by considering the physical space as a surface V_4 in a pseudoeuclidean space S_n of a higher dimension, in which the definition of spinors presents no difficulties [3].

As is well known (cf., e.g., [4]), any four dimensional Riemann space V_4 can be considered as a surface in a ten dimensional Euclidean space. For special forms of V_4 a smaller number of dimensions is sufficient. We write out the number r of components of a spinor in spaces of various dimension n:

$$\begin{matrix} n: & 10 & 9 & 8 & 7 & 6 & 5 \\ r: & 32 & 16 & 16 & 8 & 8 & 4 \end{matrix} \qquad (1.3)$$

We consider a region on the surface V_4 which is remote from the point $x_0(|x^k - x_0^k| \gg l)$ where V_4 becomes a plane S_4. With respect to the rotations and reflections of this plane the spinors of the higher dimensional space decompose into a corresponding number of four-component spinors which transform among themselves only. There appears the possibility of describing several particles by a single spinor of the higher dimensional space. In particular, the four leptons can be unified either into one spinor in the spaces S_9 or S_8, or into two distinct spinors in the spaces S_7 or S_6. In the region $|x^k - x_0^k| \lesssim l$, where the space becomes curved, these four-component spinors will transform into each other, which leads to the possibility of the appearance of interactions among them.

We will assume that the particle itself is the cause of the modification of the metric of the space, and therefore we will construct geometric quantities which characterize the surface V_4 in terms of the spinors of the space S_n. Besides, in what follows an equation for the values of the spinor on the surface V_4 will be postulated.

Let us summarize the fundamental assumptions of the model:

1) the physical four-dimensional space is considered as a surface with metric (1.2) in a multidimensional pseudoeuclidean space;

2) the geometric quantities which determine this surface are constructed in terms of the spinors of the multidimensional space, and the values of these spinors on the surface satisfy a certain equation;

3) for large distances, i.e. for $|x^k - x_0^k| \gg l$, the surface has to become a plane.

2. THE EQUATION OF THE SURFACE V_4

In this section we collect several formulae from the theory of surfaces in n-dimensional space (cf., e.g., [4]), which will be used in what follows. Let z^α denote the coordinates in an n-dimensional pseudoeuclidean space S_n, endowed with the metric

$$ds^2 = \sum_{\alpha=0}^{n-1} c_\alpha (dz^\alpha)^2, \qquad c_0 = -1, \qquad c_\alpha = 1, \qquad (2.1)$$

and let x^i (i = 0, 1, 2, 3) denote the coordinate on the surface V_4 with the metric (1.2). Then

$$\sum_\alpha c_\alpha \frac{\partial z^\alpha}{\partial x^i}\frac{\partial z^\alpha}{\partial x^j} = g_{ij}. \qquad (2.2)$$

Let $\eta^\beta_{\sigma|}$ be the components of n − 4 mutually orthogonal unit vectors, which are orthogonal to V_4:

$$\sum_\beta c_\beta \eta^\beta_{\sigma|}\eta^\beta_{\mu|} = \delta_{\sigma\mu}, \qquad \sigma, \mu = 4 \ldots n-1; \qquad (2.3)$$

$$\sum_\beta c_\beta \eta^\beta_{\sigma|}\frac{\partial z^\beta}{\partial x^i} = 0. \qquad (2.4)$$

The equations of the surface V_4 have the following form (cf. [4]):

$$\left(\frac{\partial z^\alpha}{\partial x^i}\right)_{,j} = \sum_{\sigma=4}^{n-1} b_{\sigma|ij}\eta^\alpha_{\sigma|}; \qquad (2.5)$$

$$\eta^\alpha_{\sigma|,j} = -b_{\sigma|lj} g^{lm}\frac{\partial z^\alpha}{\partial x^m} + \sum_{\tau=4}^{n-1} \nu_{\tau\sigma|j}\eta^\alpha_\tau. \qquad (2.6)$$

Here $b_{\sigma|ij}$ and $\nu_{\tau\sigma|j}$ are quantities which determine the geometry of the surface, up to rotations and motions of the entire V_4 in S_n. The symbol $f_{,j}$ denotes the covariant derivative of f computed with the tensor g_{ij}. It is clear from (2.5) and (2.6) that $b_{\sigma|ij}$ is a symmetric second rank tensor in V_4 and $\nu_{\tau\sigma|j}$ is a vector in V_4.

