TRANSPORT PHENOMENA IN A PLASMA IN A STRONG MAGNETIC FIELD

Yu. M. ALIEV and A. R. SHISTER

P. N. Lebedev Physics Institute, Academy of Sciences U.S.S.R.

Submitted to JETP editor April 29, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 1499-1508 (November, 1963)

Expressions for particle and heat fluxes in a plasma perpendicular to a magnetic field are obtained on the basis of a kinetic equation in which the effect of the magnetic field on particle collisions is taken into account. In the case of strong magnetic fields the expressions significantly differ from those of the usual theory.

INTRODUCTION

TRANSPORT transverse to a strong magnetic field was investigated by many workers. In most of the investigations no account was taken of the influence of the magnetic field on the particle-collision act. It is clear, however, that such an account is necessary under conditions when the average Larmor radii of the particles are small compared with the radius of the Debye screening. A theoretical study of diffusion under such conditions was carried out in many investigations [1-3]. Temperature relaxation and diffusion, with allowance for the influence of the magnetic field on the particle collision, was investigated by Kihara, Midzuno, and Kaneko [4].

Silin [5] obtained a kinetic equation which takes into account the influence of the magnetic field on the particle-collision act. Silin [6] and Lovetskiĭ [7] considered with the aid of such an equation the high-frequency dielectric constant of a plasma, while Silin [8] studied the temperature relaxation. In the present paper we also use this equation to study the processes of diffusion, thermo-diffusion, and heat conduction of plasma transversely to a magnetic field.

Interest in the theory of such processes has increased in view of recently initiated experiments with a quiet plasma under conditions where the Larmor radii of the particles are of the same order as the Debye radius [9].

1. INITIAL KINETIC EQUATION

The kinetic equation obtained in [5] for the distribution function $f_\alpha(t, \mathbf{v}_\alpha, \mathbf{r}_\alpha)$ of particles of kind α with charge e_α and mass m_α, situated in a homogeneous electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$, is of the form

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}_\alpha \frac{\partial f_\alpha}{\partial \mathbf{r}_\alpha} + \frac{e_\alpha}{m_\alpha}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_\alpha \mathbf{H}]\right)\frac{\partial f_\alpha}{\partial \mathbf{v}_\alpha} = I_{\text{st}}, \qquad (1.1)^*$$

where

$$I_{\text{st}} = \frac{1}{m_\alpha}\frac{\partial}{\partial \mathbf{v}_\alpha}\sum_\beta \int d\mathbf{r}_\beta d\mathbf{v}_\beta \frac{\partial}{\partial \mathbf{r}_\alpha} \times \left(\frac{e_\alpha e_\beta}{|\mathbf{r}_\alpha - \mathbf{r}_\beta|}\right)\int_0^\infty \mathscr{E}(t+\tau, \mathbf{v}_\alpha, \mathbf{v}_\beta, \mathbf{r}_\alpha, \mathbf{r}_\beta)\, d\tau, \qquad (1.2)$$

$$\mathscr{E}(t+\tau, \mathbf{v}_\alpha, \mathbf{v}_\beta, \mathbf{r}_\alpha, \mathbf{r}_\beta) = S_\tau \frac{\partial}{\partial \mathbf{r}_\alpha}\left(\frac{e_\alpha e_\beta}{|\mathbf{r}_\alpha - \mathbf{r}_\beta|}\right)\left(\frac{1}{m_\alpha}\frac{1}{\partial \mathbf{v}_\alpha} - \frac{1}{m_\beta}\frac{\partial}{\partial \mathbf{v}_\beta}\right) f_\alpha f_\beta. \qquad (1.3)$$

