MINIMAL INTERACTIONS BETWEEN SPIN 0, ½, AND 1 FIELDS

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The most general Lagrangian describing class A interactions [1] for an arbitrary system of fields of spin 0, ½, and 1 is derived under the sole restriction that the coupling constants are dimensionless (minimal interactions). Some new general groups of phase transformations are indicated under which all the theories are invariant as a result of their belonging to class A. With a suitable choice of masses, the free-field Lagrangians are also invariant under these transformations.

EARLIER[1] we investigated the theories of class A for interacting fields with spins 0, ½, and 1 under the assumptions that the number of spinor particles is conserved and the coupling constants are dimensionless, and established a deep connection between such concepts as electrical charge, baryon charge, and isospin, and the property of a vector field having spin 1. In the present paper these results are generalized to the case when conservation of the number of spinor particles is not assumed (i.e., the bosons can make virtual transitions to two fermions), but the coupling constants are dimensionless, as formerly. As before, the interactions are considered successively. The analysis of the self-coupling of the vector fields that was carried out in [1] is not needed in the modification. The interactions with fields with spin ½ and 0 and their symmetry properties are examined in Secs. 2 and 3, respectively; the general results and discussion are given in Secs. 4 and 5.

We note that the groups of transformations that are found remain invariant, in particular the free Lagrangian of the spinor fields for a suitable choice of masses. They can be considered as generalizations of the Pauli-Gürsey group [2] to the case of multi-spinor fields.

In the Appendix are presented some concrete realizations of the Lie algebra and the corresponding interactions. Among the examples considered in which there is nonconservation of the number of spinor particles, we note the interaction of the "deuteron field." It would be possible to speak of such an interaction if, closing our eyes to obvious difficulties, we describe a friable deuteron (a stable boson with spin 1 and baryon charge B = 2) by a field in the Lagrangian formalism. Then of necessity we should arrive at a "triplet" field with equal masses, consisting of a deuteron (1^+, B = 2), a neutral boson (1^-, B = 0), and an antineutron (1^+, B = −2). This symmetry, however, is destroyed by strong π-meson interactions (and by some others as well). Therefore, the mass of the neutral boson, if in general it exists, can differ from the mass of the deuteron by several hundred MeV.

2. We begin with the interaction of a system of vector fields b_μ^i (i = 1, ..., k) with spinors ψ^r (r = 1, ..., m) taking into account the possibility that the number of spinor particles is not conserved. All symbols used in the present paper are the same as those used in [1]. In particular, by ψ we shall mean a column consisting of the individual fields ψ^r. The most complete Lagrangian with dimensionless coupling constants is written in the form

$$L_{1/2,1} = L_1 - \bar\psi(\gamma\partial + M)\psi + i\bar\psi\gamma_\mu(T_j^{(1)} + \gamma_5 T_j^{(2)})\psi b_\mu^j + \frac{i}{2}\bar\psi\gamma_\mu(T_j^{(3)} + \gamma_5 T_j^{(4)})\psi_C b_\mu^j + \frac{i}{2}\bar\psi_C\gamma_\mu \times (T_j^{(3)+} + \gamma_5 T_j^{(4)+})\psi b_\mu^j, \quad (1)$$

where the index $^+$ indicates Hermitian conjugate, L_1 is the Lagrangian of the self-action of the vector fields (cf. Eq. (3) in [1]), and ψ_C is the charge-conjugated spinor.

The matrices $T^{()}$ are a compact way of writing the corresponding coupling constants. To the terms involving the Hermitian matrices $T_j^{(1)}$ and $T_j^{(2)}$ have been added new terms with matrices $T_j^{(3)}$ and $T_j^{(4)}$, which describe the possible interactions with nonconservation of the number of spinor particles (e.g., interactions with vector bosons having a double baryon charge, if such existed). The matrices $T_j^{(3)}$ are antisymmetric, whereas the $T_j^{(4)}$ are symmetric. It is clear that the symmetric parts of the matrices $T_j^{(3)}$ and the asymmetric parts in $T_j^{(4)}$ automatically give zero. In fact, taking into account the anti-commutative property of the fields,

$$\bar{\psi}\gamma_\mu T\psi_C = -\bar{\psi}\gamma_\mu\tilde{T}\psi_C, \qquad \bar{\psi}\gamma_\mu\gamma_5 T\psi_C = \bar{\psi}\gamma_\mu\gamma_5\tilde{T}\psi_C,$$

no matter what the matrix T is (the sign ~ indicates transposition).

