ONE-MESON EXCHANGE AND THE ASYMPTOTIC BEHAVIOR OF NUCLEON-NUCLEON AND PION-NUCLEON SCATTERING

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The properties of the vertex functions in one-meson exchange graphs for NN and πN interactions are investigated. It is concluded that the one-meson exchange approximation is valid at very high energies and that head-on NN and πN collisions do not contribute significantly.

DEFINITE progress is now being made in the interpretation of the experimental data on nucleon-nucleon (NN) and pion-nucleon (πN) scattering and the photoproduction of π mesons at energies of a few BeV with the help of the one-meson exchange approximation (OMA). The extension of the OMA to the region of high and ultrahigh energies is complicated by the fact that, beginning at some energy, the cross sections of the OMA exceed the experimental values for the total cross sections (σ_{NN}, $\sigma_{\pi N}$) and continue to increase beyond limit as the energy of the colliding particles increases. This absurd rise in the cross sections must be regarded as indication that the vertex (cut-off) functions must be taken into account.

The aim of the present paper is to investigate the conditions which the cut-off functions have to satisfy in order that the OMA be applicable to interactions at very high energies. It will turn out that the one-meson interactions must be described by one generalized graph and not by two, as was assumed earlier.

1. ONE-MESON GRAPHS FOR NN AND πN SCATTERING

The most general one-meson graphs for inelastic NN scattering are the graphs 1 and 2, and for πN scattering, the graphs 3 and 4 of Fig. 1. Let us study the properties of these graphs without in advance making any assumptions about the contribution of the associated processes at very high energies.

The cross sections for the processes 1 and 2 at very high energies, where we can set $\sigma_{\pi N}$ = const, are given by the following expressions written in the c.m.s. ($\hbar = c = M_{\mathrm{nucl}} = 1$):

$$\sigma_{NN}^{(1)} = \frac{\tau_\alpha^2 g^2 \sigma_{\pi N}}{4\pi^2 j_{NN} \varepsilon_0} \int \frac{\Delta_1^2 \varepsilon_\pi^{(1)} j_{\pi N}^{(1)} F_1^2 F_2^2 F_{\pi N} d^4\Delta_1}{\varepsilon_1 (\Delta_1^2 + \mu^2)^2}, \tag{1}$$

$$\sigma_{NN}^{(2)} = \frac{4\sigma_{\pi N}^2}{(2\pi)^4 j_{NN}} \int \frac{\varepsilon_\pi j_{\pi N}^{(1)} \varepsilon_\pi j_{\pi N}^{(2)} F_1^2 F_{\pi N}^{(1)} F_{\pi N}^{(2)}\, d^4\Delta}{(\Delta^2 + \mu^2)^2}, \tag{2}$$

where ϵ_0 is the energy of the primary nucleon, ϵ_π is the energy of the virtual pion, ϵ_1 is the energy of the secondary nucleon emitted at the vertex containing the operator $\tau_\alpha g \gamma_5$, τ_α is the isospin matrix of the nucleon, g is the pion-nucleon coupling constant ($g^2 = 15$), Δ is the four-momentum of the virtual pion, j_{NN} is the relative current of the colliding nucleons, $j_{\pi N}$ is the relative current of the pion and the nucleon, $s_{\pi N}$ is the square of the center-of-mass energy of the pion and nucleon, and F_1, F_2, and $F_{\pi N}$ are unknown functions. $F_1(\Delta^2)$ takes account of the deviation of the Green's function of the virtual pion from the pole propagator $(\Delta^2 + \mu^2)^{-1}$, $F_2(\Delta^2)$ is the form factor of the proper vertex part (for the vertex $\tau_\alpha g \gamma_5$), and $F_{\pi N}(\Delta^2, s_{\pi N})$ takes account of the possible difference in the interaction of virtual and real pions with nucleons.[1] F_1 and F_2 are functions of the "virtuality" Δ^2 alone. $F_{\pi N}$ can, in prinicple, depend on Δ^2 as well as on the other invariant $s_{\pi N}$.