The integrability conditions for the Eqs. (2.5) and (2.6) have the form [4]

$$R_{ijkl} = \sum_{\sigma=4}^{n-1} (b_{\sigma|ik} b_{\sigma|jl} - b_{\sigma|il} b_{\sigma|jk}); \qquad (2.7)$$

$$b_{\sigma|ij,k} - b_{\sigma|ik,j} = \sum_{\tau=4}^{n-1} (\nu_{\tau\sigma|k} b_{\tau|ij} - \nu_{\tau\sigma|j} b_{\tau|ik}); \qquad (2.8)$$

$$\nu_{\tau\sigma|j,k} - \nu_{\tau\sigma|k,j} + \sum_{\rho=4}^{n-1} (\nu_{\rho\tau|j}\nu_{\rho\sigma|k} - \nu_{\rho\tau|k}\nu_{\rho\sigma|j}) + g^{lm}(b_{\tau|lj} b_{\sigma|mk} - b_{\tau|lk} b_{\sigma|mj}) = 0. \qquad (2.9)$$

Here R_{ijkl} is the Riemannian curvature tensor of the surface V_4. From (2.7) and (2.8) it is easy to see that the Bianchi identity (cf. [4]) imposes on the coefficients the condition of being antisymmetric with respect to the indices τ and σ

$$\nu_{\tau\sigma|j} = -\nu_{\sigma\tau|j}. \qquad (2.10)$$

It will be more convenient to use the formulation of the equations of the surface V_4, Eqs. (2.5) and (2.6) in an orthogonal local base, introduced in the space V_4. We choose an orthogonal local base in V_4, consisting of unit vectors with components $\lambda^i_{h|}$:

$$g_{ij}\lambda^i_{h|}\lambda^j_{k|} = \begin{cases} a_h, & h = k \\ 0, & h \neq k \end{cases},$$

$$a_0 = -1, \qquad a_i = 1, \qquad i = 1, 2, 3. \qquad (2.11)$$

The coordinates of the base in S_n are defined as follows:

$$\frac{\partial z^\alpha}{\partial x^i} = \sum_{h=0}^{3} a_h \eta^\alpha_{h|}\lambda_{k;i}; \qquad \eta^\alpha_{h|} = \frac{\partial z^\alpha}{\partial x^i}\lambda^i_{h|};$$

$$\sum_{\alpha=0}^{n-1} c_\alpha \eta^\alpha_{h|}\eta^\alpha_{k|} = \begin{cases} a_h, & h = k \\ 0, & h \neq k \end{cases}. \qquad (2.12)$$

Introducing the operation of differentiation along the vectors of the base

$$\frac{\partial f}{\partial s^k} = \lambda^i_{k|}\frac{\partial f}{\partial x^i}, \qquad (2.13)$$

it is easy to transform the equations (2.5) and (2.6) to the form

$$\frac{\partial \eta^\alpha_{\beta|}}{\partial s^k} = \sum_{\mu=0}^{n-1} c_\mu \omega_{\beta\mu k}\eta^\alpha_{\mu|}$$

$$(\beta = 0, 1 \ldots n-1;\ k = 0, 1, 2, 3), \qquad (2.14)$$

where

$$\omega_{mnk} = \gamma_{mnk} = \lambda_{m|i,j}\lambda^i_{h|}\lambda^j_{k|}, \qquad k, m, n = 0, 1, 2, 3;$$

$$\gamma_{mnk} = -\gamma_{nmk} \qquad (2.15)$$

are the well known rotation coefficients, and

$$\omega_{\alpha hk} = -\omega_{h\alpha k} = -\lambda^i_{h|}\lambda^j_{k|}b_{\alpha|ij}$$