In (1.2) Σ denotes summation over the kinds of particles, that is, over the electrons and ions. The integration with respect to τ is over the entire particle interaction time. In (1.3) the operator S_τ replaces the coordinates and velocities by the coordinates and velocities of the particles moving in homogeneous constant magnetic field at the instant $t + \tau$, if they were situated at the instant t at the point $\mathbf{r}_\alpha$ and had a velocity $\mathbf{v}_\alpha$, that is, the velocities $\mathbf{v}_\alpha$ are replaced by

$$\mathbf{V}_\alpha(\tau, \mathbf{v}_\alpha) = \mathbf{h}(\mathbf{h}\mathbf{v}_\alpha) - [\mathbf{h}\mathbf{v}_\alpha]\sin\Omega_\alpha\tau - [\mathbf{h}[\mathbf{h}\mathbf{v}_\alpha]]\cos\Omega_\alpha\tau, \qquad (1.4)$$

and the coordinates $\mathbf{r}_\alpha$ by

$$\mathbf{R}_\alpha(\tau, \mathbf{v}_\alpha, \mathbf{r}_\alpha) = \mathbf{r}_\alpha + \int_0^\tau \mathbf{V}_\alpha(\tau', \mathbf{v}_\alpha)\, d\tau', \qquad (1.5)$$

where $\mathbf{h}$ is a unit vector directed along the magnetic field and $\Omega_\alpha = e_\alpha H/m_\alpha c$ is the Larmor frequency.

Assuming the inhomogeneity of the plasma and

*$[\mathbf{v}_\alpha \mathbf{H}] = \mathbf{v}_\alpha \times \mathbf{H}$.

the external electric field to be weak, we seek a distribution function f_α in the form

$$f_\alpha = f_\alpha^{(0)} (1 + \Phi_\alpha), \quad (1.6)$$

where $f_\alpha^{(0)}$ is the "local" Maxwellian function

$$f_\alpha^{(0)} = n_\alpha (\mathbf{r}_\alpha, t) \frac{m_\alpha^{3/2}}{[2\pi T (\mathbf{r}_\alpha, t)]^{3/2}} \times \exp\left\{-\frac{m_\alpha [\mathbf{v}_\alpha - \mathbf{v}_0 (\mathbf{r}_\alpha, t)]^2}{2T}\right\}. \quad (1.7)$$

Here n_α is the number of particles of kind α per unit volume, T is the temperature in energy units, and $\mathbf{v}_0$ is the average mass velocity, defined by

$$\mathbf{v}_0 (\mathbf{r}, t) = \sum_\alpha \int m_\alpha v_\alpha f_\alpha d\mathbf{v}_\alpha \Big/ \sum_\alpha \int m_\alpha f_\alpha d\mathbf{v}_\alpha, \quad (1.8)$$

and Φ_α is a correction due to the presence of the electric field and the weak inhomogeneity of the plasma.

It is convenient to change over in (1.1) to new independent variables, by transforming to the velocities of the particles of a given kind relative to $\mathbf{v}_0$. Taking into account the weak dependence of the distribution function on the coordinates and on the time, and taking the Fourier transforms with respect to the coordinates in the collision term, we obtain ultimately

$$f_\alpha^{(0)} \left[\frac{\partial}{\partial \mathbf{r}} \ln n_\alpha T - \frac{e_\alpha}{T}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_0 \mathbf{H}]\right) + \left(\frac{m_\alpha v_\alpha^2}{2T} - \frac{5}{2}\right)\frac{\partial \ln T}{\partial \mathbf{r}}\right]\mathbf{v}_\alpha$$
$$= -f_\alpha^{(0)}\left[\frac{m_\alpha v_\alpha}{\sum_\beta n_\beta m_\beta}[\mathbf{j}\mathbf{H}] + \frac{e_\alpha H}{m_\alpha c}[\mathbf{v}_\alpha \mathbf{h}]\frac{\partial \Phi_\alpha}{\partial \mathbf{v}_\alpha}\right]$$
$$+ \frac{2}{\pi m_\alpha}\frac{\partial}{\partial \mathbf{v}_\alpha}\sum_\beta (e_\alpha e_\beta)^2 \int d\mathbf{v}_\beta f_\alpha^{(0)} f_\beta^{(0)} \int \frac{d\mathbf{k}}{k^4}\mathbf{k}\int d\tau \times \left(\frac{\mathbf{k}}{m_\alpha}\frac{\partial \Phi_\alpha}{\partial \mathbf{V}_\alpha} - \frac{\mathbf{k}}{m_\beta}\frac{\partial \Phi_\beta}{\partial \mathbf{V}_\beta}\right)\exp(i\mathbf{k}\mathbf{M}_{\alpha\beta}), \quad (1.9)$$

