# ASYMPTOTIC RELATIONS BETWEEN POLARIZATIONS IN CROSSED REACTIONS

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Asymptotic relations between polarizations in crossed reactions are derived. In particular, it is shown that for large energies the polarizations of the recoil protons in elastic scattering processes of positive and negative pions on protons are of the same magnitude but opposite sign. Also considered are the polarizations in scattering processes of nucleons and antinucleons on nucleons and in reactions in which strange particles participate.

1. Recently the well-known Phragmén–Lindelöf theorem from the theory of functions of a complex variable[1] has been used to derive asymptotic relations between cross sections for various reactions. Thus Meĭman[2] derived relations between the total cross sections for the interactions of particles and antiparticles at high energies on the basis of this theorem, relations which had been obtained earlier by Pomeranchuk[3] on the basis of the dispersion relation technique. In the papers by Logunov et al.[4-6] the Pomeranchuk relations for the differential cross sections at angle 0° are generalized by the Phragmén–Lindelöf theorem to non-vanishing momentum transfers. We derive below, on the basis of this theorem, asymptotic relations between the polarizations in crossed reactions. We note that the realization of polarized hydrogen targets[7,8] facilitates essentially the measurement of polarizations at high energies and may make possible a verification of these relations in the not too far future. We restrict our considerations to the simplest cases of reactions in which spin 0 and ½ particles participate. Our treatment is purely phenomenological and we will not consider the mechanism by which polarization is produced at high energies.

We enumerate the essential results of our work. For large energies:

1) the polarizations of the recoil protons in $\pi^+p$ and $\pi^-p$ scattering at the same values of energy and angle are equal in magnitude but of opposite signs;

2) the polarization of the neutron in the charge exchange process $\pi^- + p \to \pi^0 + n$ vanishes;

3) the polarizations of the hyperons in the processes $\pi + p \to K + Y$ and $\bar{K} + p \to \bar{\pi} + Y$ are opposite, independently of the relative intrinsic parities of the particles; the same is true also for the processes $K^- + p \to K^0 + \Xi^0$ and $\bar{K}^0 + p \to K^+ + \Xi^0$;

4) the polarizations of the final particles in reactions of the type $\Sigma^+ + He \to He_\Lambda + p$ and $\bar{p} + He \to He_\Lambda + \bar{\Sigma}^+$ are opposite if the relative parity of the $\Sigma$ and $\Lambda$ particles is $+1$ and the polarizations are equal if the relative parity is $-1$;

5) the polarizations of the final particles in the elastic scattering processes $N + N \to N + N$ and $\bar{N} + N \to \bar{N} + N$, and also in the elastic scattering processes of strange particles $Y + N \to Y + N$ and $\bar{Y} + N \to \bar{Y} + N$ are opposite; the polarizations of the recoil neutron, e.g., in the processes $\Sigma^- + p \to \Lambda + n$ and $\bar{\Lambda} + p \to \bar{\Sigma}^- + n$, are also opposite;

6) the polarization of the $\Xi$ hyperons in the process $K^- + p \to K^+ + \Xi^-$ vanishes;

7) the polarization of the recoil protons in elastic scattering of gamma-quanta on protons also vanishes.

All the above statements refer to polarizations which appear in collisions of unpolarized particles. The first of the results listed above has been obtained previously by Levintov[11] on the basis of dispersion relations.

2. We start from a detailed analysis of the simplest case of pion scattering on nucleons. The amplitudes of the processes

$$\pi^+ + p \to \pi^+ + p, \qquad \pi^- + p \to \pi^- + p \tag{1}$$

have the following form:

$$M_\pm(p'\,q'; pq) = a_\pm + \tfrac{1}{2} i b_\pm (\hat{q} + \hat{q}'), \tag{2}$$

where q and q′ are the initial and final momenta of the meson, p and p′ the corresponding momenta of the proton, $a_\pm$ and $b_\pm$ are functions of $s = -(p+q)^2$ and $t = -(p-p')^2$, and the signs plus or minus refer to the scattering of positive and negative pions, respectively.