The Lagrangian (1) is conveniently written in more compact form. We introduce for this a two-dimensional column

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} \psi + \psi_C \\ i(\psi - \psi_C)\end{pmatrix} \tag{2}$$

with the property

$$\bar{\Psi} = \Psi C^{-1} \tag{3}$$

and matrices

$$\hat{M} = \begin{pmatrix} M & 0 \\ 0 & M\end{pmatrix}, \tag{4}$$

$$\hat{T}_j^{(1)} = \frac{1}{2}\begin{pmatrix} T_j^{(1)} - \tilde{T}_j^{(1)} + T_j^{(3)} + T_j^{(3)+} & -i(T_j^{(1)} + \tilde{T}_j^{(1)} - T_j^{(3)} + T_j^{(3)+}) \\ i(T_j^{(1)} + \tilde{T}_j^{(1)} + T_j^{(3)} - T_j^{(3)+} & T_j^{(1)} - \tilde{T}_j^{(1)} - T_j^{(3)} - T_j^{(3)+}\end{pmatrix}, \tag{5}$$

$$\hat{T}_j^{(2)} = \frac{1}{2}\begin{pmatrix} T_j^{(2)} + \tilde{T}_j^{(2)} + T_j^{(4)} + T_j^{(4)+} & -i(T_j^{(2)} - \tilde{T}_j^{(2)} - T_j^{(4)} + T_j^{(4)+}) \\ i(T_j^{(2)} - \tilde{T}_j^{(2)} + T_j^{(4)} - T_j^{(4)+}) & T_j^{(2)} + \tilde{T}_j^{(2)} - T_j^{(4)} - T_j^{(4)+}\end{pmatrix}. \tag{6}$$

Then the Lagrangian $L_{1/2,1}$ takes the form

$$L_{1/2,1} = L_1 - \frac{1}{2}\bar{\Psi}(\gamma\partial + \hat{M})\Psi + \frac{i}{2}\bar{\Psi}\gamma_\mu(\hat{T}_j^{(1)} + \gamma_5\hat{T}_j^{(2)})\Psi b_\mu^j. \tag{7}$$

In accordance with the construction of the interaction term in Eq. (7) and the property of Ψ with respect to charge conjugation (3), the matrices $\hat{T}_j^{(1)}$ are antisymmetric, and $T_j^{(2)}$ are symmetric; in addition they are both Hermitian.

The Lagrangian (7) is practically the same in form as the Lagrangian (24) in [1]. The main difference is that the matrices $T_j^{(1)}$ and $T_j^{(2)}$ are here replaced by the two-dimensional matrices $\hat{T}_j^{(1)}$ and $\hat{T}_j^{(2)}$. This leads immediately to the conclusion that to fulfill the alternatives

$$\partial_\mu b_\mu^j = \begin{cases} 0, & \text{if } m_i^2 \neq 0 \\ \text{arbitrary}, & \text{if } m_i^2 = 0\end{cases}, \tag{8}$$

which cuts off spin 0 in the vector field and gives significance only to spin 1, it is necessary and sufficient that

$$[\hat{T}_i^{(1)}, \hat{M}] = 0, \tag{9}$$

$$[\hat{T}_i^{(2)}, \hat{M}]_+ = 0, \tag{10}$$

$$[\hat{T}_i^{(1)} + \gamma_5\hat{T}_i^{(2)}, \hat{T}_j^{(1)} + \gamma_5\hat{T}_j^{(2)}] = i\alpha_{ijk}(\hat{T}_k^{(1)} + \gamma_5\hat{T}_k^{(2)}). \tag{11}$$