where the current j is equal to

$$\mathbf{j} = \sum_\beta e_\beta \int f_\beta^{(0)} \Phi_\beta \mathbf{v}_\beta d\mathbf{v}_\beta, \quad (1.10)$$

and

$$\mathbf{M}_{\alpha\beta} = \mathbf{R}_\alpha - \mathbf{R}_\beta - (\mathbf{r}_\alpha - \mathbf{r}_\beta). \quad (1.11)$$

Integration with respect to k in (1.9) must be carried out, as usual, from $k_{min} \approx 1/r_D$ to $k_{max} \approx 1/r_{min}$, where $r_D = (T/4\pi n e^2)^{1/2}$ is determined by the screening of the Coulomb interaction at large distances, $r_{min} = e^2/T$ results from the inapplicability of perturbation theory, with the aid of which Equation (1.1) was derived, at short distances.

2. SOLUTION OF THE KINETIC EQUATION

Taking into account the form of the left part of (1.9), we seek Φ_α in the form

$$\Phi_\alpha = (A_\alpha^{\mathrm{I}} \mathbf{v}_\alpha + A_\alpha^{\mathrm{II}} [\mathbf{v}_\alpha \mathbf{h}])\, \partial \ln T/\partial \mathbf{r} + (B_\alpha^{\mathrm{I}} \mathbf{v}_\alpha + B_\alpha^{\mathrm{II}} [\mathbf{v}_\alpha \mathbf{h}])\, \mathbf{d}, \quad (2.1)$$

where

$$\mathbf{d} = \frac{n_e}{n}\left[\frac{\partial \ln n_e T}{\partial \mathbf{r}} + \frac{e}{T}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_0 \mathbf{H}]\right)\right]. \quad (2.2)$$

The coefficients A and B as functions of the velocity are expanded in a series of Laguerre polynomials of degree $^3\!/_2$:

$$A_\alpha^{\mathrm{I}} = \sum_{\mu=0}^{\infty} a_{\alpha,\mu}^{\mathrm{I}} L_\mu \left(\frac{m_\alpha v_\alpha^2}{2T}\right), \quad B_\alpha^{\mathrm{I}} = \sum_{\mu=0}^{\infty} b_{\alpha,\mu}^{\mathrm{I}} L_\mu \left(\frac{m_\alpha v_\alpha^2}{2T}\right),$$
$$A_\alpha^{\mathrm{II}} = \sum_{\mu=0}^{\infty} a_{\alpha,\mu}^{\mathrm{II}} L_\mu \left(\frac{m_\alpha v_\alpha^2}{2T}\right), \quad B_\alpha^{\mathrm{II}} = \sum_{\mu=0}^{\infty} b_{\alpha,\mu}^{\mathrm{II}} L_\mu \left(\frac{m_\alpha v_\alpha^2}{2T}\right). \quad (2.3)$$

The Laguerre polynomials are normalized by the condition

$$\int_0^\infty e^{-x} x^{3/2} L_n (x)\, L_m (x)\, dx = \delta_{mn} n!\, \Gamma(n + {}^5\!/_2).$$

We consider the case of sufficiently strong magnetic fields, when the particle Larmor frequencies Ω are much larger than the collision frequency ν, which will be determined below. This enables us to seek the coefficients $a_{\alpha,\mu}$ and $b_{\alpha,\mu}$ in the form of a series in powers of ν/Ω. In the zeroth approximation in ν/Ω the only nonzero coefficients are

$$a_{e,1}^{\mathrm{II}} = -\Omega_e^{-1}, \quad a_{i,1}^{\mathrm{II}} = -\Omega_i^{-1},$$
$$b_{e,0}^{\mathrm{II}} = n/n_e \Omega_e, \quad b_{i,0}^{\mathrm{II}} = -n/n_i \Omega_i. \quad (2.4)$$

