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COMPOSITION OF NUCLEAR-ACTIVE COMPONENT OF COSMIC RAYS IN THE ATMOSPHERE

N. L. GRIGOROV

Institute of Nuclear Physics, Moscow State University

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The flux of high energy pions at various heights of the atmosphere is calculated. It is shown that at mountain heights and at sea level the pion fluxes should comprise, according to conservative estimates, 25 and 40 per cent of the nucleon fluxes at energies of 10^{11} and 10^{12} eV respectively.

HIGH-APERTURE arrays such as the ionization calorimeter with cloud chamber [1,2] have made it possible to investigate in detail the interaction between high-energy cosmic particles and atomic nuclei at low altitudes, where the low intensity of the particles can be compensated for by the large dimensions of the arrays. [3] In this connection, the question of the composition of the nuclear-active component of cosmic rays at different altitudes of the atmosphere becomes most timely, particularly the question of the fraction of $\pi^{\pm}$ mesons in the flux of high-energy nuclear-active particles.

This question can be considered by making definite assumptions with respect to the mechanism whereby the pions are generated in the elementary interaction act.

We denote by $n_1(E, E_0)dE$ and $n_2(E, E_0)dE$ the spectra of the pions generated in interactions between the nucleon and pion, respectively, and the atomic nucleus. We further denote by $P_\pi(E, X)dE$ the intensity of the vertical stream of pions with energy E, $E+dE$ at a depth X of the atmosphere (X is measured in nucleon-interaction ranges).

The equation for $P_\pi(E, X)$ is of the form

$$\frac{\partial P_\pi}{\partial X} + \frac{L_n}{L_\pi} P_\pi + \frac{C}{EX} P_\pi = \int_E^\infty N_n(E_0, x)\, n_1(E, E_0)\, dE_0 + \frac{L_n}{L_\pi} \int_E^\infty P_\pi(E_0, X)\, n_2(E, E_0)\, dE_0,$$

where $N_n(E_0, X)dE_0$ is the flux of nucleons with energy E_0, E_0+dE_0 at a depth X. L_n and L_π are the ranges for the interactions of the nucleons and pions; $C = 1.8 \times 10^{11}$ eV.

Putting $L_n/L_\pi = 1 - a$, we obtain

$$\frac{\partial P_\pi}{\partial X} + \left(1 - a + \frac{C}{EX}\right) P_\pi = \int_E^\infty N_n(E_0, X)\, n_1(E, E_0)\, dE_0 + (1 - a) \int_E^\infty P_\pi(E_0, X)\, n_2(E, E_0)\, dE_0. \quad (1)$$

The solution of (1) should satisfy the boundary condition $P_\pi(E, X=0) = 0$.

We shall seek the solution of (1) in the form

$$P_\pi(E, X) = \sum_{i=1}^{\infty} P_i(E, X)$$

with boundary conditions $P_i(E, X=0) = 0$. Then (1) assumes the form

$$\frac{\partial P_1}{\partial X} + \left(1 - a + \frac{C}{EX}\right) P_1 - \int_E^\infty N_n(E_0, X)\, n_1(E, E_0)\, dE_0 + \sum_{i=2} \left\{ \frac{\partial P_i}{\partial X} + \left(1 - a + \frac{C}{EX}\right) P_i - (1 - a) \int_E^\infty P_{i-1} n_2\, dE_0 \right\} = 0.$$

If we stipulate that

$$\frac{\partial P_1}{\partial X} + \left(1 - a + \frac{C}{EX}\right) - \int_E^\infty N_n n_1 dE_0 = 0,$$

$$\frac{\partial P_i}{\partial X} + \left(1 - a + \frac{C}{EX}\right) P_i - (1 - a) \int_E^\infty P_{i-1} n_2\, dE_0 = 0 \quad (i = 2, 3, \ldots), \quad (2)$$

then the solution of the system of equations (2) will be a solution of the initial equation (1).