The relations for the interaction constants of the vector fields obtained in [1] remain unchanged. We are again led to a Lie algebra, but for the matrices $T_k^{(1)} + \gamma_5 T_k^{(2)}$, and to invariance of the theory about the transformation groups, which have the following infinitesimal form:

$$b_\mu^{i'} = b_\mu^i + \alpha_{ijk}\omega_j b_\mu^k, \quad \Psi' = \Psi - i\omega_j(\hat{T}_j^{(1)} + \gamma_5\hat{T}_j^{(2)})\Psi \tag{12}$$

where ω_j are infinitesimal parameters.

We present Eqs. (9) to (11) written directly in terms of the matrices $T^{(1)}$, $T^{(2)}$, $T^{(3)}$, and $T^{(4)}$:

$$[T_j^{(1)}, M] = 0, \quad [T_j^{(2)}, M]_+ = 0, \quad [T_j^{(3)}, M] = 0, \quad [T_j^{(4)}, M]_+ = 0, \tag{13}$$

$$[T_i^{(1)}, T_j^{(1)}] + [T_i^{(2)}, T_j^{(2)}] + T_i^{(3)}T_j^{(3)+} - T_j^{(3)}T_i^{(3)+} + T_i^{(4)}T_j^{(4)+} - T_j^{(4)}T_i^{(4)+} = i\alpha_{ijk}T_k^{(1)}, \tag{14}$$

$$[T_i^{(1)}, T_j^{(2)}] + [T_i^{(2)}, T_j^{(1)}] + T_i^{(3)}T_j^{(4)+} - T_j^{(3)}T_i^{(4)+} + T_i^{(4)}T_j^{(3)+} - T_j^{(4)}T_i^{(3)+} = i\alpha_{ijk}T_k^{(2)}, \tag{15}$$

$$T_i^{(1)}T_j^{(3)} - T_j^{(1)}T_i^{(3)} + T_i^{(2)}T_j^{(4)} - T_j^{(2)}T_i^{(4)} - T_i^{(3)}\tilde{T}_j^{(1)} + T_j^{(3)}\tilde{T}_i^{(1)} + T_i^{(4)}\tilde{T}_j^{(2)} - T_j^{(4)}\tilde{T}_i^{(2)} = i\alpha_{ijk}T_k^{(3)}, \tag{16}$$

$$T_i^{(1)}T_j^{(4)} - T_j^{(1)}T_i^{(4)} + T_i^{(2)}T_j^{(3)} - T_j^{(2)}T_i^{(3)} - T_i^{(4)}\tilde{T}_j^{(1)} + T_j^{(4)}\tilde{T}_i^{(1)} + T_i^{(3)}\tilde{T}_j^{(2)} - T_j^{(3)}\tilde{T}_i^{(2)} = i\alpha_{ijk}T_k^{(4)}. \tag{17}$$

The transformation law (12) for the spinor particles is accordingly written in the form

$$\psi' = \psi - i\omega_j\{(T_j^{(1)} + \gamma_5 T_j^{(2)})\psi + (T_j^{(3)} + \gamma_5 T_j^{(4)})\psi_C\}. \tag{18}$$

The spinor fields with zero mass and the spinor fields with non-zero mass, as before, do not interact directly, as can be easily shown by displaying the matrices in block form, as was done in [1].

The term with γ_5 enters into the transformation law (18) only when among the spinor fields there is one with zero mass. In this case the transformation of (18), with account taken of the relations (14)—(17) is a generalization of the Pauli and Gürsey group [2] to the case of multi-spinor fields with mass 0. If the masses of all the spinor fields are non-zero, then $\hat{T}^{(2)} = 0$, i.e., $T_j^{(2)} = 0$ and $T_j^{(4)} = 0$. In this case, however, it is possible to transform ψ and ψ_C into each other

$$\psi' = \psi - i\omega_j (T_j^{(1)}\psi + T_j^{(3)}\psi_C). \qquad (19)$$

This class of transformations is the maximum generalization of the phase transformation $\psi \to e^{i\Lambda}\psi$ to the case of many fields with non-zero mass.

