INTERACTION BETWEEN PARTICLES WITH SPINS AND THE REGGE POLES

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The contribution of Regge poles with arbitrary quantum numbers to scattering of particles with spins is determined with aid of helicity amplitudes. The cross sections and polarization effects in πN and NN scattering are found. Production of the ρ meson and the nucleon isobar is considered. The isotopic spin is taken into account.

1. CONTRIBUTION OF REGGE POLE TO THE SCATTERING AMPLITUDE IN THE HELICITY REPRESENTATION

THE description of the interaction of particles with spins at high energies, using Regge poles, yields information not only on the behavior of the cross sections but also on the polarization effects. The contribution from the Regge pole with arbitrary quantum numbers can be obtained with the aid of the helicity amplitudes (HA)[1].

Usually the problem of the contribution of the Regge poles to the amplitude for nucleon-nucleon scattering is solved in the following manner[2-5]. Assume that we are interested in scattering in the s channel; we then construct an expansion of the HA in the t-channel in states with definite angular momentum[1]. This expansion is represented in the form of a contribution from the poles in the complex angular momentum plane. Then one uses the connection between the HA and the invariant functions[6], and the coefficients of the Fermi invariants are determined, and the measured quantities are then determined in terms of these coefficients. A much simpler method of calculating the contributions of the Regge poles, particularly valuable in the case of high spins, is a direct examination of the HA. In this case there is no need for a cumbersome transition from the HA, for which the expansion in partial waves is made, to the invariant functions. In addition, the measured quantities have a simpler form in terms of the HA. For example, for particles with spin ½ the calculation includes the operation of taking the trace of a product of Pauli σ matrices, and not of the 4×4 γ matrices.

The contribution from the Regge pole in the t-channel to the amplitude of the scattering of particles $1 + 2 \to 3 + 4$ in the s-channel (Fig. 1) is calculated in the following manner.

Let the particles have masses m_i and spins s_i

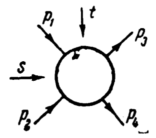

FIG. 1

($i = 1, \ldots, 4$). We write the expansion of the HA for the scattering $1 + \overline{3} \to \overline{2} + 4$ in the t-channel in terms of states with definite angular momentum J. We denote the amplitude in the c.m.s. system, referred to the coordinate system in which the relative momentum of particles 1 and $\overline{3}$ is directed along the z axis, and the relative momentum of the particles $\overline{2}$ and 4 lies in the xz plane and makes an angle θ_t with the z axis ($K_{1\overline{3}}$ system), by $M^{\lambda_2\lambda_4}_{(t)\lambda_1\lambda_3}(s, t)$ (the helicity indices of the incoming particles are the lower ones, and those of the outgoing are the upper ones, in accordance with the transformation properties of the amplitude[7,8]). Expansion in J takes the form[1]

$$M^{\lambda_2\lambda_4}_{(t)\lambda_1\lambda_3} = \frac{1}{4\pi}\sum_J (2J+1)\, M^{\lambda_2\lambda_4}_{\lambda_1\lambda_3}(t, J)\, d^J_{\lambda\mu}(\theta_t);$$
$$\lambda = \lambda_1 - \lambda_3, \quad \mu = \lambda_2 - \lambda_4.$$

We now make the usual transition from a sum over J to a sum over the poles in the complex J plane:

$$M^{\lambda_2\lambda_4}_{\lambda_1\lambda_3} = \frac{1}{4}\sum_{\{j\}} R^{\lambda_2\lambda_4}_{\lambda_1\lambda_3}(t, j)\frac{2j+1}{\sin \pi j}[d^j_{\lambda\mu}(\theta_t)$$
$$+ P_j d^j_{\lambda\mu}(\pi - \theta_t)] + \frac{1}{8\pi i}\int\limits_{\mathrm{Re}\, j = -1/2} dj. \qquad (1)$$

Here $j = j(t)$—position of the pole of the amplitude M(J), $P_j = \pm 1$—signature of the pole, R(j)—residue of the function M(J) at the pole $J = j(t)$, and d^j—analytical continuation in J of the generalized spherical function d^J.

By virtue of the unitarity condition, the residue at the pole factors: $R^{\lambda_2\lambda_4}_{\lambda_1\lambda_3} = \widetilde{B}^{\lambda_2\lambda_4}\widetilde{A}_{\lambda_1\lambda_3}$, where $\widetilde{B}$ and $\widetilde{A}$ depend respectively only on the final and initial states[9].

We assume, as usual, that the pole has definite parity π. Then, starting from the properties of helicity states with definite angular momentum under space inversion[1]

$$P\,|J,\,\lambda_a,\,\lambda_b\rangle = \eta_a\eta_b\,(-1)^{J-s_a-s_b}\,|J,\,-\lambda_a,\,-\lambda_b\rangle,$$

where η_a and η_b are the intrinsic parities of the particles a and b, we obtain for a boson trajectory (J—integer)

$$\widetilde{A}_{\lambda_1\lambda_3} = \pi\eta_1\eta_{\bar{3}}P_j\,(-1)^{s_1+s_3}\widetilde{A}_{-\lambda_1,\,-\lambda_3}; \tag{2}$$

For fermion trajectories (J—half-integer) the multiplier $P_j(-1)^{s_1+s_3}$ must be replaced by $P_j(-1)^{s_1+s_3-1/2}$.

The function $d^J_{\lambda\mu}(z)$ is connected with the hypergeometric function, and by virtue of this we determine its analytic continuation in the complex J-plane:

$$d^J_{\lambda\mu}(z) = i^{\lambda-\mu}\,[\Gamma(2\alpha+1)\,2^{\alpha+\beta}]^{-1}$$
$$\times\left[\frac{\Gamma(J+\alpha+\beta+1)\,\Gamma(J+\alpha-\beta+1)}{\Gamma(J-\alpha-\beta+1)\,\Gamma(J-\alpha+\beta+1)}\right]^{1/2}$$
$$\times(z-1)^{\alpha}(z+1)^{\beta}F\Big(+\alpha+\beta+1,\,-J$$
$$+\alpha+\beta;\,2\alpha+1\,\Big|\frac{1-z}{2}\Big);$$
$$2\alpha = |\lambda-\mu|,\quad 2\beta = |\lambda+\mu|. \tag{3}$$

For arbitrary masses, the argument of the hypergeometric function in (3) is connected with the invariants in the following fashion:

$$\frac{\pm 1+\cos\theta_t}{2} = \frac{s-(E_1-E_2)^2+(p_1\pm p_2)^2}{4p_1p_2}, \tag{4}$$

where E_1, E_2 and p_1, p_2—energy and momenta of particles 1 and $\bar{2}$ in the c.m.s. of the t-channel, with $(E_i^2 - p_i^2 = m_i^2)$

$$2\sqrt{t}\,p_1(t) = \sqrt{[t-(m_1+m_3)^2]\,[t-(m_1-m_3)^2]},$$
$$2\sqrt{t}\,p_2(t) = \sqrt{[t-(m_2+m_4)^2]\,[t-(m_2-m_4)^2]}.$$