We assume that upon interaction between a nucleon and a nucleus, the pions are formed in two ways: a) by pionization and b) by generation of energetically favored pions. The first mechanism (pionization) can be replaced in first approximation by generation, in one elementary act, of n charged pions with equal energy $E = \bar{K}E_0/1.5\,\bar{n}$, where $\bar{K}$ is the average energy lost by the nucleon to pion production, and $\bar{n} = bE^{\nu}$. For the second process we assume that the form of the spectrum of the generated pions does not depend on the nucleon energy E_0, i.e., it has the form $n_1 f_1(E/E_0)\, dE/E_0$.

When these two assumptions are satisfied, we can write

$$n_1(E, E_0)\, dE = \bar{n}_1 f_1\left(\frac{E}{E_0}\right)\frac{dE}{E_0} + bE_0^{\nu}\delta\left(E - \frac{KE_0}{1.5\bar{n}}\right)\frac{dE}{E_0}.$$

Assume that $N_n(E_0, X) = Be^{-\beta X}/E_0^{\gamma}$, we obtain for $P_1(E, X)$ an expression in the form

$$P_1(E, X) = N_n(E, X)\, F(E)\, \varphi(E, X), \qquad (3)$$

where

$$F(E) = A_1 + \frac{\bar{K}^{(\gamma-\nu-1)/(1-\nu)}}{1.5\,(1.5b)^{(\gamma-2)/(1-\nu)}\, E^{\nu(\gamma-2)/(1-\nu)}}, \qquad (4)$$

$$A_1 = \bar{n}_1 \int_0^1 u^{\gamma-1} f_1(u)\, du;$$

$$\varphi(E, X) = X \int_0^1 y^{C/E}\, e^{X(1-a-\beta)(y-1)}\, dy. \qquad (5)$$

When $b = 3$, $\nu = 1/4$, $K = 1/3$, and $\gamma = 2.8$ we obtain (E is measured in BeV)

$$F(E) = A_1 + 0{,}013/E^{0.27}. \qquad (4')$$

By comparing the flux of high-energy muons with the flux of primary cosmic rays we have found[4] that $A_1 = 0.11$—0.13. From (4′) we see that when $E \geq 10$ BeV we have $0.013/E^{0.27} \ll A_1$, i.e., the pionization can be neglected in the production of first-generation pions, and

$$P_1(E, X) = A_1 N_n(E, X)\, \varphi(E, X). \qquad (6)$$

Knowing $P_1(E, X)$ we can estimate the role of the second generation $P_2(E, X)$, assuming that the pions generate pions only in pionization processes. In this case for $(1.5bE/K_\pi)^{1/(1-\nu)} \lesssim 10$ BeV we have

$$\frac{P_2(E, X)}{P_1(E, X)} = \frac{X\bar{K}_\pi^{(\gamma-2-\nu)/(1-\nu)}}{1.5\,(1.5b)^{(\gamma-3)/(1-\nu)}\, CE^{[\nu(\gamma-2)-1]/(1-\nu)}};$$

For $(1.5bE/K_\pi)^{1/(1-\nu)}$ 10^3 BeV we have

$$\frac{P_2(E, X)}{P_1(E, X)} = \left[1 - \frac{XEe^{-(1-a-\beta)X}}{(C+E)\,\varphi(E, X)}\right] \times \frac{K_\pi^{(\gamma-1-\nu)/(1-\nu)}}{1.5\,(1-a-\beta)\,(1.5b)^{(\gamma-2)/(1-\nu)}E^{\nu(\gamma-2)/(1-\nu)}}.$$

Assuming that for pions the average inelasticity coefficient is $K_\pi = 1$, $\gamma = 2.8$, $b = 3$, and $\nu = 1/4$ we obtain for mountain altitudes $P_2/P_1 = 7\%$ for $E = 10^{12}$ eV, 13% for $E = 10^{11}$ eV, 20% for $E = 2 \times 10^{10}$ eV and 7% for $E = 1.3 \times 10^9$ eV.