3. We turn now to the general system of interacting fields with spins 1, ½, and 0 (the latter are symbolized by φ_a ($a = 1, \ldots, l$)). In order to encompass all interactions with nonconservation of the number of spinor particles, it is necessary to add to the general Lagrangian (37) in [1] the terms

$$\tfrac{1}{2}\overline{\psi}(G_a^{(3)} + i\gamma_5 G_a^{(4)})\psi_C\varphi^a + \tfrac{1}{2}\overline{\psi}_C(G_a^{(3)+} + i\gamma_5 G_a^{(4)})\psi\varphi^a, \qquad (20)$$

where the coupling matrices $G_a^{(3)}$ and $G_a^{(4)}$ are symmetric (antisymmetric matrices give zero contribution: $\overline{\psi}(G - \widetilde{G})\psi_C \equiv \overline{\psi}\gamma_5(G - \widetilde{G})\psi_C \equiv 0$).

We introduce again the spinors $\Psi(2)$ and the two-dimensional symmetric Hermitian matrices

$$\hat{G}_a^{(1)} = \frac{1}{2}\begin{pmatrix} G_a^{(1)} + \widetilde{G}_a^{(1)} + G_a^{(3)} + G_a^{(3)+} & -i(G_a^{(1)} - \widetilde{G}_a^{(1)} - G_a^{(3)} + G_a^{(3)+}) \\ i(G_a^{(1)} - \widetilde{G}_a^{(1)} + G_a^{(3)} - G_a^{(3)+}) & G_a^{(1)} + \widetilde{G}_a^{(1)} - G_a^{(3)} - G_a^{(3)+} \end{pmatrix}, \qquad (21)$$

$$\hat{G}_a^{(2)} = \frac{1}{2}\begin{pmatrix} G_a^{(2)} + \widetilde{G}_a^{(2)} + G_a^{(4)} + G_a^{(4)+} & -i(G_a^{(2)} - \widetilde{G}_a^{(2)} - G_a^{(4)} + G_a^{(4)+}) \\ i(G_a^{(2)} - \widetilde{G}_a^{(2)} + G_a^{(4)} - G_a^{(4)+}) & G_a^{(2)} + \widetilde{G}_a^{(2)} - G_a^{(4)} - G_a^{(4)+} \end{pmatrix}. \qquad (22)$$

Then the most general Lagrangian of the interacting fields with spins 1, ½, and 0 with dimensionless coupling constants is written as:

$$L_{0,\,^1\!/_2,\,1} = L_{^1\!/_2,\,1} - \tfrac{1}{2}\partial_\mu\varphi^a\cdot\partial_\mu\varphi^a - \tfrac{1}{2}\varphi^a(\mu^2)_{ab}\varphi^b + \xi_{abcd}\varphi^a\varphi^b\varphi^c\varphi^d + \eta^i_{ab}\varphi^a\partial_\mu\varphi^b\cdot b^i_\mu + \zeta^{ij}_{ab}\varphi^a\varphi^b b^i_\mu b^j_\mu + \tfrac{1}{2}\overline{\Psi}(\hat{G}_a^{(1)} + i\gamma_5\hat{G}_a^{(2)})\Psi\varphi^a. \qquad (23)$$

It basically coincides in form with the Lagrangian (37) in [1], differing only in the replacement of the matrices G by the two-dimensional matrices $\hat{G}$, such that the choice of the new matrices $\hat{G}$ is limited by the relation

$$(\hat{T}_j^{(1)} - \gamma_5\hat{T}_j^{(2)})(\hat{G}_a^{(1)} + i\gamma_5 G_a^{(2)}) - (\hat{G}_a^{(1)} + i\gamma_5 G_a^{(2)}) \times (\hat{T}_j^{(1)} + i\gamma_5\hat{T}_j^{(2)}) = -i\eta^j_{ab}(\hat{G}_b^{(1)} + i\gamma_5\hat{G}_b^{(2)}), \qquad (24)$$

similar to the relation (46) in [1]. For $\hat{T}^{(1)}$ and $\hat{T}^{(2)}$ the relations (9) to (11) just introduced remain in force, and the remaining relations for α and η maintain the same form as in [1].