Assuming a finite constant contribution from the process 2 at $\epsilon_0 \to \infty$, Dremin and Chernavskiĭ[1] found that the dependence of $F_{\pi N}$ on Δ^2 and $s_{\pi N}$ is not factorable. We recall that as a

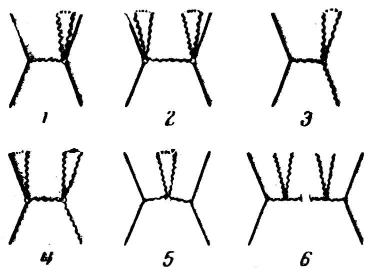

FIG. 1.

consequence of this result, the cross section for process 1 must go to zero for $\epsilon_0 \to \infty$. However, some properties of $F_{\pi N}$ can also be obtained without the introduction of any additional assumptions.

Let us consider the cross section for process 5. It can be derived in two ways. If we write the total cross section $\sigma_{\pi N}$ on the right-hand side of (1) as a sum, $\sigma_{\pi N} = \sigma_{\pi N}^{(3)} + \sigma'_{\pi N}$, where $\sigma_{\pi N}^{(3)}$ is the πN cross section according to graph 3 and $\sigma'_{\pi N}$ is the summed cross section of all the remaining processes, and substitute the expression (obtained by general rules)

$$\sigma_{\pi N}^{(3)} = \frac{\tau_\alpha^2 g^2 \sigma_{\pi\pi}}{4\pi^2 j_{\pi N}\varepsilon_0} \int \frac{\Delta^2 \varepsilon_\pi j_{\pi\pi} F_1^2(\Delta^2) F_2^2(\Delta^2) F_{\pi\pi}(\Delta^2, s_{\pi\pi})\, d^4\Delta}{\varepsilon(\Delta^2+\mu^2)^2} \tag{3}$$

in (1), we obtain

$$\sigma_{NN}^{(1)} = \frac{\tau_\alpha^2 \tau_\beta^2 g^4 \sigma_{\pi\pi}}{(4\pi^2)^2 j_{NN}\varepsilon_0^2} \int \frac{d^4\Delta_2 \Delta_1^2 \Delta_2^2 \varepsilon_\pi^{(1)} j_{\pi N} F_1^2(\Delta_1^2) F_2^2(\Delta_1^2) F_{\pi N}(\Delta_1^2, s_{\pi N})\, \varepsilon_\pi^{(2)} j_{\pi\pi} F_1^2(\Delta_2^2) F_2^2(\Delta_2^2) F_{\pi\pi}(s_{\pi\pi}, \Delta_2^2)\, d^4\Delta_1}{j_{\pi N}^{(2)} \varepsilon_1 \varepsilon_2 (\Delta_1^2+\mu^2)^2 (\Delta_2^2+\mu^2)^2}$$
$$+ \frac{\tau_\alpha^2 g^2}{4\pi^2 j_{NN}\varepsilon_0} \int \frac{\Delta_1^2 \varepsilon_\pi^{(1)} j_{\pi N} F_1^2(\Delta_1^2) F_2^2(\Delta_1^2) F_{\pi N}(\Delta_1^2, s_{\pi N})\, \sigma'_{\pi N}(s_{\pi N})\, d^4\Delta_1}{\varepsilon_1 (\Delta_1^2+\mu^2)^2}. \tag{4}$$

The first term in (4) represents the cross section for process 5. On the other hand, an expression for $\sigma_{\pi N}^{(5)}$ can be obtained directly:

$$\sigma_{NN}^{(5)} = \frac{\tau_\alpha^2 \tau_\beta^2 g^4 \sigma_{\pi\pi}}{(4\pi^2)^2 j_{NN}\varepsilon_0^2} \int \frac{d^4\Delta_2 \Delta_1^2 \Delta_2^2 \varepsilon_\pi^{(1)} F_1^2(\Delta_1^2) F_2^2(\Delta_1^2)\, \varepsilon_\pi^{(2)} F_1^2(\Delta_2^2) F_2^2(\Delta_2^2)\, j_{\pi\pi} F'_{\pi\pi}(\Delta_1^2, \Delta_2^2, s_{\pi\pi})\, d^4\Delta_1}{\varepsilon_1\varepsilon_2 (\Delta_1^2+\mu^2)^2 (\Delta_2^2+\mu^2)^2}, \tag{5}$$

where $F'_{\pi\pi}(s_{\pi\pi}, \Delta_1^2, \Delta_2^2)$ is a function which takes account of the difference between the scattering cross sections for two virtual pions and two real pions. Comparing (5) with the first term of (4), we find

$$F_{\pi N}(\Delta_1^2, s_{\pi N}) F_{\pi\pi}(\Delta_2^2, s_{\pi\pi}) = F'_{\pi\pi}(\Delta_1^2, \Delta_2^2, s_{\pi\pi}). \tag{6}$$

The left-hand side of (6) depends on $s_{\pi N}$, whereas the right-hand side does not contain the variable $s_{\pi N}$. Since $s_{\pi N}$ and $s_{\pi\pi}$ are independent variables, it follows that $F_{\pi N}$ is not a function of $s_{\pi N}$.