Thus, pion production due to pionization can be neglected in first approximation for all generations. On the other hand, if an energetically favored pion is produced in an interaction between pions and nuclei, this is equivalent to an increase in the range for pion interactions, accounted for by the coefficient a in the equation for the first-generation pions.

With these stipulations, we can write

$$P_\pi(E, X) \cong P_1(E, X) = A_1 N_n(E, X)\, \varphi(E, X).$$

Hence

$$P_\pi(E, X)/N_n(E, X) = A_1 \varphi(E, X).$$

The function $\varphi(E, X)$ can be calculated for different values of the parameter a. Therefore in order to determine $P_\pi(E, X)/N_n(E, X)$ it is necessary to know the value of A_1(4). This quantity enters in the equation that determines the intensity of the high-energy muon flux at sea level[4]

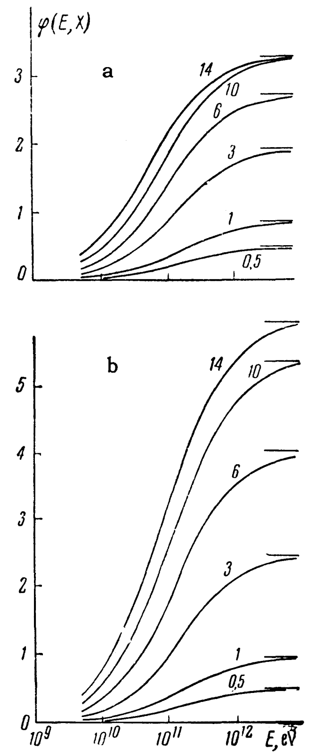
$$N_\mu(E)\, dE = A_1 \frac{\psi(E, a)\, N_n(E, X=0)\, dE}{(1.3)^{\gamma-1}},$$

$$\psi(E, a) = \frac{1}{1-a}\sum_{n=0}^{\infty} \frac{d^n}{1 + 0.72\,(n+1)\, E/10^{11}\,[\mathrm{eV}]},$$

$$d = \frac{1-a-\beta}{1-a}.$$

As we have already noted above, from a comparison of the muon fluxes and the primary cosmic radiation we have obtained a value $A_1 = 0.11$—0.13.

The function $\varphi(E, X)$ was calculated by us for $a = 0$ and $a = 0.15$ (see the figure, cases a and b respectively) for different depths of the atmosphere X, viz., 0.5, 1, 3, 6, 10, and 14. The minimum estimate of the pions in the flux of nuclear-active particles is obtained for $a = 0$ ($a = 0$ denotes that the ranges for the interaction between the pions and the nucleons are identical and that when the pions interact with the atomic nuclei they do not generate energetically favored secondary pions). As can be seen from the figure (case a), even at $a = 0$ and at mountain altitudes ($X = 10$) we get $P_\pi/N_n = 0.25$ or 0.39 for particle energy 10^{11} or 10^{12} eV, respectively. Thus, the pions should constitute an appreciable fraction of the total flux of nuclear-active particles at mountain altitudes and at sea level, and can greatly influence the results of some experiments.



Value of the function $\varphi(E, X)$ calculated: a – for a = 0 and b – for a = 0.15. The numbers on the curves – depth of the atmosphere X, expressed in ranges for the nuclear interaction.

Let us examine the effects that can result from such a fraction of pions in the flux of the nuclear-active particles. It is natural to assume for the interaction inelasticity coefficient of pions K = 1. If the interaction inelasticity coefficient of nucleons is $\bar{K}_n = 0.5$, then interactions of pions and nucleons of equal energy will differ in that the pions will transfer, in the mean, twice as much energy in each interaction to the π^0 mesons, i.e., to the electron-photon component.