Through the use of matrices T and G, Eq. (24) can be written directly in the form

$$i[T_i^{(1)}, G_a^{(1)}] + [T_i^{(2)}, G_a^{(2)}]_+ + iT_i^{(3)}G_a^{(3)+} - iG_a^{(3)}T_i^{(3)+} + T_i^{(4)}G_a^{(4)+} + G_a^{(4)}T_i^{(4)+} = \eta^i_{ab}G_b^{(1)}, \qquad (25)$$

$$-[T_i^{(1)}, G_a^{(2)}] - i[T_i^{(2)}, G_a^{(1)}]_+ - T_i^{(3)}G_a^{(4)+} + G_a^{(4)}T_i^{(3)+} - iT_i^{(4)}G_a^{(3)+} - iG_a^{(3)}T_i^{(4)+} = i\eta^i_{ab}G_b^{(2)}, \qquad (26)$$

$$i(T_i^{(1)}G_a^{(3)} + G_a^{(3)}\widetilde{T}_i^{(1)}) + T_i^{(2)}G_a^{(4)} + G_a^{(4)}\widetilde{T}_i^{(2)} + G_a^{(2)}T_i^{(4)} + T_i^{(4)}\widetilde{G}_a^{(2)} - i(G_a^{(1)}T_i^{(3)} - T_i^{(3)}\widetilde{G}_a^{(1)}) = \tfrac{1}{2}\eta^i_{ab}G_b^{(3)}, \qquad (27)$$

$$-T_i^{(1)}G_a^{(4)} - G_a^{(4)}\widetilde{T}_i^{(1)} - i(T_i^{(2)}G_a^{(3)} + G_a^{(3)}\widetilde{T}_i^{(2)}) + G_a^{(2)}T_i^{(3)} - T_i^{(3)}\widetilde{G}_a^{(2)} - i(G_a^{(1)}T_i^{(4)} + T_i^{(4)}\widetilde{G}_a^{(1)}) = \tfrac{1}{2}\eta^i_{ab}G_b^{(4)}. \qquad (28)$$

In the most interesting case of non-zero masses in a all the spinor fields $\hat{T}_i^{(2)} = 0$, and Eq. (24) takes the form

$$[\hat{T}_i^{(1)}, \hat{G}_a^{(1)}] = -i\eta^i_{ab}\hat{G}_b^{(1)}, \qquad (29)$$

$$[\hat{T}_i^{(1)}, \hat{G}_a^{(2)}] = -i\eta^i_{ab}\hat{G}_b^{(2)}. \qquad (30)$$

4. Thus, the most complete Lagrangian of class A for a system of fields with spin 0, ½, and 1 with dimensionless coupling constants has the form

$$L_{0,\,^1\!/_2,\,1} = -\tfrac{1}{4}G_{\mu\nu}G_{\mu\nu} - \tfrac{1}{2}b_\mu m^2 b_\mu - \tfrac{1}{2}\overline{\Psi}\{\gamma_\mu[\partial_\mu - i(\hat{T}_j^{(1)} + \gamma_5\hat{T}_j^{(2)})b^j_\mu] + \hat{M}\}\Psi - \tfrac{1}{2}(\partial_\mu - \eta^i b^i_\mu)\varphi\cdot(\partial_\mu - \eta^j b^j_\mu)\varphi - \tfrac{1}{2}\varphi\mu^2\varphi + \xi_{abcd}\varphi^a\varphi^b\varphi^c\varphi^d + \tfrac{1}{2}\overline{\Psi}\{\hat{G}_a^{(1)} + i\gamma_5\hat{G}_a^{(2)}\}\Psi\varphi^a. \qquad (31)$$