The asymptotic value of the contribution to the s-channel amplitude from the pole j(t) is determined by the asymptotic behavior of the function d^j

$$d^j_{\lambda\mu}(z) \approx i^{\lambda-\mu}\,(z/2)^j\,\Gamma(2j+1)\,[\Gamma(j+\lambda+1)\,\Gamma(j-\lambda+1)$$
$$\times\Gamma(j+\mu+1)\,\Gamma(j-\mu+1)]^{-1/2}$$
$$\text{as}\quad z\to\infty,\quad \operatorname{Re} j > -\tfrac{1}{2}, \tag{5}$$

The helicity amplitude in the s-channel, referred to some frame K, is obtained from the HA in the t-channel in the following manner[8]:

$$M^K_{(s)} = D^{(s_1)}(R_1)\,D^{(s_2)}(R_2)\,C^{T(s_2)}M_{(t)}C^{(s_3)}D^{+(s_3)}(R_3)\,D^{+(s_4)}(R_4), \tag{6}$$

where $D^{(s_i)\lambda_i'}_{\lambda_i}(R_i)$—matrices which represent the rotation corresponding to the (unphysical) Lorentz transformation $K_{1\bar{3}} \to K$ and the momentum p_i, while $C^{(s)\lambda\lambda'} = (-1)^{s-\lambda}\delta_{\lambda,-\lambda'}$ is a matrix which raises the helicity indices; "T" denotes the transpose. Summation over the helicity indices is implied in (6).

The contribution $M_{(s)}$ from the pole j(t) to the HA is represented in the $K_{1\bar{3}}$ system in the following form:

$$M^{\lambda_3\lambda_4}_{(s)\lambda_1\lambda_2} = \frac{1}{2\pi}\,s^jC(j)\,A^{\lambda_3}_{\lambda_1}B^{\lambda_4}_{\lambda_2}; \tag{7}$$

$$C(j(t)) = \sqrt{\pi}\,\frac{\Gamma(j+3/2)}{\Gamma(j+1)}\,\frac{1+P_je^{-i\pi j}}{\sin\pi j}, \tag{7a}$$

$$A^{\lambda_3}_{\lambda_1}(t) = i^{\lambda}\,(-1)^{s_3-\lambda_3}\left[\frac{\Gamma^2(j+1)}{\Gamma(j+\lambda+1)\,\Gamma(j-\lambda+1)}\right]^{1/2}\frac{\widetilde{A}_{\lambda_1,\,-\lambda_3}}{|p_1(t)|^j}, \tag{7b}$$

$$B^{\lambda_4}_{\lambda_2}(t) = i^{\mu}\,(-1)^{s_2+\lambda_2}\left[\frac{\Gamma^2(j+1)}{\Gamma(j-\mu+1)\,\Gamma(j+\mu+1)}\right]^{1/2}\frac{\widetilde{B}_{-\lambda_2,\,\lambda_4}}{|p_2(t)|^j},$$
$$\lambda = \lambda_1+\lambda_3,\quad \mu = \lambda_2+\lambda_4. \tag{7c}$$

Equation (7) allows us to calculate measurable quantities such as polarization cross sections, etc.

2. ISOTOPIC SPIN AND CONNECTION BETWEEN CHANNELS

Allowance for the isotopic properties of the scattering amplitude entails no difficulty. The isotopic indices which pertain to the incoming particles are conveniently written as subscripts, while those pertaining to the outgoing particles as superscripts, the connection between the upper and lower indices being given by the matrix $C^{(T)qq'} = (-1)^{T-q}\delta_{q,-q'}$, $C^{qq''}C_{q''q'} = \delta^q_{q'}$, analogous to the matrix $C^{(s)}$ for the helicity representation.

We denote by T_i and q_i (i = 1, . . . , 4) the isospin and its third projection for the particle i. The amplitude in the s-channel $M^{(s)q_3q_4}_{q_1q_2}$ is expressed in terms of the amplitude for the scattering in a state with isospin T′:

$$M^{(s)q_3q_4}_{q_1q_2} = \sum_{T'}\begin{pmatrix} T_1 & T_2 & q \\ q_1 & q_2 & T' \end{pmatrix}\begin{pmatrix} T' & q_4 & q_3 \\ q & T_4 & T_3 \end{pmatrix}M^{(s)}_{T'}, \tag{8}$$

where $\begin{pmatrix} T_1 & T_2 & q \\ q_1 & q_2 & T' \end{pmatrix}$ is the Wigner 3j-symbol with superior index q and lower indices q_1 and q_2. Formula (8) is the consequence of the Wigner-Eckart theorem for isospace. Analogously we have for the amplitude in the t-channel

$$M^{(t)q_2q_4}_{q_1q_3} = \sum_T \begin{pmatrix} T_1 & T_3 & q \\ q_1 & q_3 & T \end{pmatrix} \begin{pmatrix} T & q_4 & q_2 \\ q & T_4 & T_2 \end{pmatrix} M^{(t)}_T. \quad (9)$$

The connection between these amplitudes is of the form

$$M^{(s)} = C^{\mathrm{T}(T_2)} M^{(t)} C^{(T_3)}. \quad (10)$$

From (8), (9), and (10) we can easily establish the connection between $M^{(s)}_{T'}$ and $M^{(t)}_T$. To this end we use the formula [7]

$$\begin{pmatrix} T_1 & q_3 & q \\ q_1 & T_3 & T \end{pmatrix} \begin{pmatrix} T & q_4 & T_2 \\ q & T_4 & q_2 \end{pmatrix} = (-1)^{2T_4} \sum_{T'} (2T'+1) \begin{Bmatrix} T_1 & T_2 & T' \\ T_4 & T_3 & T \end{Bmatrix} \begin{pmatrix} T_1 & T_2 & q' \\ q_1 & q_2 & T' \end{pmatrix} \begin{pmatrix} T' & q_4 & q_3 \\ q' & T_4 & T_3 \end{pmatrix}, \quad (11)$$

where $\{\ldots\}$ is the Wigner 6j-symbol. As a result we get

$$M^{(s)}_{T'} = \sum_T \beta_{T'T} M^{(t)}_T,$$

where

$$\beta_{T'T} = (-1)^{2T_4} (2T+1) \begin{Bmatrix} T_1 & T_2 & T' \\ T_4 & T_3 & T \end{Bmatrix}. \quad (12)$$

For the cases $T_i = \frac{1}{2}$ and $T_i = 1$ these matrices were calculated by Goldberger et al[6] and by Chew and Mandelstam[10] respectively and are of the form

$\beta_{(1/2)} =$ (12a)