We denote by $f_\pi(\alpha)d\alpha$ and $f_n(\alpha)d\alpha$ the probability that all π^0 mesons will be transferred a fraction α, $\alpha+d\alpha$ of the primary-particle energy upon interaction with a π^+ meson or nucleon, respectively.

If the apparatus registers the total energy ϵ transferred to all the π^0 mesons in one interaction of the nuclear-active particle, then the number of registered interactions can be written in the form

$$N_{int}(\varepsilon, X)\,d\varepsilon = N^{n}_{int}(\varepsilon, X)\,d\varepsilon + N^{\pi}_{int}(\varepsilon, X)\,d\varepsilon. \quad (7)$$

Here $N^n_{int}(\epsilon, X)d\epsilon$ and $N^\pi_{int}(\epsilon, X)d\epsilon$ is the number of interactions per unit time, due to nucleons and $\pi^\pm$ mesons, with

$$N^{n}_{int}(\varepsilon, X)\,d\varepsilon = w\,d\varepsilon\int_0^1 N_n\left(\frac{\varepsilon}{\alpha}, X\right) f_n(\alpha)\,\frac{d\alpha}{\alpha},$$

$$N^{n}_{int}(\varepsilon, X)\,d\varepsilon = w\,d\varepsilon\int_0^1 P_\pi\left(\frac{\varepsilon}{\alpha}, X\right) f_\pi(\alpha)\,\frac{d\alpha}{\alpha}$$

$$= A_1\,d\varepsilon\int_0^1 N_n\left(\frac{\varepsilon}{\alpha}, X\right)\varphi\left(\frac{\varepsilon}{\alpha}, X\right) f_\pi(\alpha)\,\frac{d\alpha}{\gamma}$$

$$N_n\left(\frac{\varepsilon}{\alpha}, X\right) = \frac{B\alpha^\gamma}{\varepsilon^\gamma}e^{-\beta\lambda}$$

(w — interaction probability).

Therefore

$$\frac{N^{\pi}_{int}(\varepsilon, X)}{N^{n}_{int}(\varepsilon, X)} = A_1\int_0^1 \alpha^{\gamma-1}\varphi\left(\frac{\varepsilon}{\alpha}, X\right) f_\pi(\alpha)\,d\alpha \Big/ \int_0^1 \alpha^{\gamma-1} f_H(\alpha)\,d\alpha$$

$$= A_1 C(\varepsilon, X)\,\varphi(\varepsilon, X)\;\overline{\alpha_\pi^{\gamma-1}}/\overline{\alpha_H^{\gamma-1}},$$

where

$$C(\varepsilon, X) = \int_0^1 \alpha^{\gamma-1}\frac{\varphi(\varepsilon/\alpha, X)}{\varphi(\varepsilon, X)} f_\pi(\alpha)\,d\alpha \Big/ \int_0^1 \alpha^{\gamma-1} f_\pi(\alpha)\,d\alpha.$$

The total number of interactions in the array, accompanied by the transfer of energy ϵ, $\epsilon+d\epsilon$ to the π^0 mesons, can be represented in the form

$$N_{int}(\varepsilon, X)\,d\varepsilon = N^{n}_{int}(\varepsilon, X)\,d\varepsilon\left\{1 + \frac{N^{\pi}_{int}(\varepsilon, X)}{N^{n}_{int}(\varepsilon, X)}\right\}$$

$$= w\,\frac{Be^{-\beta x}\,d\varepsilon}{\varepsilon^\gamma}\{1 + A_1\varkappa C(\varepsilon, X)\,\varphi(\varepsilon, X)\}, \quad (8)$$

where

$$\varkappa = \overline{\alpha_\pi^{\gamma-1}}\Big/\overline{\alpha_n^{\gamma-1}},$$

$$\overline{\alpha_\pi^{\gamma-1}} = \int_0^1 \alpha^{\gamma-1} f_\pi(\alpha)\,d\alpha, \qquad \overline{\alpha_n^{\gamma-1}} = \int_0^1 \alpha^{\gamma-1} f_n(\alpha)\,d\alpha.$$

Assuming that the form of the distribution functions with respect to α is the same for pion and nucleon interactions, we get $\kappa = (2)^{\gamma-1}$, so that $\kappa = 3.5$ when $\gamma - 1 = 1.8$ and $\kappa = 3.7$ when $\gamma - 1 = 1.9$.