On the basis of this result we may assume that $F_{\pi\pi}$ and $F'_{\pi\pi}$ also are independent of $s_{\pi\pi}$. Then

$$F'_{\pi\pi}(\Delta_1^2, \Delta_2^2) = F_3(\Delta_1^2) F_3(\Delta_2^2). \tag{7}$$

The calculated $\sigma_{NN}^{(2)}$ increases beyond limit as ϵ_0 increases if $F_{\pi N}$ is independent $s_{\pi N}$, no matter how strong the dependence on Δ^2.

As will be shown in Sec. 3, the cross sections calculated according to graphs 1 and 3 with account of the nucleon from factor F_2 are finite and constant for $\epsilon_0 \to \infty$. Thus the one-meson exchange NN interaction must be described by the one-meson graph 1 and the πN interaction by the graph 3.

Noting that all relative currents entering in the expression for the cross section are equal at very high energies ($j_{NN} = j_{\pi N} = j_{\pi\pi} = 2$) and introducing the notation

$$B = \frac{\tau^2 g^2}{4\pi^2 \varepsilon_0} \int \frac{\Delta^2 \varepsilon_\pi F_1^2(\Delta^2) F_2^2(\Delta^2) F_3(\Delta^2)\, d^4\Delta}{\varepsilon(\Delta^2+\mu^2)^2}, \tag{8}$$

we can rewrite (1), (3), and (5) in the form

$$\sigma_{NN}^{(1)} = B\sigma_{\pi N}, \tag{9}$$

$$\sigma_{NN}^{(5)} = B^2\sigma_{\pi\pi}, \tag{10}$$

$$\sigma_{\pi N}^{(3)} = B\sigma_{\pi\pi}. \tag{11}$$

Eliminating B from (11) and (12), we obtain the equation

$$\sigma_{NN}^{(5)}\sigma_{\pi\pi} = (\sigma_{\pi N}^{(3)})^2. \tag{12}$$

Comparing (12) with the relation between the total πN and NN cross sections [2]

$$\sigma_{NN}\sigma_{\pi\pi} = \sigma_{\pi N}^2, \tag{13}$$

we see that (13) is not in contradiction with the assumption of the predominance of the contribution from the one-meson exchanges in NN and πN scattering at very high energies.

However, although (13) is not a necessary condition, it is not yet a sufficient basis for concluding that the one-meson interactions play an exclusive role. One sufficient condition for such a conclusion would be the agreement of the calculated values of $\sigma_{NN}^{(5)}$ and $\sigma_{\pi N}^{(3)}$ with experiment. To verify this, we must specify the form of the functions F_1, F_2, and F_3 entering in the expressions for the cross sections, and in particular, consider the nucleon form factor $F_2(\Delta^2)$.

2. FORM FACTOR FOR THE NUCLEON CORE

The introduction of the nucleon form factor $F_2(\Delta^2)$ in the calculations reflects the fact that the nucleon, as the source of the pion field, is not point-like. The account of the finite extension of the field source implies the transition to a nonlocal interaction. The invariant form factor is subject to the general requirement that it must not correspond to the transfer of a wave packet over macroscopic distances and hence, cannot be a function of the type of a propagator. As shown by Chrétien and Peierls,[3] the form factors may be represented by functions with poles at complex values of Δ^2 (whereas propagators have poles at real values of Δ^2).