T' \ T	0	1
0	$-1/2$	$3/2$
1	$1/2$	$1/2$

$\beta_{(1)} =$ (12b)

T' \ T	0	1	2
0	$1/3$	1	$5/3$
1	$1/3$	$1/2$	$-5/6$
2	$1/3$	$-1/2$	$1/6$

It follows from isotopic invariance that the dependence of the contribution from the Regge pole in the state with isospin T on the isotopic indices is determined by the Wigner 3j-symbols

$$A^{q_3}_{q_1} = \begin{pmatrix} T_1 & q_3 & q \\ q_1 & T_3 & T \end{pmatrix} A, \quad B^{q_4}_{q_2} = \begin{pmatrix} T & q_4 & T_2 \\ q & T_4 & q_2 \end{pmatrix} B. \quad (13)$$

A and B no longer depend on q. Thus, the contribution from the pole with isospin T to the scattering amplitude in the s-channel is proportional to the quantity

$$\begin{pmatrix} T_1 & q_3 & q_1-q_3 \\ q_1 & T_3 & T \end{pmatrix} \begin{pmatrix} T & q_4 & T_2 \\ q_4-q_2 & T_4 & q_2 \end{pmatrix}.$$

3. CALCULATION OF MEASURED QUANTITIES WITH THE AID OF HELICITY AMPLITUDES

Before we proceed to concrete processes, let us consider briefly the technique for calculating the measured quantities with the aid of the HA. We note first the following.

1. The unitarity of the D matrices is expressed by the equality

$$D^+D = \sum_{\lambda''} (D^{\lambda}_{\lambda''})^* D^{\lambda'}_{\lambda''} = \delta^{\lambda'}_{\lambda}. \quad (14)$$

In summing and averaging the probabilities over the polarizations we encounter just such combinations, and therefore the probability does not depend on the choice of the coordinate system to which the HA is referred, as should be the case.

2. The polarization of a particle with spin σ in its rest system is described by a polarization density matrix

$$\rho^{(\sigma)} = \sum_{n=0}^{2\sigma} s^{(n)}_{i_1\ldots i_n} H^{(\sigma)}_{i_1} \ldots H^{(\sigma)}_{i_n} = \sum_{n=0}^{2\sigma} s^{(n)}_{\{i\}} \prod_{p=1}^{n} H^{(\sigma)}_{i_p}, \quad (15)$$

where $H^{(\sigma)}_i$ ($i = 1, 2, 3$)—matrices of infinitesimal rotations of the representation with weight σ [1], $s^{(n)}_{\{i\}}$ are "statistical tensors," which are symmetrical in the indices, and

$$\sum_i s_{iii_3\ldots i_n} = 0, \quad s^{(0)} = (2\sigma+1)^{-1},$$

as follows from the condition $\mathrm{Sp}\,\rho = 1$. The spin state of a particle with velocity $\mathbf{v}$ in the helicity representation is described by a matrix ρ for that rest frame, which is obtained from the given frame by means of the pure Lorentz transformation with velocity $-\mathbf{v}$. In calculating the probabilities of the processes with polarized particles, expressions are encountered of the type

$$(D^{\lambda_1'}_{\lambda_1}(R))^* \rho^{\lambda_1'}_{\lambda_2'} D^{\lambda_2'}_{\lambda_2}(R) = \rho'^{\lambda_1}_{\lambda_2}. \quad (16)$$

From the properties of the matrix $D^{(\sigma)}$, as a matrix representation of a rotation group, it follows that

$$D^+(R) H_i D(R) = \sum_k r_{ik} H_k,$$

where r_{ik} is a matrix by which the three vectors are transformed under the rotation R. Therefore the quantities $s'^{(n)}$, which define the matrix ρ' in (16), are obtained from the values of $s^{(n)}$ of the matrix ρ in accordance with the usual rule for the transformation of three-tensors under rotation,

[1]In the case $\sigma = 1/2$ one has $H^{1/2}_i = 1/2\sigma_i$ (where σ_i are Pauli matrices) and the density matrix has the familiar form $\rho = 1/2(1 + \mathbf{s}\cdot\boldsymbol{\sigma})$, where s is the polarization vector.

thus explaining their name. Recalling now that the D matrix which relates the helicity states in certain systems K_1 and K_2 corresponds precisely to the rotation which relates the rest systems obtained from the systems K_1 and K_2, we verify that the foregoing description of the spin states is self-consistent.

Let, for example, a particle with spin σ be produced in the reaction and let the amplitude of this process be M^λ. The creation probability is $w \sim \mathrm{Sp} M^+ M$, and the polarization of the particle is described by the matrix

$$\rho_{\lambda'}^{\lambda} = (M^{\lambda'})^* M^\lambda / \mathrm{Sp}\, M^+ M, \tag{17}$$

where the values of the statistical tensors correspond to the coordinate system for the particle at rest, obtained by a pure Lorentz transformation from the reference frame in which the Ha M^λ was found.

Analogously, the probability of a process with a particle in initial state, the polarization of which is specified by matrix ρ, is expressed in terms of the Ha N_λ in the following fashion:

$$w \sim N^+ \rho N, \tag{18}$$

and the coordinate system in which ρ should be written is determined by the system in which the HA N_λ is calculated.

In final analysis, one measures not the components of the statistical tensors but invariant quantities (intensities). The spin dependences come into play in the case when two processes, each of which depends on the spin, occur in sequence. Let, for example, the polarization be measured with the aid of double scattering and let the HA of the first process M^λ be known in a certain reference frame K_1, as well as the HA of the second process N_λ in the frame K_2. The probability of double scattering is

$$w = N^+ D^+(R) M^+ M D(R) N, \tag{19}$$

and the rotation R is determined by the momentum of the particle and by the Lorentz transformation $K_2 \to K_1$; its parameters depend, in particular, on both scattering angles and on the angle between the planes of the reactions.

The spin states of a system consisting of two particles with spins σ_1 and σ_2 are described by a density matrix $\rho_{\lambda\mu}^{\lambda'\mu'}$, $|\lambda| \le \sigma_1$, $|\mu| \le \sigma_2$. This matrix can be represented in a form analogous to (15):

$$\rho^{(\sigma_1,\sigma_2)} = \sum_{n=0}^{2\sigma_1} \sum_{m=0}^{2\sigma_2} S^{(n,m)}_{i_1\ldots i_n j_1\ldots j_m} H^{(1)}_{i_1} \ldots H^{(1)}_{i_n} \times H^{(2)}_{j_1} \ldots H^{(2)}_{j_m} \tag{15a}$$

(The multiplication sign denotes the direct product). The properties of $S^{(n,m)}_{\{i\}\{j\}}$ with respect to each of the two aggregates of indices coincide with the properties of $s^{(n)}_{\{i\}}$ and $s^{(m)}_{\{j\}}$, and in particular the condition $\mathrm{Sp}\,\rho = 1$ means that $S^{(0,0)} = (2\sigma_1 + 1)^{-1}(2\sigma_2 + 1)^{-1}$.