In order to ascertain the degree to which C depends on the form of the function $f_\pi(\alpha)$, we have calculated this function for three types of distribution:

1) $f_\pi(\alpha) = \delta(\alpha - 1/3)$,
2) $f_\pi(\alpha) = 17.5\,[\alpha(1-\alpha)^3 + 0.007]$,
3) $f_\pi(\alpha) = 1$

(distribution 2) describes the distribution of the inelasticity coefficients of the interaction [5], if we replace α by K). The values of the coefficient C(E, X) for a depth X = 10, calculated for these three forms of distributions, C_1, C_2, and C_3 respectively), are

E, eV:	10^{10}	10^{11}	10^{12}
C_1:	2.2	1.4	1.06
C_2:	1.7	1.26	1.00
C_3:	1.39	1.14	1.00

If the quantity $A_1\kappa C\varphi(\epsilon, X)$ in the expression for $N_{int}(\epsilon, X)$ were much smaller than unity, then the spectrum of the registered interactions and their altitude variation would duplicate the spectrum and altitude variation of the nucleon component (as is the case when $E < 10^{10}$ eV or when $X \lesssim 3$). However, deep in the atmosphere (at mountain altitudes or at sea level), at high particle energies of the order of 10^{11} eV or more, this condition is not satisfied. Therefore the flux of the pions calculated even under minimum assumptions ($a = 0$) will cause the measured spectrum $N_{int}(\epsilon, X)d\epsilon$ and the altitude variation of the interactions not to coincide with the spectrum and altitude variation of the high energy nucleons.

The pions will influence the spectrum of the nuclear-active particles deep in the atmosphere most in the energy range 10^{10}—10^{12} eV. If the nuclear-active particle spectrum determined from the energy transferred to the π^0 mesons (from the ionization bursts or from the electromagnetic cascades in nuclear emulsion) is approximated by a power law, then we can write

$$N_{\rm int}(\varepsilon, X)\, d\varepsilon = B^*(X)\, \varepsilon^{-\gamma^*} d\varepsilon$$

$$\sim \varepsilon^{-\gamma} \{1 + A_1 \varkappa C(\varepsilon, X)\, \varphi(\varepsilon, X)\}\, d\varepsilon. \quad (9)$$

Using the calculated coefficients $C(E, X=10)$ (see above) we can calculate by formula (9) the change of the spectral exponent in different energy intervals. Calculation has yielded (different variants of the function $f_\pi(\alpha)$ change $\Delta\gamma = \gamma - \gamma^*$ by 0.004—0.005):

Range of E, eV:	10^{10}—10^{11}	10^{11}—10^{12}	$>10^{12}$
$\Delta\gamma = \gamma - \gamma^*$:	0.18	0.17	0

Thus, even for a pure power-law spectrum of the high-energy nucleons in the atmosphere, the spectrum exponent γ of the nuclear-active particles determined from the energy transferred in one interaction to the π^0 mesons will change by 0.2 at $E \approx 10^{12}$ eV.

If the absorption range of the particles of the nuclear-active components of the cosmic rays is determined from the intensities of the interactions measured near the top of the atmosphere ($X = 0$) and at large depths in the atmosphere ($X = 10$—14), using the ratio

$$N_{\rm int}(\varepsilon, X)/N_{\rm int}(\varepsilon, X=0) = e^{-X\beta^*},$$

then

$$e^{-\beta^* X} = e^{-\beta X}\{1 + A_1 \varkappa C(\varepsilon, X)\, \varphi(\varepsilon, X)\}, \quad (10)$$

where $1/\beta^* = L_{n.a}/L_{int}$ and $1/\beta = L_n/L_{int}$ ($L_{n.a}$ and L_n are the absorption ranges of all the nuclear-active particles and the nucleons, respectively).