The polarization of the recoil proton which is produced in the scattering of mesons on unpolarized protons can be easily found by using the following formula[9,10]:

$$\xi_\mu = \frac{\mathrm{Sp}\, i\gamma_5\gamma_\mu M\Lambda(p)\,\overline{M}\Lambda(p')}{\mathrm{Sp}\, M\Lambda(p)\,\overline{M}\Lambda(p')}, \qquad (3)$$

where $\xi_\mu$ is the polarization four-vector, which is orthogonal to the momentum p′ and $\Lambda(p)$ and $\Lambda(p')$ are projection operators. We note that in the laboratory system (l.s.) and in the center-of-mass system (c.m.s.) $\xi_4$ vanishes, and $\boldsymbol{\xi}$ is equal to the expectation value of the operator $\sigma$ in the rest system of the recoil nucleon. From (2) and (3) we derive the following expression for the polarization:

$$\xi_\mu^\pm = \frac{2\mathrm{Im}\, a_\pm b_\pm^* \,[t\,(su-(M^2-\mu^2)^2]^{1/2} n_\mu}{|a_\pm|^2(4M^2-t)+2\mathrm{Re}\,a_\pm b_\pm^* M\,(u-s)+{}^1\!/_4\,|b_\pm|^2\,[(u-s)^2-t\,(t-4\mu^2)]}, \qquad (4)$$

where M and $\mu$ are the masses of the proton and pion, $u = -(p-q')^2 = 2(M^2+\mu^2) - s - t$, and $n_\mu$ is a space-like unit vector proportional to $i\epsilon_{\mu\nu\rho\sigma}p_\nu q_\rho p'_\sigma$. In the c.m.s. $n_4 = 0$ and $\mathbf{n} = (\mathbf{p}\times\mathbf{p}')/|\mathbf{p}\times\mathbf{p}'|$.

For $s \gg t$ and $M^2$, Eq. (4) implies

$$\xi_\mu^\pm = \frac{2\mathrm{Im}\, a_\pm b_\pm^* s\sqrt{-t}}{|2Ma_\pm - sb_\pm|^2 - t\,|a_\pm|^2}\, n_\mu. \qquad (5)$$

From this expression it can be seen that the polarization differs from zero for $s \to \infty$ and fixed t only in the case when $a_\pm$ and $M^{-1}sb_\pm$ have the same behavior in the indicated region of values of the variables s and t.

Let us assume that asymptotically $a_+$ and $M^{-1}sb_+$ behave like $s^{\alpha(t)}\varphi(s,t)$, where $\alpha(t)$ and $\varphi(s,t)$ are functions which are defined in [4-6]. Then, as shown in [4], the Phragmén–Lindelöf theorem implies that

$$a_+(-s,t) = e^{i\pi\alpha(t)}a_+(s,t), \qquad (6)$$

$$b_+(-s,t) = -e^{i\pi\alpha(t)}b_+(s,t). \qquad (7)$$

We now utilize the condition of crossing symmetry which connects the amplitudes $M_+$ and $M_-$:

$$M_-(p'q';pq) = \gamma_4 M_+^+(p-q';p'-q)\gamma_4. \qquad (8)$$

From (8) follow the following relations for the functions $a_\pm$ and $b_\pm$;

$$a_-(s,t) = a_+^*(u,t), \qquad b_-(s,t) = -b_+^*(u,t). \qquad (9)$$

Combining (9) and (7) we obtain for $s \to \infty$ and fixed t

$$a_-(s,t) = e^{-i\pi\alpha(t)}a_+^*(s,t), \quad b_-(s,t) = e^{-i\pi\alpha(t)}b_+^*(s,t). \qquad (10)$$

From (10) and (5) we find the following asymptotic relation between the polarizations

$$\xi_\mu^+(s,t) = -\xi_\mu^-(s,t). \qquad (11)$$

Here we suppose that the reactions (1a) and (1b) are compared at the same values of energy and angle.

Thus, if for large energies the polarization of the recoil protons in the scattering of positive pions on nucleons does not vanish, the polarization of the recoil protons in negative pion scattering on protons will also not vanish and differs only in sign from the polarization in $\pi^+p$ scattering.

We go over to the consideration of charge exchange process

$$\pi^- + p \to \pi^0 + n. \qquad (12)$$

Utilizing crossing symmetry we connect the amplitude for the reaction (12) in the unphysical region with the amplitude of the process

$$\pi^+ + n \to \pi^0 + p. \qquad (13)$$

However, owing to charge symmetry, the amplitudes for the processes (12) and (13) coincide. With the aid of crossing symmetry and the Phragmén–Lindelöf theorem we obtain for the charge exchange process

$$a_0(s,t) = e^{-i\pi\alpha_0(t)}a_0^*(s,t), \quad b_0(s,t) = e^{-i\pi\alpha_0(t)}b_0^*(s,t). \qquad (14)$$

These equalities signify that $\mathrm{Im}\, a_0b_0^* = 0$ and that the polarization of the recoil neutron in (12) vanishes at large energies and in the case of identical asymptotic behavior of the functions $a_0$ and $M^{-1}sb_0$.