$$(\alpha = 4 \ldots n-1;\ h, k = 0, 1, 2, 3); \qquad (2.16)$$

$$\omega_{\alpha hk} = \omega_{\alpha kh}; \quad \omega_{\alpha\beta k} = \lambda^i_{k|}\nu_{\alpha\beta|i}; \quad \omega_{\alpha\beta k} = -\omega_{\beta\alpha k}$$

$$(\alpha, \beta = 4 \ldots n-1;\ k = 0, 1, 2, 3). \qquad (2.17)$$

The integrability conditions take the following form:

$$\frac{\partial \omega_{\mu\lambda h}}{\partial s^k} - \frac{\partial \omega_{\mu\lambda k}}{\partial s^h} = \sum_{\sigma=0}^{n-1} c_\sigma(\omega_{\sigma\mu h}\omega_{\sigma\lambda k} - \omega_{\sigma\mu k}\omega_{\sigma\lambda h});$$

$$\mu, \lambda = 0 \ldots n-1. \qquad (2.18)$$

We note, that for infinitesimal displacements ds_l along the directions of the base vectors, the components of a vector in this base vary by the following quantities

$$\delta B_{h|} = -\sum_{k,\,l=0}^{3} a_k a_l \omega_{hkl} B_{k|}\, ds_l. \qquad (2.19)$$

In the following we will denote by latin letters the subscripts of the coefficients $\omega_{\alpha\beta k}$ if they take on the values 0, 1, 2, 3, and by greek letters for the values 4, ..., n − 1. In the following section we will construct these coefficients as linear combinations of spinors of the space S_n.

3. THE FUNDAMENTAL EQUATIONS OF THE MODEL

We consider an n-dimensional pseudoeuclidean space S_n with the metric (2.1). We define in S_n spinors ψ and n matrices γ_α[3]. The number of components of the spinors, which is also equal to the rank of the matrices γ_α, is given by Eq. (1.3). The γ_α obey the anticommutation relations

$$\gamma_\alpha\gamma_\beta + \gamma_\beta\gamma_\alpha = \begin{cases} 2c_\alpha, & \alpha = \beta \\ 0, & \alpha \neq \beta \end{cases}. \qquad (3.1)$$

We will use a representation of the γ matrices, which has the following properties:

$$\gamma_0^+ = -\gamma_0, \qquad \gamma_\alpha^+ = \gamma_\alpha; \quad \alpha = 1 \ldots n-1.$$

The conjugate spinor is defined as[3]

$$\bar{\psi} = \psi^+\gamma_0. \qquad (3.2)$$

We introduce the matrices β_α which realize a representation equivalent to (3.1)

$$\beta_\alpha = \sum_\delta \eta^\delta_{\alpha|}\gamma_\delta; \qquad \alpha = 0, 1 \ldots n-1;$$

$$\beta_\alpha\beta_\gamma + \beta_\gamma\beta_\alpha = \begin{cases} 2c_\alpha, & \alpha = \gamma \\ 0, & \alpha \neq \gamma \end{cases}. \qquad (3.3)$$

We will construct the coefficients ω with the help of the matrices β_α.

We first consider the quantity $\omega_{\alpha nk}$, which according to (2.16) and (2.7) is related to the curvature tensor of the surface V_4. It would be most natural to write $\omega_{\alpha nk}$ in the form

$$\omega_{\alpha nk} = c\bar{\psi}\beta_\alpha(\beta_n\beta_k + \beta_k\beta_n)\psi. \qquad (3.4)$$

In this case the curvature tensor takes the simple form:

$$R_{ijkl} = C(x)(g_{ik}g_{jl} - g_{il}g_{jk}).$$

Then, according to Schur's theorem (cf.[4]) $C(x)$ = const and the space V_4 has constant curvature. This contradicts the condition, according to which the surface V_4 has to become flat for $|x^k - x^k_0| \gg l$, except if the curvature is zero everywhere. Thus, the representation (3.4) does not work, and one has to find another form for the function $\omega_{\alpha nk}$.