It is convenient to represent the matrix ρ in the form

$$\rho = \sum_{n=0}^{2\sigma_1} S^{(n,0)}_{\{i\}} \prod H^{(1)}_i \times \sum_{m=0}^{2\sigma_2} S^{(0,m)}_{\{j\}} \prod H^{(2)}_j + \sum_{n,m} [S^{(n,m)}_{\{i\}\{j\}} - S^{(n,0)}_{\{i\}} S^{(0,m)}_{\{j\}}] \prod H_i \times \prod H_j \equiv \rho^{(1)} \times \rho^{(2)} + \sum_{n,m=1} c^{(n,m)}_{\{i\}\{j\}} \prod H_i \times \prod H_j, \tag{15b}$$

where $\rho^{(1)}$ and $\rho^{(2)}$ are the polarization density matrices for each of the particles with respective statistical tensors $s^{(n)}_{\{i\}} = S^{(n,0)}_{\{i\}}$ and $s^{(m)}_{\{j\}} = S^{(0,m)}_{\{j\}}$.

The quantities $c^{(n,m)}_{\{i\}\{j\}}$ describe the spin correlations and can be called the spin-correlation tensors. It should be noted that $c_{\{i\}\{j\}}$ is not a tensor in the sense of the statistical tensors $s_{\{j\}}$, for on going from one reference frame to the other $c_{\{i\}\{j\}}$ transforms in the indices of the first and second group differently. This is the consequence of the fact that the rotations by which the helicity quantities are transformed under a Lorentz transformation depend not only on this transformation but also on the momenta of the corresponding particles, and are therefore different for different particles. This circumstance, of course, in no way interferes with the covariant description of the measured quantities.

Formulas (17) and (18) can be extended in trivial fashion to include states containing two or even an arbitrary number of particles possessing spin. In formula (19) it is necessary to substitute in place of one D matrix the direct product of D matrices of all particles.

4. SCATTERING OF PIONS AND NUCLEONS

Let us consider the structure of the "vertex functions" for pions and nucleons assuming, as is customary, that the pole is characterized by a definite isospin T and a G-parity g.

Two pions can go over into a state with $g = \pm 1$ and T = 0, 1, 2; this transition is characterized by a "vertex function"

$$\widetilde{A}_{qq'} = \begin{pmatrix} 1 & 1 & q+q' \\ q & q' & T \end{pmatrix} \widetilde{a}(t),$$

where q, q′ = 0, ±1—isotopic indices of the pions. The transition of a pair $N\bar{N}$ into a state with isospin T is determined by the expression

$$\widetilde{B}_{qq',\lambda\lambda'} = \begin{pmatrix} 1/2 & 1/2 & q+q' \\ q & q' & T \end{pmatrix} \widetilde{B}_{\lambda\lambda'}(t).$$

Let us find the structure $\widetilde{B}_{\lambda\lambda'}$ for different poles. From parity conservation [formula (2)] we obtain ($\eta_N \eta_{\bar{N}} = -1$)

$$\widetilde{B}_{\lambda\lambda'} = \pi P_j \widetilde{B}_{-\lambda-\lambda'}. \tag{20}$$

As is well known[11], the G-parity of an $N\bar{N}$ pair in a state with isospin T is determined by the formula

$$G = (-1)^T C = -(-1)^T P_{12}, \tag{21}$$

where C is the charge conjugation operator and P_{12} is the space-spin exchange operator. It is shown in [1] that

$$P_{12}|J, \lambda_1, \lambda_2\rangle = P_J(-1)^{2s}|J, \lambda_2, \lambda_1\rangle. \tag{22}$$

From conservation of G parity and from (21) and (22) it follows that

$$\widetilde{B}_{\lambda\lambda'} = g(-1)^T P_j \widetilde{B}_{\lambda'\lambda}. \tag{23}$$

Formulas (20) and (23) enable us to find the connection between the quantum numbers of the pole and the spin structure of the amplitude. All the poles can be broken up into two classes[4].

Class A. $\pi P_j = +1$. Here

$$\widetilde{B} = \begin{pmatrix} \widetilde{b}_0 & i\widetilde{b}_1 \\ i\widetilde{b}_1 & \widetilde{b}_0 \end{pmatrix} = \widetilde{b}_0 + i\sigma_1 \widetilde{b}_1. \tag{24}$$

In the case when

$$g(-1)^T P_j = g(-1)^T \pi = +1, \quad \widetilde{b}_0 \neq 0, \quad \widetilde{b}_1 \neq 0$$

the contribution from the pole depends on two functions. This type includes trajectories with quantum numbers of vacuum, the ω and the ρ mesons (if ω and ρ are vector particles).

In the case when

$$g(-1)^T \pi = -1, \qquad \widetilde{b}_0 = \widetilde{b}_1 = 0.$$

The transition of the $N\bar{N}$ pair into a state with such quantum numbers is forbidden.

Class B. $\pi P_j = -1$. Here

$$\widetilde{B} = i\sigma_3 \widetilde{b}_3 + i\sigma_2 \widetilde{b}_2. \tag{25}$$

Case a):

$$g(-1)^T \pi = -g(-1)^T P_j = -1, \quad \widetilde{b}_2 = 0, \quad \widetilde{B} = i\sigma_3 \widetilde{b}_3.$$

This type includes trajectories with the quantum numbers of π and η mesons (0^{-+}).

Case b):

$$g(-1)^T \pi = +1, \quad \widetilde{b}_3 = 0, \quad \widetilde{B} = i\sigma_2 \widetilde{b}_2.$$

To determine the contribution from the pole belonging to class B, it is sufficient to specify one function.

Let us find with the aid of (7) the contribution of an arbitrary Regge pole to the measured quantities.

1. $\pi\pi$ scattering. Only poles with g = +1 and T = 0, 1, 2 make any contribution.

The total cross section is of the form

$$\sigma = 16\pi s^{j_0 - 1}\, \mathrm{Im}\, C(j)\, |a(0)|^2 \gamma_T,$$

$$a(t) = \widetilde{a}(t)\, |4/(t - 4m_\pi^2)|^{1/2j}, \tag{26}$$

where

$$j_0 = j(0), \qquad \gamma_T = \begin{pmatrix} 1 & q_1 & 0 \\ q_1 & 1 & T \end{pmatrix} \begin{pmatrix} T & q_2 & 1 \\ 0 & 1 & q_2 \end{pmatrix}$$

is a factor that depends on the isospin of the trajectory and on the charge of the interacting mesons. The values of γ_T are:

Interaction \ T	0	1	2
$\pi^+\pi^+$ and $\pi^-\pi^-$	$1/3$	$-1/6$	$1/10$
$\pi^+\pi^0$ and $\pi^-\pi^0$	$1/3$	0	$-1/5\sqrt{3}$
$\pi^+\pi^-$	$1/3$	$1/6$	$1/10$
$\pi^0\pi^0$	$1/3$	0	$2/15$

The differential scattering cross section in a state with isospin T′ is of the form

$$d\sigma/d\Omega = 4\,|\beta_{T'T}|^2\, |C(j)|^2\, |a(t)|^2\, s^{2j-1}, \tag{27}$$

$\beta_{T'T}$ is an element of the matrix written in (12b).