**3.** Let us consider the reactions

$$\pi + p \to Y + K, \qquad (15a)$$

$$\overline{K} + p \to Y + \overline{\pi}. \qquad (15b)$$

If the intrinsic parities $I_i$ and $I_f$ of the initial and final particles coincide, the amplitude of the process (15a) is

$$M(p'q';pq) = a + \tfrac{1}{2} ib\,(\hat{q} + \hat{q}'), \qquad (16)$$

where q and q′ are the momenta of the pions or K mesons, and p and p′ are the momenta of the nucleon and the hyperon.

In the case of parity-flip ($I_f = -I_i$) the amplitude of this process can be written in the form

$$M(p'q';pq) = c\gamma_5 + \tfrac{1}{2} id\gamma_5 (\hat{q} + \hat{q}'). \tag{17}$$

For $s \to \infty$ and fixed $t$, in the case $I_i = I_f$ the polarization is

$$\xi_\mu = \frac{2\,\mathrm{Im}\, ab^* s \sqrt{-t}}{|a(M+M') - sb|^2 - t|a|^2} n_\mu, \tag{18}$$

where $M$ and $M'$ are the masses of the nucleon and hyperon, and $n_\mu$ has the same significance as above. Similarly, the asymptotic expression of the polarization for $I_i = -I_f$ becomes

$$\xi_\mu = -\frac{2\,\mathrm{Im}\, cd^* s \sqrt{-t}}{|c(M-M') - sd|^2 - t|c|^2} n_\mu. \tag{19}$$

A crossing relation of the form (8) connects the amplitude of the reaction (15a) in the unphysical region with the amplitude for the reaction $\bar{\pi} + Y \to \bar{K} + p$ which is the inverse of the reaction (15b). The amplitude of this process can be connected with the amplitude of the reaction (15b) by making use of PT invariance. The crossing symmetry (8) and PT invariance yield

$$M_c(p'q';pq) = \eta\gamma_4 UM^*(p'-q;\,p-q')\,U^{-1}\gamma_4. \tag{20}$$

Here $M_c$ is the amplitude of the process (15b), $U$ is a matrix satisfying the condition $U\gamma_\mu^T U^{-1} = \gamma_\mu$, and $\eta$ is a phase factor which appears in the PT transformation. In the case $I_f = I_i$ Eq. (20) yields

$$a_c(s,t) = \eta a^*(u,t), \qquad b_c(s,t) = -\eta b^*(u,t). \tag{21}$$

For $I_f = -I_i$ we obtain

$$c_c(s,t) = -\eta c^*(u,t) \qquad d_c(s,t) = \eta d^*(u,t). \tag{22}$$

In these equations the functions $a_c$, $b_c$, and $c_c$, $d_c$ are related to the amplitude $M_c(p',q';p,q)$ of the process (15b) by relations similar to (16) and (17).

Assuming as in Sec. 2 identical asymptotic behavior of the amplitudes $a$, $M^{-1}sb$ and $c$, $M^{-1}sd$ and applying the Phragmén–Lindelöf theorem for $s \to \infty$ and fixed $t$, we obtain

$$I_f = I_i: \qquad a_c(s,t) = \eta e^{-i\pi\alpha_1(t)} a^*(s,t),$$
$$b_c(s,t) = \eta e^{-i\pi\alpha_1(t)} b^*(s,t); \tag{23}$$
$$I_f = -I_i: \quad c_c(s,t) = -\eta e^{-i\pi\alpha_2(t)} c^*(s,t),$$
$$d_c(s,t) = -\eta e^{-i\pi\alpha_2(t)} d^*(s,t). \tag{24}$$

Expressions for the polarization $\xi_\mu^c$, which is produced in reaction (15b), can be obtained from (18) and (19) by the substitution $a \to a_c$, etc. Taking this into account, we obtain from (18), (19), (23), and (24) that, independently of the relative parity, the polarizations in the reactions (15a) and (15b) have equal magnitude and opposite signs:

$$\xi_\mu^c = -\xi_\mu. \tag{25}$$

This naturally refers also to the reactions

$$K^- + p \to K^0 + \Xi^0, \qquad \bar{K}^0 + p \to K^+ + \Xi^0,$$

if the spin of the $\Xi$ hyperon is $\frac{1}{2}$.