(If all the spinor fields have masses different from zero, then the term with $\gamma_5\hat{T}^{(2)}$ falls out.) In Eq. (31) the tensors of the vector fields are

$$G^i_{\mu\nu} = \partial_\mu b^i_\nu - \partial_\nu b^i_\mu + \alpha_{ijk}b^j_\mu b^k_\nu,$$

and the coupling matrices α, η, ξ and masses m^2, μ^2 satisfy Eqs. (20) to (22), (42)–(45) from [1]. The new matrices $\hat{T}$, $\hat{G}$, and $\hat{M}$ satisfy relations (9) to (11) and (24) of the present paper.

In the preceding paper [1] the general Lagrangian for the class A theory was written out with two limitations: dimensionless coupling constants and conservation of the number of spinor particles. As we have shown above, when the latter limitation is removed, the conclusions made in [1] remain in force. Only now they are formulated in the language of two-dimensional spinors and matrices: Ψ, $\hat{T}$, $\hat{M}$, and $\hat{G}$. The corresponding transformation groups relative to which the theory is invariant are isomorphous with the groups considered in [1], and the infinitesimal transformations are written in the form

$$b_\mu^{i'} = b_\mu^i + \alpha_{ijk}\omega_j b_\mu^k, \qquad \Psi = \Psi - i\omega_j(\hat{T}_j^{(1)} + \gamma_5\hat{T}_j^{(2)})\Psi,$$
$$\varphi^{a'} = \varphi^a + \omega_j\eta_{ab}^j\varphi^b. \tag{32}$$

We note that the calculation of terms with non-conservation of spinor particles, generally speaking, can lead to non-conservation of parity in the interaction of spinor fields having non-zero mass with vector fields. (See the interaction A7 in the Appendix.)

5. We make one further general observation. Usually the fields are classified according to the homogeneous Lorentz group (scalars, spinors, vectors, etc.), whereas physical concepts (momentum, spin, mass) are concepts of the inhomogeneous Lorentz group. By going through the program designated in [3], we eliminate in [1] and in this paper superfluous components of the vector fields so that all the fields would be proper functions of one of the invariants of the inhomogeneous group (spin); this is responsible also for the generality and elegance of the symmetry properties inherent in the class A theories. One can express the hope that if an apparatus could be constructed, adequate for the inhomogeneous Lorentz group and with a suitable classification of the interacting fields, in which the fields would not be cluttered with superfluous components, then these symmetry properties would become plainly obvious.

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APPENDIX

EXAMPLES OF THE REALIZATION OF THE GENERAL STRUCTURE RELATIONS (14)–(17)

We shall present a few examples of unusual interactions that do not conserve the number of spinor particles. We consider the possible realizations of the relations (14) to (17) by 2×2 matrices for two choices of α_{ijk}. This means that there are only two spinor fields. We assume that their masses are different from zero, so that $T_i^{(2)} = T_i^{(4)} = 0$.

A. Let $\alpha_{ijk} = g\varepsilon_{ijk}$ (i, j, k = 1, 2, 3). We enumerate all possibilities.

1) $T_i^{(1)} = g\tau_i/2$, $T_i^{(3)} = 0$—the usual isotopically invariant interaction:

$$i\frac{g}{2}\bar{\psi}\gamma_\mu\tau_i\psi b_\mu^i. \tag{A1}$$

It is interesting that because of its vector property it is also invariant relative to the three-parameter transformation group

$$b_\mu^{i'} = b_\mu^i, \qquad \psi' = \left(\cos\omega - \frac{i\omega_3}{\omega}\sin\omega\right)\psi - \frac{i\omega_1 + \omega_2}{\omega}\sin\omega\,\psi_G, \tag{A2}$$

where

$$\omega = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}, \qquad \psi_G = i\tau_2\psi_C.$$