For a determination of the explicit form of $F_2(\Delta^2)$ we must give the density distribution of the source. In the case of an exponential distribution we have

$$\rho(r) = \frac{3\sqrt{3}}{\pi a^3}\exp\left(-\frac{\sqrt{12}\,r}{a}\right), \tag{14}$$

where $a = \langle r^2\rangle^{1/2}$ is the mean square radius of the distribution. Going over to momentum space, we obtain

$$F(p^2) = \left(1 + \tfrac{1}{12}a^2p^2\right)^{-2}. \tag{15}$$

The variable p^2 is the square of the three-momentum of the virtual meson (or recoil nucleon) entering in a given nucleon vertex in the rest system.

To go from $F(p^2)$ to $F_2(\Delta^2)$ we must express p^2 in terms of Δ^2. In the rest system of the nucleon

$$\Delta^2 = \mathbf{\Delta}^2 - \Delta_0^2 = p^2 - \Delta_0^2.$$

In the same system

$$\Delta_0 = -\Delta^2/2,$$

and hence

$$p^2 = \Delta^2 + \Delta^4/4.$$

Thus the form factor for an exponential distribution is

$$F_2(\Delta^2) = [1 + \tfrac{1}{12}a^2(\Delta^2 + \Delta^4/4)]^{-2}. \tag{16}$$

The poles of this form factor are located at the points

$$\Delta^2 = -2 \pm \sqrt{4 - 4(12/a^2)}.$$

The above-mentioned requirement that Δ^2 be complex at the poles is satisfied if $12/a^2 > 1$, i.e., the mean square radius of an exponential distribution must satisfy the condition

$$\langle r^2\rangle^{1/2} < \sqrt{12}\,\hbar/Mc = 0.72\ \text{F}.$$

In the case of a Yukawa distribution of the form

$$\rho(r) = \frac{3}{2\pi a^3}\,\frac{\exp(-\sqrt{6}r/a)}{r} \tag{17}$$

we have the form factor

$$F(\Delta^2) = [1 + \tfrac{1}{6}a^2(\Delta^2 + \Delta^4/4)]^{-1}, \tag{18}$$

where the mean square radius must satisfy the condition $\langle r^2\rangle^{1/2} < 0.51$ F.

The estimates for the core radius of the nucleon were obtained with the help of a selection of phenomenological potentials for the nucleon-nucleon interaction which yield the correct phase shifts at low energies. According to Brueckner and Watson,[4] the core radius is equal to 0.17 to 0.19 F. In the later work of Lassila et al.[5] the core radius was taken to be 0.24 F. However, the quoted authors are considering the "hard" core, while we are concerned with an estimate of the mean square radius. The latter was found to have the value 0.2 F in the interpretation of the electromagnetic form factors of the nucleons.[6] We shall use this value in our further calculations. It is interesting to note that it is very close to the value of the Compton wave length of the nucleon, 0.21×10^{-13} cm.

3. ASYMPTOTIC BEHAVIOR OF THE CROSS SECTIONS

Besides F_2, we must also know the functions $F_1(\Delta^2)$ and $F_3(\Delta^2)$ in order to calculate the cross sections. The explicit form of F_1 is unknown, and we can only expect that $F_1^2(\Delta^2) \lesssim 1$ from the general properties of the spectral decomposition of the Green's function of the virtual boson. The results of the work of Okun' and Pomeranchuk[7] indicate that the large masses make a small contribution to the propagation function. For definiteness, we set $F_1^2(\Delta^2) = 1$, and return to this point somewhat later. As to the function F_3, there are no reasons to expect it to depend on Δ^2. We recall that a dependence of F_3 on $s_{\pi N}$ is not indicated. Let us therefore set $F_3 = 1$. Thus the cross sections will be computed on the basis of two assumptions and a single given parameter $\langle r^2\rangle^{1/2}$.

With these conditions and $\sigma_{\pi N} = 23$ mb, the cross section for process 1 for energies higher than 10^{11} ev is calculated to be equal to 29 mb. The behavior of the cross section $\sigma_{NN}^{(1)}$ at energies from 10 to 30 Bev, where there are sufficient experimental data (with an accuracy of ~4%) is in good agreement with the experimental curves for the inelastic scattering cross section (Fig. 2).