We also note that the use of the crossing symmetry condition (20) and of the Phragmén–Lindelöf theorem in the case of reaction $K^- + p \to K^+ + \Xi^-$ leads to the conclusion that for $s \to \infty$ and fixed $t$ the polarization of the $\Xi$ hyperon vanishes independently of the assumptions about the asymptotic behavior of the individual parts of the amplitude.

We show now that in the reactions

$$Y_1 + A \to Y_2 + B, \tag{26a}$$
$$\bar{Y}_2 + A \to \bar{Y}_1 + B, \tag{26b}$$

where $A$ and $B$ are spin-0 particles and $Y_1$ and $Y_2$ are spin-$\frac{1}{2}$ particles, the polarizations $\xi_\mu$ and $\xi_\mu^c$ of the particles $Y_2$ and $\bar{Y}_1$ are opposite,

$$\xi_\mu^c = -\xi_\mu, \tag{27}$$

if $I_f = I_i$, and are equal

$$\xi_\mu^c = \xi_\mu, \tag{28}$$

if $I_f = -I_i$. For this we write the matrix elements of the processes (26a) and (26b) in the form

$$\bar{u}(p')\,N(p'q';pq)\,u(p), \qquad \bar{u}(p')\,N_c(p'q';pq)\,u(p),$$

where $u(p)$ and $u(p')$ are positive energy spinors. The amplitudes $N$ and $N_c$ have the form (16) or (17), depending on the relative parity of the particles. The crossing symmetry condition has in this case the form

$$N_c(p'q';pq) = \eta'\gamma_4 N^+(p-q';\,p'-q)\,\gamma_4, \tag{29}$$

where $\eta'$ is a phase factor appearing due to charge conjugation. If the parity does not change, (29) leads to relations of the type (21). If the intrinsic parity changes there will appear relations which differ from (22) only by the sign of the second equality. Since the expressions of the polarizations will have the forms (18) and (19), we obtain (27) and (28). Examples of reactions (26) are the following pairs:

$$\Sigma^+ + \mathrm{He} \to \mathrm{He}_\Lambda + p, \qquad \bar{p} + \mathrm{He} \to \mathrm{He}_\Lambda + \bar{\Sigma}^+;$$
$$\Xi^- + \mathrm{He} \to \mathrm{He}_\Lambda + \Sigma^-, \qquad \bar{\Sigma}^- + \mathrm{He} \to \mathrm{He}_\Lambda + \bar{\Xi}^-.$$

4. We go over now to the more complicated case of reactions with particles of spin $\frac{1}{2}$. We consider first the reactions

$$\Sigma^- + p \to \Lambda + n, \tag{30}$$
$$\bar{\Lambda} + p \to \bar{\Sigma}^- + n. \tag{31}$$

The amplitude of the process (30) can be written in the form

$$M(p_1'p_2';\ p_1p_2) = a + b\gamma_5^{(2)} + c\gamma^{(2)}K_1 + d\gamma_5^{(2)}\gamma^{(2)}K_1, \quad (32)$$

where $p_1$ and $p_1'$ are the momenta of the proton and the neutron, $p_2$ and $p_2'$ are the momenta of the $\Sigma$ and $\Lambda$ hyperons, $K_1 = \frac{1}{2}(p_1 + p_1')$, and a, b, c, d are matrices in the spin indices of the nucleons.

Assuming identical intrinsic parities for the $\Sigma$ and $\Lambda$, we have

$$a = a_1 + ia_2\gamma^{(1)}K_2, \qquad b = b_1\gamma_5^{(1)} + ib_2\gamma_5^{(1)}\gamma^{(1)}K_2,$$
$$c = c_1 + ic_2\gamma^{(1)}K_2, \qquad d = d_1\gamma_5^{(1)} + id_2\gamma_5^{(1)}\gamma^{(1)}K_2,$$
$$K_2 = \tfrac{1}{2}(p_2 + p_2'). \quad (33)$$