2) The second possibility[1)]:

$$T_j^{(1)} = -\frac{g}{2}(\xi^+\tau_j\xi), \qquad T_j^{(3)} = \frac{g}{2}(\xi\tau_2\tau_j\xi)\tau_2,$$

where ξ is a two-component quantity depending on three parameters, $\xi = (\sin\theta e^{i\varphi_1}, \cos\theta e^{i\varphi_2})$. One or another choice of parameters secures a realization of the Lie algebra. In particular, for $\theta = 0$, $\varphi_2 = 0$

$$T_1^{(1)} = T_2^{(1)} = 0, \qquad T_3^{(1)} = \frac{g}{2}, \qquad T_1^{(3)} = \frac{ig}{2}\tau_2,$$
$$T_2^{(3)} = \frac{g}{2}\tau_2, \qquad T_3^{(3)} = 0.$$

In this case the interaction of the vector field with the spinors has the form

$$\frac{ig}{2\sqrt{2}}[\bar{\psi}\gamma_\mu\psi_G b_\mu^+ + \bar{\psi}_G\gamma_\mu\psi b_\mu^- + \sqrt{2}\,\bar{\psi}\gamma_\mu\psi b_\mu^3], \tag{A3}$$

where ψ_G is the G-conjugated spinor $\psi_G = i\tau_2\psi_C$, and

$$b_\mu^+ = (b_\mu^1 - ib_\mu^2)/\sqrt{2}, \qquad b_\mu^- = (b_\mu^1 + ib_\mu^2)/\sqrt{2}.$$

This interaction is not only invariant relative to the transformation groups[2)]

$$b_\mu^{i'} = b_\mu^i + g\varepsilon_{ijk}\omega_j b_\mu^k, \tag{A4}$$

$$\psi' = \psi - \frac{1}{2}ig\omega_3\psi - \frac{1}{2}g(i\omega_1 + \omega_2)\psi_G, \tag{A5}$$

but also relative to the usual isotopic transformations of the field ψ, if we consider the field b_μ^i to be isoscalar. We note that transition to the

[1)]The matrices $T_i^{(3)}$ are asymmetric by definition, and therefore they are by necessity multiples of the matrix τ_2.

[2)]The second transformation on ψ is written in form (A2) with the replacement there $\omega \to g\omega/2$.

spinors $\chi = \frac{1}{2}[(1+\tau_3)\psi + (1-\tau_3)\psi_C]$ gives the interaction (A3) the form (A1).

3) Finally, there remains the last possibility

$$T_j^{(1)} = \frac{1}{2} g\,[\tau_j - (\xi^+\tau_j\xi)], \qquad T_j^{(3)} = \frac{1}{2} g\,(\xi\tau_2\tau_j\xi)\,\tau_2. \quad \text{(A6)}$$

This case is a completely determined combination of interactions of the type 1) and 2). In particular, for $\theta = 0$, $\varphi_2 = 0$

$$i\frac{g}{2}[\bar{\psi}\gamma_\mu\tau_i\psi b_\mu^i + \frac{1}{\sqrt{2}}(\bar{\psi}\gamma_\mu\psi_G b_\mu^+ + \bar{\psi}_G\gamma_\mu\psi b_\mu^- + \sqrt{2}\bar{\psi}\gamma_\mu\psi b_\mu^3)]. \quad \text{(A7)}$$

The interaction (A7) is invariant relative to the transformations of b_μ according to (A4) and ψ according to

$$\psi' = \exp\left(-\frac{i}{2}g\boldsymbol{\omega\tau}\right)\left\{\left[\cos\left(\frac{g}{2}\omega\right) - i\frac{\omega_3}{\omega}\sin\left(\frac{g}{2}\omega\right)\right]\psi - \frac{i\omega_1+\omega_2}{\omega}\sin\left(\frac{g}{2}\omega\right)\psi_G\right\}. \quad \text{(A8)}$$