If the elastic cross section goes to zero in the asymptotic limit,[8] we find from the above-mentioned result that the total cross section has

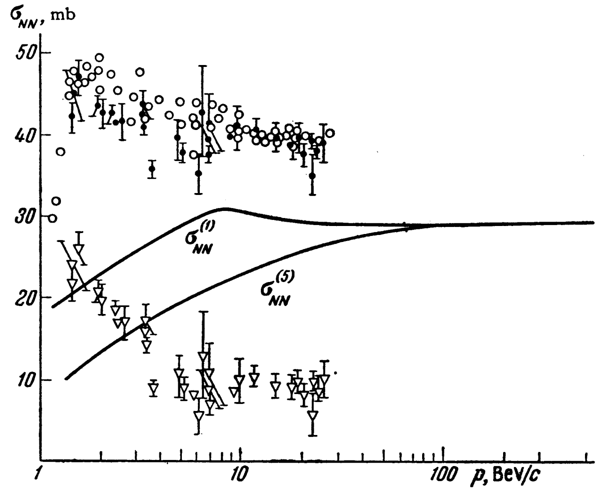

FIG. 2. Dependence of the inelastic NN scattering cross section on the momentum of the primary nucleon (in the lab system): ∇ experimental value of the elastic NN cross section, ○ experimental value of the total cross section, ● value of the total NN cross section obtained by addition of the experimental elastic scattering cross section and the curve for $\sigma_{NN}^{(1)}$ (p).

the value $\sigma_{NN} = 29$ mb (to the extent that $\sigma_{NN}^{(1)}$ agrees with experiment at high energies). Substituting the values $\sigma_{\pi N} = 23$ mb and $\sigma_{NN} = 29$ mb in (13), we obtain the estimate $\sigma_{\pi\pi} = 18.2$ mb for the total $\pi\pi$ scattering cross section at ultra-high energies. Using this value for $\sigma_{\pi\pi}$, we can compute the cross sections $\sigma_{NN}^{(5)}$ and $\sigma_{\pi N}^{(3)}$. At energies above 10^{11} ev the calculation gives $\sigma_{NN}^{(5)} = 29$ mb and $\sigma_{\pi N}^{(3)} = 23$ mb.

Thus the processes 5 and 3 (Fig. 1) not only satisfy condition (13), but also have cross sections which agree with the total NN and πN scattering cross sections within the limits of experimental and theoretical errors (the latter caused by the inaccuracy of the parameters). On this basis we arrive at the conclusion that the one-pion exchange processes play the dominant role, admitting, at the same time, a small contribution from other processes.

It follows from the results shown in Fig. 2 that the one-pion exchange processes are very important also at high energies (the curve for $\sigma_{NN}^{(1)}$ at these energies is calculated with account of all resonances in the total cross section for the πN scattering; at energies above 20 Bev the total cross section $\sigma_{\pi N}$ was taken to be equal to 23 mb). In this way one can understand the agreement of the calculations[9,10] with the experimental data on the inelastic scattering of protons on neutrons at 9 BeV. It was by including the graph 1 that one could explain the asymmetry in the emission of the secondary protons in the c.m.s.[10,11]

These results also show that $F_1^2 = 1$ and $F_3 = 1$ are good approximations. The deviation of F_1^2 from unity could be estimated by using more exact values for σ_{NN} and $\langle r^2 \rangle^{1/2}$. As to the function F_3, there are evidently no experimental indications that it should depend on Δ^2, and we can neglect this dependence in all calculations.

Despite the fact that not all one-meson exchange graphs play a distinct role at very high energies, but only those in which the scattering goes through the $\pi\pi$ interaction without "excitation" of the nucleons, the picture of the elementary process remains complicated. The center of attention becomes the intermediate $\pi\pi$ scattering. About this we remark the following. The graph 6 (Fig. 1) in which the $\pi\pi$ vertices are described by the total cross sections $\sigma_{\pi\pi}$ is a special case of graph 2, when the πN interactions in the nucleon vertices are described by the graph 3. Since $F_{\pi\pi}$ is independent of $s_{\pi\pi}$, the calculations gives arbitrarily high values for $\sigma_{NN}^{(6)}$ at $\epsilon_0 \to \infty$. It follows from this that it is impossible to express the interaction probabilities in the separate vertices in terms of the total cross sections $\sigma_{\pi\pi}$ by representing the $\pi\pi$ scattering by a chain of several $\pi\pi$ scatterings.