2. πN scattering. Contributions are made by poles of class A, g = +1, T = 0 and 1.

If we take xz as the scattering plane, then the structure of the HA of πN scattering is described by the formula [1]

$$M = M_0 + i\sigma_2 M_2.$$

The contribution from the Regge pole is of the form [equation (7)]

$$M = \frac{1}{2\pi} s^j C(j)\, a(t)\, (b_0(t) + i\sigma_2 b_2(t)), \tag{28}$$

where

$$b_0 = -\sqrt{\frac{j}{j+1}}\, \widetilde{b}_1 \left| \frac{4}{t - 4m_N^2} \right|^{1/2j},$$

$$b_2 = -\widetilde{b}_0 \left| \frac{4}{t - 4m_N^2} \right|^{1/2j},$$

and m_N is the nucleon mass.

The total cross section is written in the form

$$\sigma = 16\pi s^{j_0-1}\,\mathrm{Im}\,C(j)\,a(0)\,b_0(0)\,\gamma_T. \quad (29)$$

The values of

$$\gamma_T = \begin{pmatrix} 1 & q_1 & 0 \\ q_1 & 1 & T \end{pmatrix} \begin{pmatrix} T & q_2 & 1/2 \\ 0 & 1/2 & q_1 \end{pmatrix}$$

(q_1 and q_2 are the isotopic indices of the pion and nucleon) are

Interaction \ T	0	1
π^+p and π^-n	1/6	1/6
π^-p and π^+n	1/6	−1/6
π^0p and π^0n	1/6	0

We present now formulas for the differential cross section for the scattering of a pion by a polarized nucleon in a state with isospin T′. The nucleon polarization is described by a matrix $\rho^i = (1 + \mathbf{s}^i \cdot \sigma)/2$. We have

$$d\sigma/d\Omega = (2\pi/p)^2\,\mathrm{Sp}\,M^+\rho^i M$$
$$= 4\beta^2_{T'T} s^{2j-1} |C(j)|^2 |a(t)|^2 [|b_0|^2 + |b_2|^2 + 2s_2^i\,\mathrm{Im}\,b_0 b_2^*]. \quad (30)$$

Here s_2^i is the degree of polarization of the nucleon in a direction perpendicular to the scattering plane—a quantity which is invariant relative to rotation in this plane; this is also true of the quantities $|b_0|^2 + |b_2|^2$ and $\mathrm{Im}\,b_0b_2^*$, so that the reference frame in which the amplitude is determined is immaterial.

The matrix $\beta_{T'T}$ [equation (12)] determines the isospin dependence of the cross section:

T′ \ T	0	1
1/2	$-\sqrt{1/6}$	1
3/2	$\sqrt{1/6}$	1/2

The term in the differential cross section due to the interference between the two poles a and b (for example vacuum and ω meson) is of the form

$$d\sigma/d\Omega = 4\beta_{T'T_a}\beta_{T'T_b} s^{j_a+j_b-1}$$
$$\times\,\mathrm{Re}\,\{C(j_a)\,C^*(j_b)\,a_a a_b^*\,[b_{0a}b_{0b}^* + b_{2a}b_{2b}^*]\}. \quad (31)$$

We now consider the polarization of nucleons in the final state. The polarization of the recoil nucleon is described by a vector $\mathbf{s}^f$, which is determined from

$$\rho^f = \tfrac{1}{2}(1 + s^f\sigma) = M^+\rho^i M/\mathrm{Sp}\,M^+\rho^i M.$$

We get from this for the polarization in a direction perpendicular to the scattering plane

$$\mathcal{F}s_2^f = 2\,\mathrm{Im}\,b_0b_2^* + s_2^i(|b_0|^2 - |b_2|^2), \quad (32a)$$

where

$$\mathcal{F} = |b_0|^2 + |b_2|^2 + 2s_2^i\,\mathrm{Im}b_0b_2^*.$$

For longitudinal polarization in the l.s. we have

$$\mathcal{F}s_3^f = s_3^i(|b_0|^2 - |b_2|^2) - 2s_1^i\,\mathrm{Re}\,b_0b_2^*. \quad (32b)$$

The transverse polarization in the reaction plane is equal to

$$\mathcal{F}s_1^f = s_1^i(|b_0|^2 - |b_2|^2) + 2s_3^i\,\mathrm{Re}\,b_0b_2^*. \quad (32c)$$

Here $s_3^i s_1^i$ are the components of the target polarization vector in the direction of the momentum of the recoil nucleon and in a direction perpenducular to it and lying in the scattering plane.

3. Nucleon-nucleon scattering. It is easy to show that owing to P and T invariance and the principle of indistinguishability of identical particles, the HA for NN scattering takes the form

$$M = \sum_{i=0}^{3} M_{ii}\sigma_i^{(1)}\sigma_i^{(2)} + M_{02}\sigma_0^{(1)}\sigma_2^{(2)} + M_{02}^*\sigma_2^{(1)}\sigma_0^{(2)},$$

where $\sigma_i^{(1)}$ and $\sigma_i^{(2)}$ are Pauli matrices with indices λ_1, λ_3 and λ_2, λ_4, respectively.

The three types of poles give the following contributions to the NN scattering amplitude.

Class A pole

$$\frac{1}{2\pi}C(j)\,s^j\,(b_0\sigma_0^{(1)} + i\sigma_2^{(1)}b_2)\,(b_0^*\sigma_0^{(2)} - i\sigma_2^{(2)}b_2^*). \quad (33a)$$

Class B pole, case a):

$$\frac{1}{2\pi}C(j)\,s^j b_1 b_1^*\sigma_1^{(1)}\sigma_1^{(2)}, \quad b_1(t) = -\widetilde{b}_3(t)\,|4/(t-4m_N^2)|^{1/2j}. \quad (33b)$$

Class B pole, case b):

$$\frac{1}{2\pi}C(j)\,s^j b_3 b_3^*\sigma_3^{(1)}\sigma_3^{(2)}, \quad b_3 = \sqrt{j/(j+1)}\,\widetilde{b}_2\,|4/(t-4m_N^2)|^{1/2j}. \quad (33c)$$