The expression for the polarization of the final neutron can be obtained by means of an equation similar to (3). We give only the final result for $s \to \infty$ and fixed t:

$$\xi_\mu = 2\sigma^{-1}s\sqrt{-t}\,n_\mu \{[(m+m')^2 - t]\ \mathrm{Im}\ a_1a_2^* - [(m'-m)^2 - t]\ \mathrm{Im}\ b_1b_2^* + s^2\ \mathrm{Im}\ c_1c_2^* - s^2\ \mathrm{Im}\ d_1 d_2^* + (m+m')\,s\ \mathrm{Re}\,(a_2c_1^* - a_1c_2^*) + (m-m')\,s\ \mathrm{Re}\,(b_1 d_2^* - b_2 d_1^*)\}, \quad (34)$$

where

$$\sigma = [(m+m')^2 - t]\{|a_1|^2(4M^2 - t) - 4Ms\ \mathrm{Re}\ a_1a_2^* + s^2|a_2|^2\} + [(m'-m)^2 - t]\{-t\,|b_1|^2 + 2(m'^2 - m^2)M\ \mathrm{Re}\ b_1b_2^* + s^2|b_2|^2\} + s^2[|c_1|^2(4M^2 - t) - 4Ms\ \mathrm{Re}\ c_1c_2^* + s^2|c_2|^2] + s^2[-t\,|d_1|^2 + 2M(m'^2 - m^2)\ \mathrm{Re}\ d_1 d_2^* + s^2|d_2|^2] + (m+m')\,s\{(4M^2 - t)\ \mathrm{Im}\ a_1c_1^* - 2Ms\ \mathrm{Im}\,(a_1c_2^* + a_2c_1^*) + s^2\ \mathrm{Im}\ a_2c_2^*\} + (m-m')\,s\{-t\ \mathrm{Im}\ b_1 d_1^* + (m'^2 - m^2)\ \mathrm{Im}\,(b_2 d_1^* + b_1 d_2^*) + s^2\ \mathrm{Im}\ b_2d_2^*\}, \quad (35)$$

m and m′ are the masses of the $\Sigma$ and $\Lambda$ hyperons, respectively, and M is the nucleon mass.

The crossing condition has the form

$$M_c(p_1'p_2';\ p_1p_2) = \gamma_4^{(2)}\gamma_4^{(1)}C^{(1)}M^{*T^{(2)}}(-p_1p_2';\ -p_1p_2')\,C^{(1)-1}\gamma_4^{(1)}\gamma_4^{(2)}. \quad (36)$$

Here $M_c(p_1'p_2';\ p_1p_2)$ is the amplitude of the reaction (31), $T^{(2)}$ means transposition in the spin indices of the hyperons, and C is the charge conjugation matrix, satisfying the conditions $C\gamma_\mu^T C^{-1} = -\gamma_\mu$ and $C^T = -C$. We note that in writing (36) we have omitted a phase factor which is inessential for what follows and which is due to the charge conjugation.

Obviously the amplitude $M_c(p_1'p_2';\ p_1p_2)$ has the same form as the amplitude (32). We denote the corresponding coefficients by $a_1^c$, $a_2^c$, ... etc. From (36) we obtain

$$a_1^c(s,t) = a_1^*(u,t), \qquad a_2^c(s,t) = -a_2^*(u,t),$$
$$b_1^c(s,t) = b_1^*(u,t), \qquad b_2^c(s,t) = -b_2^*(u,t),$$
$$c_1^c(s,t) = c_1^*(u,t) \qquad c_2^c(s,t) = -c_2^*(u,t),$$
$$d_1^c(s,t) = -d_1^*(u,t), \qquad d_2^c(s,t) = d_2^*(u,t), \quad (37)$$

where

$$u = 2M^2 + m^2 + m'^2 - s - t.$$

The neutron polarization in reaction (31) is obtained from (34) and (35) by means of the substitution $a_1 \to a_1^c$ etc, and also $m \rightleftarrows m'$.