We introduce the matrix $\tilde{\gamma}$, which is a generalization of the ordinary γ_5 matrix. The definition of this matrix admits some arbitrariness, and we will require for the moment that for a flat space the following conditions be satisfied:

$$\tilde{\gamma} \to -i\beta_0\beta_1\beta_2\beta_3, \qquad (\tilde{\gamma})^2 \to 1;$$

$$\tilde{\gamma}\beta_n + \beta_n\tilde{\gamma} = f_n \to 0, \qquad n = 0, 1, 2, 3; \qquad (3.5)$$

$$\tilde{\gamma}\beta_\alpha - \beta_\alpha\tilde{\gamma} = \varphi_\alpha \to 0, \qquad \alpha = 4 \ldots n-1.$$

In the general case of a curved space f_n and φ_α

will not vanish, in general. We assume that in the general case the following relation holds

$$\gamma_0\tilde{\gamma}^+\gamma_0 = \tilde{\gamma}. \qquad (3.6)$$

Other properties of the matrix $\tilde{\gamma}$ will be postulated below.

The use of the matrix $\tilde{\gamma}$ allows us to satisfy the condition that the curvature tensor vanish at infinity in a noncontradictory manner. Indeed, let us replace the matrices β_α by the matrices

$$\alpha_\alpha = \beta_\alpha \pm \tfrac{1}{2}(\beta_\alpha\tilde{\gamma} - \tilde{\gamma}\beta_\alpha). \qquad (3.7)$$

Then for values $|x^k - x_0^k| \gg l$, for which the surface V_4 has to become flat, the quantities $\omega_{\alpha nk}$ and R_{ijkl} vanish, due to (3.5). Thus the use of the matrices α_δ turns out to be more natural than the use of the matrices β_δ.

We assume that all the matrices β_α which occur in our model ($\alpha = 0, \ldots, n-1$), have to be replaced by the matrices (3.7). We can now write out the expressions for the rotation coefficients $\omega_{\alpha\beta n}$, $\omega_{\alpha mn}$, ω_{mnk}:

$$\omega_{nmk} = ic_1\bar{\psi}(\alpha_n\alpha_k\alpha_m - \alpha_m\alpha_k\alpha_n)\psi;$$
$$\omega_{\alpha mn} = c_2\bar{\psi}(\alpha_n\alpha_\alpha\alpha_m + \alpha_m\alpha_\alpha\alpha_n)\psi;$$
$$\omega_{\alpha\beta n} = ic_3\bar{\psi}(\alpha_\alpha\alpha_n\alpha_\beta - \alpha_\beta\alpha_n\alpha_\alpha)\psi, \qquad (3.8)$$

where c_1, c_2, c_3 are real coefficients having the dimension l^2. In (3.8) the real character of the coefficients ω has been taken into account. The values of the spinors ψ have been taken on the surface V_4.

Let us consider the equation that the values of the spinor ψ satisfy on the surface V_4. In defining the notion of covariant derivatives for spinors we make use of Fock's method.[5] Taking into account the fact that the quantities $b_{\sigma|ij}$, $\nu_{\sigma\tau|h}$, $\lambda^i_{h|}$ have to be a tensor and vectors, respectively, in V_4, we require that for infinitesimal displacements ds_l along the vectors of the base in V_4, the quantity $\bar{\psi}\beta_n\psi$ should acquire increments which are characteristic of the coefficients of a vector in the local base, (2.19), and the quantities $\bar{\psi}\beta_\alpha\psi$, $\bar{\psi}\tilde{\gamma}\psi$ and $\bar{\psi}\psi$ should remain invariant. Thus we obtain the following conditions:

$$\delta\bar{\psi}\beta_n\psi = -\sum_{k,\,r=0}^{3} a_k a_r \omega_{nkr}\bar{\psi}\beta_k\psi ds_r;$$
$$\delta\bar{\psi}\beta_\alpha\psi = 0, \qquad \alpha = 4\ldots n-1;$$
$$\delta\bar{\psi}\tilde{\gamma}\psi = \delta\bar{\psi}\psi = 0. \qquad (3.9)$$