The total cross section is given by the formula

$$\sigma = (2\pi/p)^2 2\,\mathrm{Im}\,\tfrac{1}{4}\,\mathrm{Sp}\,M(t=0).$$

The contribution to the total cross section is made only by poles of class A:

$$\sigma = 16\pi s^{j_0-1}\,\mathrm{Im}\,C(j_0)\,|b_0|^2\gamma_T,$$
$$\gamma_T = \begin{pmatrix} 1/2 & q_1 & 0 \\ q_1 & 1/2 & T \end{pmatrix}\begin{pmatrix} T & q_2 & 1/2 \\ 0 & 1/2 & q_2 \end{pmatrix}. \quad (34)$$

This value of γ_T is:

Interaction \ T	0	1
pp and nn	1/2	−1/6
np	1/2	1/6

The differential cross section is of the form

$$d\sigma/d\Omega = |\beta_{T'T}|\, s^{2j-1} |C(j)|^2 F^2(t), \quad (35)$$

where $\beta_{T'T}$ is an element of the matrix (12a), and

$$F(t) = \begin{cases} |b_0|^2 + |b_2|^2 & \text{For class A poles,} \\ |b_1|^2 & \text{For class B case a),} \\ |b_3|^2 & \text{For class B case b).} \end{cases}$$

For poles of class B, the differential cross section does not depend on the nucleon polarization in the initial state, while for class A the cross section is obtained by making the following substitution in (35)

$$F^2(t) \to [F(t) + s_2^{(1)}\, 2\,\mathrm{Im}\, b_0 b_2^*]\,[F(t) - s_2^{(2)}\, 2\,\mathrm{Im}\, b_0 b_2^*],$$

where $s_2^{(1)}$ and $s_2^{(2)}$ are the degrees of polarization of the first and second nucleons in a direction perpendicular to the scattering plane.

The contribution to the cross section from the interference between the class A and class B poles is proportional to the product of the degree of polarizations of the colliding nucleons in the scattering plane. For example, for the scattering of nucleons with longitudinal polarizations $s_3^{(1)}$ and $s_3^{(2)}$ and transverse polarizations in the scattering plane $s_1^{(1)}$ and $s_1^{(2)}$, the interference between a class A pole and a pole of class B, case a) yields

$$d\sigma/d\Omega = 2\beta_{T'T_a}\beta_{T'T_b}\, s^{j_a + j_b - 1}\, \mathrm{Re}\,\{C(j_a)\, C^*(j_b)\, |b_1|^2 \times (-b_0 s_1^{(1)} + b_2 s_3^{(1)})(-b_0^* s_1^{(1)} + b_2^* s_3^{(2)})\}. \quad (36)$$

We now consider the polarization of the nucleons in the final state.

If the polarizations of the initial nucleons are not correlated, that is, their spin states are described by the direct product of the polarization matrices of each of the nucleons, then in the one-pole approximation the polarization of the final nucleons is likewise uncorrelated. This is a consequence of the factorization of the contribution from one pole. For the same reason, the polarization of nucleon 3 does not depend on the polarization of nucleon 2, and depends only on the polarization of nucleon 1. An analogous statement holds for nucleon 4. Moreover, in the case of a pole of class A, the formulas relating the polarization of the final nucleons to the polarizations of the initial nucleons coincide with the formulas for πN scattering (32). For poles of class B the polarization of the final nucleon is connected with the polarization of the initial nucleon by the following formulas:

$$s_{2,3}^f = -s_{2,3}^i, \quad s_1^f = s_1^i \text{ — class B, case a)},$$
$$s_{1,2}^f = -s_{1,2}^i, \quad s_3^f = s_3^i \text{ — class B, case b)}.$$

The spin correlation of the final nucleons appears if we represent the amplitude in the form of a sum of contribution from two poles. This correlation is described by the quantity c_{ik} in accordance with (15b), and can be readily obtained. For example, for a sum of two poles of class A ($j_a(t)$ and $j_b(t)$; $j_a > j_b$) the amplitude has the form

$$A + B\sigma_2^{(1)} + D\sigma_2^{(2)} + C\sigma_2^{(1)}\sigma_2^{(2)}.$$

In this case the nonvanishing component c_{22} of the correlation tensor is equal to

$$c_{22} = \frac{\varepsilon\alpha^* + \varepsilon^*\alpha}{\Sigma^2};$$

$$\varepsilon = AC - BD, \qquad \alpha = A^2 - B^2 - D^2 + C^2,$$
$$\Sigma = |A|^2 + |B|^2 + |C|^2 + |D|^2. \quad (37)$$

It is easy to see that for large s the quantity $|c_{22}|$ decreases like $s^{j_b - j_a}$.

3. THE PROCESS $\pi + N \to \rho + N$

By way of an example of an inelastic process which can be described by Regge poles, let us consider the production of a ρ meson by a π meson on a nucleon. The contribution to this process is made by poles with $g = -1$, $T = 0$ and 1.

The polarization of the unstable particle manifests itself in the angular correlation of its decay products. In this connection, let us find the cross section of the process $\pi_i + N_i \to \rho + N_f \to (\pi_f + \pi_f') + N_f$ under the assumption that the first process is determined by a Regge pole.

The helicity amplitude of the ρ meson decay P_λ ($\lambda = 0, \pm 1$) has the simplest form in the ρ-meson rest system and in the coordinate frame in which the relative momentum of the pions is directed along the z axis (the K_ρ system):

$$P_\lambda = f\delta_{\lambda 0}. \quad (38)$$

The constant f is connected with the ρ-meson width Γ_ρ in the following fashion:

$$\Gamma_\rho = \frac{f^2}{4\pi}\frac{1}{6m_\rho}\left[1 - \frac{(2m_\pi)^2}{m_\rho^2}\right]^{1/2}.$$

The cross section of the process under consideration, averaged over the initial and summed over the final polarizations of the nucleons, is represented in the form (19):

$$d\sigma = (2\pi/p)^2\, \mathrm{Sp}\,\{MD^{(1)}(R_3)\, PP^+ D^{+(1)}(R_3)\, M^+\} \times \left(\tfrac{1}{3}\,\mathrm{Sp}\, PP^+\right)^{-1} d\Omega_N\, d\Omega/4\pi. \quad (39)$$

Here R_3 is the rotation corresponding to the Lorentz transformation from a system in which the amplitude M is determined to the system in which the amplitude P is written; $d\Omega_N$ and $d\Omega$ correspond to the angles of emission of the recoil nucleon and of one of the pions.

The contribution from the pole has the simplest form in the center of mass system of the t-channel ($K_{\pi\rho}$). We shall therefore determine the parameters of the rotation matrix corresponding to the transformation $K_{\pi\rho} \rightarrow K_\rho$. Bearing in mind the experimental conditions, we express the parameters of this rotation in terms of the following invariants: the angle φ between the plane of decay of the ρ meson in the laboratory system (l.s.) and the plane of the reaction $\pi N \rightarrow \rho N$ and the energy of one of the produced pions.