We first assume that the functions

$$a_1,\ sa_2,\ b_1,\ sb_2,\ sc_1,\ s^2c_2,\ sd_1,\ s^2d_2 \quad (38)$$

behave identically for $s \to \infty$ and fixed t. In this case, as can be seen from (34) and (35), the polarization does not vanish. Equation (37) and the Phragmén–Lindelöf theorem imply for $s \to \infty$

$$a_1^c = e^{-i\pi\alpha(t)}a_1^*, \qquad a_2^c = e^{-i\pi\alpha(t)}a_2^*,$$
$$b_1^c = e^{-i\pi\alpha(t)}b_1^*, \qquad b_2^c = e^{-i\pi\alpha(t)}b_2^*,$$
$$c_1^c = -e^{-i\pi\alpha(t)}c_1^*, \qquad c_2^c = -e^{-i\pi\alpha(t)}c_2^*,$$
$$d_1^c = e^{-i\pi\alpha(t)}d_1^*, \qquad d_2^c = e^{-i\pi\alpha(t)}d_2^*. \quad (39)$$

From (39) and the expressions for the polarizations of the neutrons in reactions (30) and (31) it is obvious that the polarizations in this case are opposite:

$$\xi_\mu^c = -\xi_\mu. \quad (40)$$

We now assume that not all the functions (38) have the same behavior as $s \to \infty$. Then it is necessary for nonvanishing polarization that at least the two functions which increase most rapidly have the same behavior (naturally, this pair of functions must occur as a product in the numerator of the expression of the polarization). Obviously, Eq. (40) holds in this case, too.

We have considered the case when the $\Sigma$ and $\Lambda$ hyperons have the same intrinsic parities. It can be shown, that the neutron polarizations in reactions (30) and (31) are connected by the relation (40) also in the case of opposite intrinsic parities.

We now consider the elastic scattering of hyperons and antihyperons by nucleons:

$$Y + p \to Y + p, \quad (41)$$
$$\bar{Y} + p \to \bar{Y} + p. \quad (42)$$

The amplitudes of these processes have the form (32) and (33), with $b_2 = d_1 = 0$. The latter conditions arise from time reversal invariance. Therefore all preceding relations are true also for the case of the elastic scattering (41) and (42) and the polarization of the recoil protons in these processes are also correlated by Eq. (40). Similarly one can show that the polarizations of the final hyperons and antihyperons in (41) and (42) satisfy Eq. (40). We note that in the case of the eight-term amplitudes which describe the processes (30) and (31) it is impossible to draw conclusions on the relations between the polarizations of the hyperons and antihyperons, independently of the assumptions about the character of the asymptotic behavior of the individual terms of the amplitude for fixed t and $s \to \infty$.

5. In conclusion we consider briefly the Compton effect on protons. The amplitude for this process can be written in the form

$$M(p'k'; pk) = \frac{(\varepsilon'P')(\varepsilon P')}{P'^2}[A_1 + iA_2\hat{K}] + \frac{(\varepsilon'N)(\varepsilon N)}{N^2}[A_3 + iA_4\hat{K}] + \frac{(\varepsilon'P')(\varepsilon N) - (\varepsilon'N)(\varepsilon P')}{\sqrt{2P'^2N^2}} i\gamma_5 A_5 + \frac{(\varepsilon'P')(\varepsilon N) + (\varepsilon'N)(\varepsilon P')}{\sqrt{2P'^2N^2}}\gamma_5\hat{K}A_6, \qquad (43)$$

where p and p′ are the momenta of the initial and final protons, while k, ϵ and k′, ϵ′ are the momenta and polarizations of the incident and scattered photons, respectively,

$$K = \tfrac{1}{2}(k + k'), \qquad P = \tfrac{1}{2}(p + p'),$$

$$P' = P - \frac{(PK)}{K^2}K, \qquad N_\alpha = i\varepsilon_{\alpha\beta\gamma\delta}P'_\beta K_\gamma (k - k')_\delta.$$

For $s \to \infty$ and fixed t the polarization of the recoil proton turns out to be

$$\xi_\mu = \frac{s\sqrt{-t}\,2\,\mathrm{Im}\,(A_1A_2^* + A_3A_4^*)\,n_\mu}{(|A_1|^2+|A_3|^2)(4M^2-t) + (|A_2|^2+|A_4|^2)s^2 - 4\mathrm{Re}(A_1A_2^* + A_3A_4^*)Ms - t|A_5|^2 + s^2|A_6|^2}. \qquad (44)$$

Applying the crossing relations

$$A_{1,3,5,6}(s, t) = A^*_{1,3,5,6}(u, t), \qquad A_{2,4}(s, t) = -A^*_{2,4}(u, t) \qquad (45)$$

and the Phragmén–Lindelöf theorem, we can verify that the polarization of the proton vanishes as $s \to \infty$ for fixed t, independently of the assumptions on the asymptotic behavior of the amplitudes.

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