The increments of the spinor components have the form

$$\delta\psi = \sum_{l=0}^{3} a_l C_l \psi ds_l. \qquad (3.10)$$

In (3.9) it is also necessary to know the increments of the matrices β_n, β_α, $\tilde{\gamma}$. Since the representations of β_α ($\alpha = 0, \ldots, n-1$) are equivalent in each point, we require that the total increments of these matrices vanish:

$$d\beta_\alpha = \sum_{k=0}^{3}\frac{\partial\beta_\alpha}{\partial s^k}ds^k + \delta\beta_\alpha = 0, \qquad \delta\beta_\alpha = -\sum_{k=0}^{3}\frac{\partial\beta_\alpha}{\partial s^k}ds^k. \qquad (3.11)$$

The increment of the matrix $\tilde{\gamma}$ is defined as follows:

$$\delta\tilde{\gamma} = -\sum_{k=0}^{3} a_k(\tilde{\gamma}C_k + \gamma_0 C_k^+\gamma_0\tilde{\gamma})\,ds_k. \qquad (3.12)$$

From (3.9)–(3.11) and (2.14) follow the relations

$$\gamma_0 C_l^+\gamma_0\beta_n + \beta_n C_l - \sum_k a_k\omega_{nkl}\beta_k + \sum_\alpha \omega_{anl}\beta_\alpha = -\sum_k a_k\omega_{nkl}\beta_k,$$
$$\gamma_0 C_l^+\gamma_0\beta_\alpha + \beta_\alpha C_l - \sum_k a_k\omega_{akl}\beta_k - \sum_\beta \omega_{\alpha\beta l}\beta_\beta = 0,$$
$$\gamma_0 C_l^+\gamma_0 + C_l = 0. \qquad (3.13)$$

These equations allow one to determine C_l:

$$C_l = -\gamma_0 C_l^+\gamma_0 = \tfrac{1}{2}\sum_{\alpha=4}^{n-1}\sum_{n=0}^{3} a_n\omega_{anl}\beta_\alpha\beta_n + \tfrac{1}{4}\sum_{\alpha,\,\beta=4}^{n-1}\omega_{\alpha\beta l}\beta_\alpha\beta_\beta + ieA_l, \qquad (3.14)$$

where A_l are the components in the local base of an arbitrary real vector, the appearance of which can be connected with the possibility of introducing an electromagnetic interaction[5].

Thus, we have determined the components of the covariant derivative of a spinor in the orthogonal local base

$$\psi_{,k} = \frac{\partial\psi}{\partial s^k} + C_k\psi; \qquad \bar{\psi}_{,k} = \frac{\partial\bar{\psi}}{\partial s^k} - \bar{\psi}C_k. \qquad (3.15)$$

In analogy with the Dirac equation, we postulate the following equation for the spinor ψ:

$$\sum_{k=0}^{3} a_k\alpha_k\psi_{,k} = \sum_k a_k\alpha_k\left\{\frac{\partial\psi}{\partial s^k} + C_k\psi\right\} = 0. \qquad (3.16)$$

This equation can be formulated also in an arbitrary coordinate system.

In the present paper we will make use of Eq. (3.16) for an investigation of the problem whether an effective four-fermion interaction can appear, which would be a model for the weak interactions.[2)] The presence of the four-fermion interaction in (3.16) is obvious. In order to find out whether (3.16) is capable of yielding anything similar to the

[2)] The problem of a possible interpretation of the modification of the Dirac equation in Riemann space from the point of view of producing an effective interaction has been considered, e.g., by Green[6].

real interactions of the leptons, e.g. the interaction responsible for muon decay, we go over to the quasieuclidean approximation, i.e., we will consider that the space is flat and will retain in the equation only the terms of lowest order in the constants c_i.