We make the transition $K_{\pi\rho} \rightarrow K_\rho$ via $K_{\pi\rho} \rightarrow K_{lab} \rightarrow K_\rho$, where K_{lab} is the system in which N_i is at rest. The first transformation corresponds to rotation in the xz plane through a certain angle κ_1, the second to rotation about the z axis through an angle φ and in the xz plane through an angle κ_2. To determine the angles κ_1 and κ_2 it is convenient to use diagrams in velocity space[12,13]. The angle κ_1 (Fig. 2) is equal to the angle in the Lobachevsky plane between the ρ-meson velocities in the systems $K_{\pi\rho}$ and K_{lab}:

$$\cos \varkappa_1 = \frac{(m_N^2 + m_\rho^2 - u)(m_\pi^2 + m_\rho^2 - t) - 2m_\rho^2 (s - m_N^2 - m_\pi^2)}{\{[(m_N + m_\rho)^2 - u][(m_N - m_\rho)^2 - u][(m_\pi + m_\rho)^2 - t][(m_\pi - m_\rho)^2 - t]\}^{1/2}},$$

$$-\cos \varkappa_1 \underset{s\to\infty}{\approx} \frac{t + m_\rho^2 - m_\pi^2}{\{[(m_\rho + m_\pi)^2 - t][(m_\rho - m_\pi)^2 - t]\}^{1/2}}. \tag{40}$$

The angle κ_2 is determined by means of the construction shown in Fig. 3, which shows the plane in which the momenta π_f and π_f' are located in the l.s. Using the cosine theorem of the Lobachevsky geometry, we obtain

$$\cos \varkappa_2 = \frac{m_\pi \operatorname{ch} \zeta \operatorname{ch} \xi - E}{m_\pi \operatorname{sh} \zeta \operatorname{sh} \xi}; \quad \operatorname{ch} \zeta = \frac{m_N^2 + m_\rho^2 - u}{2m_N m_\rho},$$

$$\operatorname{ch} \xi = \frac{m_\rho}{2m_\pi}, \tag{41*}$$

E—energy of the produced pion in the l.s. The function $D^{(1)}(R_3)$, which is contained in (39), is of the form

$$D^{(1)}_{\lambda\mu}(R_3) = \sum_\nu d^1_{\lambda\nu}(\varkappa_1)\, e^{i\nu\varphi} d^1_{\nu\mu}(\varkappa_2). \tag{42}$$

We now find the HA of the process $\pi N \rightarrow \rho N$. The amplitude of the transition of the system $\pi\rho$ into a state with parity π and signature P_j obeys condition (2): $\widetilde{A}_\lambda = -\pi P_j \widetilde{A}_{-\lambda}$, where λ is the helicity of the ρ mesons. For an ω-meson trajectory $\widetilde{A}_\lambda = -\widetilde{A}_{-\lambda}$. The contribution from the pole is represented in the form (7):

$$M^{\lambda_3\lambda_4}_{\lambda_2} = \frac{1}{2\pi} C(j)\, s^j A^{\lambda_3} B^{\lambda_4}_{\lambda_2}, \tag{43}$$

where

$$A^{+1} = A^{-1} \equiv a_{\pi\rho}(t) = -i \sqrt{\frac{j}{j+1}} \widetilde{A}_{+1} |p_\rho(t)|^{-j}, \quad A^0 = 0$$

$$B^{\lambda_4}_{\lambda_2} = b_0 \delta^{\lambda_4}_{\lambda_2} + b_2 (i\sigma_2)^{\lambda_4}_{\lambda_2}$$

[compare (24) with (28)]. Substituting (38), (42), and (43) in (39) we get

$$d\sigma = 4s^{2j-1} |C(j)|^2 |a_{\pi\rho}(t)|^2 (|b_0|^2 + |b_2|^2)$$
$$\times \sin^2 \varkappa_2 \sin^2 \varphi\, d\Omega_N d\Omega / 4\pi. \tag{44}$$

*ch = cosh, sh = sinh.

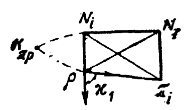

FIG. 2

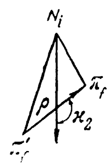

FIG. 3

The isospin is taken into account in trivial fashion. Let q_1, q_5, and q_6 be the isotopic indices of the initial and final pions, respectively, and let q_2 and q_4 be those of the initial and final nucleons. Then to the amplitude (44) we should add the factor

$$\begin{pmatrix} 1 & q_5 + q_6 & q_4 - q_2 \\ q_1 & 1 & T \end{pmatrix} \begin{pmatrix} T & q_4 & 1/2 \\ q_4 - q_2 & 1/2 & q_2 \end{pmatrix} \begin{pmatrix} 1 & q_5 & q_6 \\ q_5 + q_6 & 1 & 1 \end{pmatrix},$$

where T is the isospin of the pole. If T = 0 (ω meson) this factor is of the form

$$\frac{1}{3\sqrt{2}} \delta_{q_1, q_5+q_6} \delta_{q_2, q_4} \begin{pmatrix} 1 & q_5 & q_6 \\ q_1 & 1 & 1 \end{pmatrix}.$$

6. THE PROCESS N + N → N + N*

In conclusion let us consider the production of a third nucleon resonance (mass M = 1688 MeV, s = 5/2, P = +1, T = 1/2) in a nucleon-nucleon collision. We find the contribution made to this process by a class A pole.

Let us ascertain first the structure of the amplitude for the transition of the reggeon into the $\bar{N}N^*$ system. From parity conservation (2) we have

$$\widetilde{B}^{\lambda_2\lambda_4} = \pi P_j \widetilde{B}^{-\lambda_2 - \lambda_4}. \tag{45}$$

The formula for the production amplitude contains $B^{\lambda_4}_{\lambda_2}$ (formulas (7) and (7c)). Such a matrix is conveniently written in the form of an expansion in Clebsch-Gordan coefficients (see the appendix). This expansion is a generalization of the expansion in Pauli matrices

$$B^{\lambda_4}_{\lambda_2} = \sum_{l=2,3} (2l+1)\, b^n_{(l)} \begin{pmatrix} 1/2 & l & \lambda_4 \\ \lambda_2 & n & 5/2 \end{pmatrix}. \tag{46}$$