B. Let us now assume six vector fields, such that $\alpha_{123} = g$, $\alpha_{456} = f$, and α_{ijk} not reducing to these are zero. This representation is reducible. The matrices of the coupling with the doublet of the spinor fields can be expressed in the form

$$T_i^{(1)} = \frac{1}{2}g\tau_i\ (i = 1, 2, 3), \qquad T_4^{(1)} = T_5^{(1)} = 0, \qquad T_6^{(1)} = \frac{1}{2}f,$$
$$T_i^{(3)} = 0 \quad (i = 1, 2, 3), \qquad T_4^{(3)} = \frac{1}{2}if\tau_2, \qquad T_5^{(3)} = \frac{1}{2}f\tau_2,$$
$$T_6^{(3)} = 0.$$

To this choice of the matrices corresponds the interaction

$$\frac{ig}{2}\sum_{i=1}^{3}\bar{\psi}\gamma_\mu\tau_i\psi b_\mu^i + \frac{if}{2\sqrt{2}}[\bar{\psi}\gamma_\mu\psi_G B_\mu^+ + \bar{\psi}_G\gamma_\mu\psi B_\mu^- + \sqrt{2}\bar{\psi}\gamma_\mu\psi B_\mu^0], \quad \text{(A9)}$$

where

$$B_\mu^+ = (b_\mu^4 - ib_\mu^5)/\sqrt{2}, \qquad B_\mu^- = (b_\mu^4 + ib_\mu^5)/\sqrt{2}, \qquad B_\mu^0 = b_\mu^6.$$

This theory is invariant, when b_μ transforms according to the law (A4) and ψ according to the law

$$\psi' = \exp\left(-\frac{i}{2}g\boldsymbol{\omega\tau}\right)\left\{\left[\cos\left(\frac{f}{2}\Omega\right) - i\frac{\omega_6}{\Omega}\sin\left(\frac{f}{2}\Omega\right)\right]\psi - \frac{i\omega_4+\omega_5}{\Omega}\sin\left(\frac{f}{2}\Omega\right)\psi_G\right\},$$

where $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$, $\quad \Omega = \sqrt{\omega_4^2 + \omega_5^2 + \omega_6^2}$. (A10)

Thus the theory has invariance relative to isotopic rotations, in which the triplet b_μ^i ($i = 1, 2, 3$) transforms like an isopseudovector, and the triplet B_μ remains unchanged. In addition, it is invariant relative to the three-parameter transformation groups (isomorphous to the isotopic group) in which $B^{\pm,0}$ transform as an isotopic triplet, and the triplet b_μ is not transformed. In this, as is seen from (A10), ψ and ψ_C transform into each other. Similar transformations have been touched upon by Strel'tsov.[4]

The existence of this dual invariance is due to the vector property of the interaction. In an interaction of spinors via the combinations $\bar{\psi}\psi$, $\psi\gamma_5\psi$, or $\psi\gamma_\mu\gamma_5\psi$ this would not be possible. It is not surprising that interactions with π mesons destroy the invariance under the second group of transformations. (It can be directly verified that the relations (3) are destroyed.)

The field B_μ^+ can describe qualitatively the deuteron, which also has a spin and parity 1^+, electric charge +e, and a double baryon charge, and is an isotopic singlet. Then the neutral field B_μ^0 would have the mass of the deuteron, and the deuteron, $B_\mu^0(1^-)$, and the antideuteron would constitute the "triplet." The corresponding symmetry, however, as mentioned above, is destroyed by other strong interactions; therefore the existence of a resonance of B_μ^0 is extremely doubtful, and, in any case, its mass can differ significantly from the mass of the deuteron.

[1] V. I. Ogievetskiĭ and I. V. Polubarinov, JETP 45, 966 (1963), Soviet Phys. JETP 18, 668 (1964).

[2] W. Pauli, Nuovo cimento 6, 203 (1957); F. Gürsey, Nuovo cimento 7, 411 (1958).

[3] V. I. Ogievetskiĭ and I. V. Polubarinov, JETP 45, 237 (1963), Soviet Phys. JETP 18, 166 (1964).

[4] V. I. Strel'tsov, Preprint, Joint Institute for Nuclear Research, D-854, 1961.

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