By construction, the coefficients ω_{nmk} and $\omega_{\alpha mn}$ vanish in the limit of a flat space which is connected with the presence in these coefficients of several factors of the form $1 \pm \tilde{\gamma}$ separated by γ_n matrices. Consequently, in this approximation the following equation is obtained

$$i \sum_{n=0}^{3} a_n \gamma_n (1 \pm \tilde{\gamma}) \frac{\partial \psi}{\partial x^n} + \frac{c_3}{4} \sum_{n=0}^{3} \sum_{\alpha,\beta=4}^{n-1} a_n \gamma_\alpha \gamma_\beta \gamma_n (1 \pm \tilde{\gamma}) \times \psi \bar{\psi} (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha) \gamma_n (1 \pm \tilde{\gamma}) \psi = 0. \qquad (3.17)$$

In Eq. (3.17) all the spinors into which ψ decomposes for $|x^k - x_0^k| \gg l$, will enter only with two components. Since the model is massless, this is equivalent to the ordinary theory.

4. THE QUASIEUCLIDEAN APPROXIMATION

Let us analyze the interaction to which Eq. (3.17) leads in the quasieuclidean approximation. We choose S_n to be 9-dimensional, since this is the maximal dimension in which the spinors decompose into four four-component spinors. The matrices γ_α are defined in the standard manner [3] (cf. Appendix). Obviously the equation (3.17) is obtained in a theory with the interaction Lagrangian [3)]

$$L_{int} = \frac{c_3}{16} \sum_{\alpha,\beta,n} a_n \bar{\psi} (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha) \gamma_n (1 + \tilde{\gamma}) \times \psi \bar{\psi} (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha) \gamma_n (1 + \tilde{\gamma}) \psi. \qquad (4.1)$$

Let us see now how one can introduce the electromagnetic interaction into the model. Using the non-uniqueness in (3.14), one can obtain either four charged particles or four neutral ones. In order to be able to describe two charged particles and two neutral ones, one must define an asymptotically diagonal matrix, which would project the spinor ψ onto two four-component spinors. The matrix, e.g.,

$$E = \tfrac{1}{2} (I \pm i \beta_6 \beta_7) \qquad (4.2)$$

possesses these properties.

One can define the electromagnetic interaction by means of the matrix E, if all coefficients $\omega_{\alpha\beta n}$, in which at least one of the indices equals 6 or 7, vanish. Then E commutes with all the matrices of our model and, utilizing the ambiguity in (3.14), one can introduce the electromagnetic interaction in the following manner:

$$C_k = C_k^w + ieEA_k, \qquad (4.3)$$

where C_k^W is a term related to the four-fermion interaction.

Thus, we will sum in (4.1) over the indices α, β = 4, 5, 8. Using the explicit form of the γ_α - matrices (cf. Appendix) and decomposing the spinor ψ into the four-component spinors u_1, u_2, u_3, u_4 we obtain in place of (4.1) the expression

$$L_{int} = - c_3 \{(\bar{u}_1 u_3 \bar{u}_4 u_2 + \bar{u}_2 u_4 \bar{u}_3 u_1) + \tfrac{1}{2} (\bar{u}_1 u_1 \bar{u}_3 u_3 + \bar{u}_2 u_2 \bar{u}_4 u_4 + \bar{u}_1 u_1 \bar{u}_2 u_2 + \bar{u}_3 u_3 \bar{u}_4 u_4 - \bar{u}_1 u_1 \bar{u}_4 u_4 - \bar{u}_2 u_2 \bar{u}_3 u_3) + \tfrac{1}{4} (\bar{u}_1 u_1 \bar{u}_1 u_1 + \bar{u}_2 u_2 \bar{u}_2 u_2 + \bar{u}_3 u_3 \bar{u}_3 u_3 + \bar{u}_4 u_4 \bar{u}_4 u_4)\}, \qquad (4.4)$$

where

$$\bar{u}_i u_j \bar{u}_k u_l = \sum_{n=0}^{3} a_n \bar{u}_i \gamma_n' (1 + \gamma_5') u_j \bar{u}_k \gamma_n' (1 + \gamma_5') u_l,$$

and γ_n' and γ_5' are the usual matrices of the four-dimensional space.

Depending on the sign in (4.3), the charged particles will be described either by the spinors (u_1, u_2) or (u_3, u_4). The first two terms in (4.4) correspond to the interaction describing the decay of the muon in the V − A theory. If one does not require the possibility of introducing electromagnetic interactions and one sums over all values of α and β in (4.1), these "decay" terms will cancel out. Continuing further the analogy between the model and reality, one can associate the spinors u_i with leptons, e.g., in the following manner: u_1—electron, u_2—muon, u_3—electronic neutrino, u_4—muonic neutrino.