4. GENERAL COMPARISON WITH THE EXPERIMENTAL DATA ON NN SCATTERING AT VERY HIGH ENERGIES

Thanks to the intensive studies of recent years, a large amount of data has been obtained on the NN interaction at very high and ultra-high energies. The most detailed information is available on such features of the interaction as the coefficient of inelasticity, the angular distribution of the shower particles in the system of their center of inertia (c.m.s.), the distribution of the transverse momenta of the shower particles, and the dependence of the multiplicity of the shower particles n_s on the energy of the primary particle. It appeared possible to systematize all these data with the help of various models and phenomenological theories and, finally, to give the most probable picture for the NN interaction at very high energies.[12] However, the available information suffers from the defect that the shower selection factor is not excluded, that the criteria for the choice of the NN interactions are not sufficiently rigorous, and that the kinematical estimates of the energy of the primary particles are too approximate. An exception to this are the data on showers with a primary energy of the order of a hundred BeV obtained with the help of the ionization calorimeter.[13]

As it is not possible in this paper to make a detailed comparison between our calculations and the experimental data, we restrict ourselves to a comparison of the most general results.

A. Average coefficient of the inelasticity of the NN interaction. The distribution of the coefficient of inelasticity in the c.m.s. can be calculated exactly. The average ($\langle K \rangle$) and the most probable value of the coefficient of inelasticity are close to each other. It is noteworthy that the value $\langle K \rangle$ at energies above 10^{11} eV is independent of the energy of the primary nucleon. In the calculation based on graph 5 we have $\langle K \rangle = 0.35$ for the form factor (16) and $\langle K \rangle = 0.40$ for the form factor (18), which is in good agreement with the experimental estimates $\langle K \rangle = 0.3$ to 0.5,[12], $\langle K \rangle = 0.35$,[13] and $\langle K \rangle = 0.35$ to 0.40.[14]

B. Angular distribution of the shower particles. The angular distribution of the shower particles in the center-of-mass system is given in the case of the graph 5 by the dynamics of the $\pi\pi$ interaction. In particular, it has been noted[15] that the "two-center" structure of the showers with small inelasticity coefficients can be explained by assuming that the multiple production of particles at very high energies goes through an intermediate $\pi\pi$ interaction where, in principle, two blobs of highly excited nuclear matter can be formed. The model of the intermediate $\pi\pi$ interaction also explains the formation of "asymmetric" showers.[13]

C. Energetically distinguished pions. Recently, energetically distinguished π mesons have been observed in experiments at energies of $\gtrsim 10^{11}$ eV, which arise from the decay of nucleon isobars.[14] However, if the creation of energetically distinguished pions occurs with appreciable probability, it should, according to the above, be regarded as a result of the intermediate $\pi\pi$ interaction.

D. Angular distribution and transverse momenta of the secondary nucleons. In order to obtain the angular and momentum distributions of the shower particles in the c.m.s., it is necessary to develop the theory of the formation and decay of the excited $\pi\pi$ systems. The angular and momentum distribution in the c.m.s. as well as the distribution of the transverse momenta of the secondary nucleons can be computed. The angular distribution of the nucleons in the c.m.s. is characterized by a strong anisotropy, which increases with the primary energy. For $\epsilon_0 = 13$ (which corresponds to an energy of 300 BeV in the lab system) half of the total number of nucleons should be in the angular region from 0 to 5° and from 175 to 180°. Because of the extremely collimated emission in the c.m.s., the nucleons, which carry off 60 to 65% of the total energy on the average, have a relatively small average transverse momentum. The calculations show that the average transverse momentum of the nucleons for energies above 10^{11} eV must be equal to 0.61 BeV/c and be independent of the energy of the primary nucleon. This result is not in disagreement with the experimental data on the showers and extended atmospheric showers.[16] In the region of small energies, $\langle p_\perp \rangle$ is decreased so much that at higher energies it agrees with experiment within the limits of error.

From the results quoted above we arrive at the general conclusion: the one-meson exchange approximation is, despite the large value of the πN coupling constant, a method which gives a correct description of the basic features of the inelastic interactions of nucleons and pions with nucleons for a very wide range of energies (from several BeV upwards). The reason for the dominant role of the one-meson interactions is evidently the large probability for the existence of one-meson states in the structure of the nucleon. In the consideration of the one-meson interactions it is necessary to take account of the structure of the nucleon as the source of the pion field. The agreement between theory and experiment is bought at the price of giving up the locality of the pion-nucleon interaction.

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