It follows from (10) that

$$\frac{\delta L}{L_n} = \frac{L_{n.a}}{X} \ln\{1 + A_1 \varkappa C(\varepsilon, X)\, \varphi(\varepsilon, X)\}.$$

In measurements between $X = 0$ and $X = 10$ (mountain altitudes) at energies $\epsilon \gtrsim 10^{12}$ eV, we get $\delta L/L_n = 0.15$, i.e., owing to the presence of pions in the nuclear-active particle flux, the absorption range of the nuclear-active component particles at $\epsilon \gtrsim 10^{12}$ eV increases at least 15% compared with the absorption range of the nucleons. At $\epsilon = 10^{11}$ eV we have $\delta L/L_n = 0.09$.

In the integral spectrum for $\epsilon \geq 10^{11}$ eV this ratio is $\delta L/L_n = 0.12$, i.e., if $L_{n.a.} = 120$ g/cm^2, then we get $L_n = 107$ g/cm^2 for $\epsilon = 10^{11}$ eV and $L_n = 104$ g/cm^2 for $\epsilon = 10^{12}$ eV.

We note incidentally that this change in the range $L_{n.a.}$ must be taken into account when determining the average inelasticity coefficient of nucleon interaction from the ratio of absorption and interaction ranges. For example, a change of 4% in $L_{n.a.}$ leads to a change of 0.08 in the inelasticity coefficient $\overline{K}$, while a change of 15% in $L_{n.a.}$ leads to a change of 0.12 in $\overline{K}$.

In order to obtain the absorption range of the nucleon component for measurements of the altitude variation of the interactions of high-energy particles $N_{int}(\epsilon, X)$, it is necessary to measure the absorption of the nuclear-active particles inside the atmosphere—between sea level and mountain altitudes. In this case

$$\frac{\delta L}{L_n} = \frac{L_{n.a}}{X_1 - X_2} \ln\left\{\frac{1 + A_1\varkappa C(\varepsilon, X_1)\, \varphi(\varepsilon, X_1)}{1 + A_1\varkappa C(\varepsilon, X_2)\, \varphi(\varepsilon, X_2)}\right\}.$$

If we take $X_1 = 14$ and $X_2 = 10$, then we obtain $\delta L/L_n = 0$ for $\epsilon = 10^{12}$ eV and $\delta L/L_n = 0.03$ for $\epsilon = 10^{11}$eV.

From the foregoing analysis we can draw the following conclusions.

1. Pions should constitute a considerable fraction of the nucleon flux in the flux of nuclear-active particles inside the atmosphere, at energies $E \geq 10^{11}$ eV: not less than 25% when $E = 10^{11}$ eV and not less than 40% when $E = 10^{12}$ eV.

2. The presence of a considerable pion fraction in the flux of nuclear-active particles in the lower part of the atmosphere can change noticeably the spectrum of the nuclear-active particles in the high-energy region (by changing the spectrum ex-

ponent by ~ 0.2 and producing a "kink" in the spectrum at $E \cong 10^{12}$ eV) and in their altitude variation (by changing the absorption range by 10–15%).

3. Failure to take account of the high-energy pions in the nuclear-active component in calculations of the average inelasticity coefficient of the high-energy nucleons from the ratio of the interaction and absorption ranges of the nuclear-active component may undervalue the interaction inelasticity coefficient of the nucleons by 0.1.

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[4] N. L. Grigorov, JETP **45**, 1544 (1963), Soviet Phys. JETP **18**, 1063 (1964).

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