We emphasize the fact that in the definition of Eq. (3.16) there is an arbitrariness. In particular, one could have postulated the equation

$$\sum_{k=0}^{3} a_k \beta_k \psi_{,k} = 0, \qquad (4.5)$$

instead of (3.16). The fundamental results, in particular, the presence of the four-fermion interaction, including the "decay" interaction, parity nonconservation and the possibility of introducing electromagnetic interactions, remain valid. However, one does not obtain the V − A theory.

5. CONCLUSION

In this paper we considered a model with a noneuclidean geometry "in the small," which under certain supplementary assumptions can yield a description of the weak and electromagnetic inter-

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3)Here we have chosen the + sign in (3.7).

actions. The model describes massless particles. One may hope that the mass of the charged particles can be obtained in a self-consistent manner due to the interaction.

We indicate the problems which have to be considered in further investigations of the model. First, there is the question of compatibility of the system of equations which defines the model. Then, there is the question of the existence of divergences in the model, which is apparently also related to the problem of quantizing the spinor field ψ. It is possible that this problem will lead to supplementary conditions which will fix the explicit form of the matrix $\widetilde{\gamma}$.

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APPENDIX

We list here the explicit expressions for the matrices γ_α in nine-dimensional space. We label the components of the spinor ψ and the rows and columns of the matrices in the following order (the notation is taken from Cartan's book[3]):

0, 12, 13, 14, 23, 24, 34, 1234, 1, 2, 3, 4, 123, 124, 134, 234.

Then

$$\gamma_0 = \begin{vmatrix} 0 & I \\ -I & 0 \end{vmatrix}; \quad \gamma_\alpha = \begin{vmatrix} 0 & K_\alpha \\ K_\alpha & 0 \end{vmatrix},$$

$$\alpha = 1, 2, 3, 4, 5, 6, 7; \quad \gamma_8 = \begin{vmatrix} I & 0 \\ 0 & -I \end{vmatrix},$$

where I is the unit 8×8 matrix, and the nonvanishing matrix elements of K_α are:

$$(K_1)_{12} = -(K_1)_{35} = -(K_1)_{46} = (K_1)_{78} = 1,$$

$$(K_1)_{pq} = (K_1)_{qp};$$

$$(K_2)_{12} = -(K_2)_{35} = -(K_2)_{46} = (K_2)_{78} = i,$$

$$(K_2)_{pq} = -(K_2)_{qp};$$

$$(K_3)_{11} = -(K_3)_{22} = -(K_3)_{33} = -(K_3)_{44} = (K_3)_{55} = (K_3)_{66} = (K_3)_{77} = -(K_3)_{88} = 1;$$

$$(K_4)_{13} = (K_4)_{25} = -(K_4)_{31} = -(K_4)_{47} = -(K_4)_{52} = -(K_4)_{68} = (K_4)_{74} = (K_4)_{86} = i;$$

$$(K_5)_{13} = (K_5)_{25} = (K_5)_{31} = -(K_5)_{47} = (K_5)_{52} = -(K_5)_{68} = -(K_5)_{74} = -(K_5)_{86} = 1;$$

$$(K_6)_{14} = (K_6)_{26} = (K_6)_{37} = -(K_6)_{41} = (K_6)_{58} = -(K_6)_{62} = -(K_6)_{73} = -(K_6)_{85} = i;$$

$$(K_7)_{14} = (K_7)_{26} = (K_7)_{37} = (K_7)_{41} = (K_7)_{58} = (K_7)_{62} = (K_7)_{73} = (K_7)_{85} = 1.$$

Under transformations in the S_4 plane, the spinor ψ decomposes into spinors u_i in the following manner:

$$u_1 = (0, 12, 1, 2); \qquad u_2 = (13, 23, 3, 123);$$
$$u_3 = (14, 24, 4, 124); \qquad u_4 = (34, 1234, 134, 234).$$

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