From (45) and from the connection between $\widetilde{B}$ and B it follows that

$$b^{-n}_{(l)} = (-1)^{l-n} b^n_{(l)}. \tag{47}$$

The amplitude of the decay $N^* \to N + \pi$ in the rest system of N^* with z axis along the relative momentum of N and π (the K^* system) is of the form

$$Q^\mu_\lambda \sim \delta^\mu_\lambda$$

or in the form of (A.1) and (46)

$$Q^\mu_\lambda = \sum_{l=2,3} (2l+1)\, g^m_{(l)} \begin{pmatrix} 5/2 & l & \mu \\ \lambda & m & 1/2 \end{pmatrix}, \tag{48}$$

where only the coefficient $g^0_{(3)}$ differs from zero. We note that

$$\mathrm{Sp}\; Q^+Q = 7\,|g^0_{(3)}|^2. \tag{49}$$

The cross section of the process $N_1 + N_2 \to N_3 + N^* \to N_3 + (N_4 + \pi)$ is proportional to

$$\Phi = \mathrm{Sp}\, B^+D^+(R)\; Q^+QD\,(R)\,B\,\left(\tfrac{1}{6}\mathrm{Sp}\; Q^+Q\right)^{-1}, \tag{50}$$

where $D = D^{(5/2)}(R)$ is the matrix of the rotation corresponding to the Lorentz transformation from the system $K_{1\bar{3}}$ into the system K^* and momentum N^*. As in Sec. 5, the rotation R is represented in the form of the result of a rotation in the xz plane through an angle κ_1, in the xy plane through an angle φ, and in the xz plane through an angle κ_2 (see Fig. 4), with φ the angle between the N^* production and decay planes.

Let us consider a simpler and more interesting case when N_2 is a target, that is, when N^* acquires a small momentum in the l.s. compared with N_3. In this case $\kappa_1 = \pi$ and κ_2 is given by a formula analogous to (41):

$$\begin{aligned} \cos\varkappa_2 &= (m\,\mathrm{ch}\,\zeta\,\mathrm{ch}\,\xi - E)/m\,\mathrm{sh}\,\zeta\,\mathrm{sh}\,\xi; \\ \mathrm{ch}\,\zeta &= (M^2 + m^2 - t)/2Mm, \\ \mathrm{ch}\,\xi &= (M^2 + m^2 - \mu^2)/2Mm, \end{aligned} \tag{51}$$

M, m, and μ are the masses of N^*, N, and the pion, respectively and E is the energy of the nucleon N_4 (produced as a result of the decay of N^*) in the l.s. The matrix D contained in (50) is of the form

$$D^\mu_\lambda(R) = \sum_\nu d_{\lambda\nu}(\varkappa_1)\, e^{i\nu\varphi} d_{\nu\mu}(\varkappa_2). \tag{52}$$

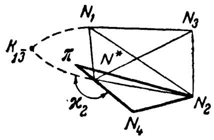

FIG. 4

The function Φ (50) is calculated with the aid of (A.2) and (A.4):

$$\Phi = \frac{6}{7} \sum_{j,m,m'} (2j+1)\, \beta_{(j)m'} D^{(j)m'}_m(R)\, \alpha^m_{(j)}, \tag{53}$$

$$\alpha^m_{(j)} = \begin{Bmatrix} 3 & 3 & j \\ 5/2 & 5/2 & 1/2 \end{Bmatrix} \begin{pmatrix} 3 & j & 3 \\ 0 & 0 & 0 \end{pmatrix} \delta_{m0}. \tag{54}$$

The number $\alpha^m_{(j)}$ differs from zero only when $m = 0$ and $j = 0, 2, 4$, otherwise one of the factors in (54) vanishes. The second quantity contained in (53) has a more complicated form:

$$\beta_{(j)m} = \sum_{\substack{l,l' \\ n,n'}} (2l+1)(2l'+1) \begin{Bmatrix} l' & l & j \\ 5/2 & 5/2 & 1/2 \end{Bmatrix} b^{n*}_{(l)} b^{n'}_{(l')} \begin{pmatrix} n & j & l' \\ l & m & n' \end{pmatrix}. \tag{55}$$

We note that by virtue of (47) we have

$$\beta_{(j)-m} = (-1)^{j-m} \beta_{(j)m}.$$

Using this equation and the properties of the D matrices, and recognizing that $\kappa_1 = \pi$, we obtain with the aid of (52)

$$\Phi = \frac{6}{7} \sum_{j=0,2,4} \sum_{m=0}^{j} (2j+1)\, \beta_{(j)m}\, \alpha^0_{(j)}\, d^{(j)}_{m0}(\pi - \varkappa_2) \cos m\varphi. \tag{56}$$

Finally the cross section for the process $N_1 + N_2 \to N_3 + (N_4 + \pi)$ averaged over the initial polarizations of the nucleons and summed over the final ones, is of the form

$$d\sigma = \pi^{-1} s^{2j-1} |C(j)|^2 (|b_0|^2 + |b_2|^2)\, \Phi(t, E, \varphi)\, d\Omega_3 d\Omega_4,$$

where $d\Omega_3$ and $d\Omega_4$ pertain to the angles of emergence of the nucleons N_3 and N_4.

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APPENDIX

Any matrix that depends on helicity indices can be expanded in Wigner matrices:

$$A^\mu_\lambda = \sum_j (2j+1)\, a^{(m)}_j \begin{pmatrix} s_1 & j & \mu \\ \lambda & m & s_2 \end{pmatrix}. \tag{A.1}$$

The product of two matrices is represented in the form

$$\sum_\lambda B^\lambda_{\mu_1} A^{\mu_2}_\lambda = \sum_l (2l+1)\, c^n_{(l)} \begin{pmatrix} s^b_2 & l & \mu_2 \\ \mu_1 & n & s^a_2 \end{pmatrix}, \tag{A.2}$$

where

$$c_{(l)}^{n} = \sum_{\substack{j,\, j' \\ m,\, m'}} (2j + 1)\,(2j' + \ \ )\begin{Bmatrix} j' & j & l \\ s_2^a & s_2^b & s_1 \end{Bmatrix} a_{(j)}^{m}\, b_{(j')}^{m'} \begin{pmatrix} j & n & j' \\ m & l & m' \end{pmatrix}.$$

We use the fundamental property of Wigner matrices[7]

$$\sum_{\substack{m_1,\, m_2 \\ m_1',\, m_2'}} \begin{pmatrix} j_1 & j_2 & m \\ m_1 & m_2 & j \end{pmatrix} D_{m_2'}^{(j_2)m_2}(R)\, D_{m_1'}^{(j_1)m_1}(R) \begin{pmatrix} m_1' & m_2' & j' \\ j_1 & j_2 & m' \end{pmatrix}$$

$$= \frac{\delta(j, j')}{2j+1} D_{m'}^{(j)\,m}(R), \tag{A.3}$$

and obtain a convenient expression for the quantity SpBD⁺(R) AD(R):

$$\sum_{\substack{\mu,\, \lambda \\ \mu',\, \lambda'}} B_{\mu'}^{\lambda'} (D_{\lambda}^{(s_1)\lambda'}(R))^{*} A_{\lambda}^{\mu} D_{\mu}^{(s_2)\mu'}(R)$$

$$= \sum_{j,\, m',\, m} (2j + 1)\, b_m^{(j)}\, D_{m'}^{(j)m}(R)\ \ a_{(j)}^{m'}. \tag{A.4}$$

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