With respect to the calculations of the next approximation (by way of an example we calculated one of the coefficients in the appendix) we shall indicate only that the obtained expressions are logarithmically divergent at large particle interaction times τ. In the integration with respect to τ in (1.2), the upper limit corresponds to the time necessary for the particles to go to infinity. In a strong magnetic field, when the particle motion in a plane perpendicular to the field is finite, the only mechanism that ensures the escape of the interacting particles to infinity is motion along the magnetic field. The time of this escape is of order of $R/v_{\|}$, where R is the impact distance and $v_{\|}$ the projection of the particle relative velocity on the direction of the magnetic field. As $v_{\|} \to 0$, the interaction (scattering) time of the particles increases without limit, and this leads in our expressions to logarithmic divergences.

It is clear, however, that at very small relative velocities, smaller than the value of $v_{\| min}$ determined from

$$m(v_{\| min})^2/2 \approx e^2/R, \tag{2.5}$$

the Coulomb interaction becomes significant. Equation (1.1) which is obtained under the assumption that this interaction is weak, becomes unsuitable for the description of the behavior of such long-interacting particles. Therefore in integrating with respect to τ in (1.2) it is necessary to cut off the interaction time at[1)]

$$\tau_{max}^{(k)} \cong R/v_{\| min}. \tag{2.6}$$

An estimate of the contribution of the particles with $\tau > \tau_{max}^{(k)}$ must be made by taking into account the Coulomb interaction. Our calculations have shown that this contribution is small and can be neglected.

3. SUMMARY OF THE RESULTS

Let us define the average diffusion velocity of the particles in the following manner:

$$\bar{\mathbf{v}}_\alpha = \frac{1}{n_\alpha}\int f_\alpha \mathbf{v}_\alpha d\mathbf{v}_\alpha. \tag{3.1}$$

Using (2.1), (2.3), and (1.8) we obtain

$$\bar{\mathbf{v}}_e = \frac{T}{m_e}\left(b_{e,0}^{\mathrm{I}}\,\mathbf{d} + a_{e,0}^{\mathrm{I}}\frac{\partial}{\partial \mathbf{r}}\ln T\right) + \frac{T}{m_e}\left(b_{e,0}^{\mathrm{II}}\,[\mathbf{hd}] + a_{e,0}^{\mathrm{II}}\left[\mathbf{h}\,\frac{\partial \ln T}{\partial \mathbf{r}}\right]\right), \tag{3.2}$$

$$\bar{\mathbf{v}}_i = -(n_e m_e/n_i m_i)\,\bar{\mathbf{v}}_e. \tag{3.3}$$

The coefficients contained in (3.2) are given below for different values of the magnetic field intensity.

The coefficient $b_{e,0}^{\mathrm{I}}$, which describes the diffusion process, is given by

$$b_{e,0}^{\mathrm{I}} = -(n\nu/n_e\Omega_e^2)\,L. \tag{3.4}$$

Here $L = L_0 = \ln(r_D/r_{min})$ if the field is weak, that is, $\rho_e \gg r_D$. At stronger fields $L = L_0 + 3L_1/4$, where L_1 has the following form, depending on the magnitude of the magnetic field, on the density, and on the plasma temperature:

$$L_1 = \ln\frac{m_i}{m_e}\ln\frac{r_D}{\rho_e} \qquad (r_0 \ll \rho_e \ll r_D \ll \rho_i), \tag{3.5}$$

$$L_1 = \ln\frac{m_i}{m_e}\ln\frac{r_D}{r_0} + \ln\frac{\sqrt{r_0\rho_e}}{r_{min}}\ln\frac{r_0}{\rho_e} \qquad (\rho_e \ll r_0 \ll r_D \ll \rho_i), \tag{3.6}$$

$$L_1 = \ln\frac{r_D}{\rho_e}\ln\frac{\sqrt{r_D\rho_e}}{r_{min}} \qquad (\rho_e \ll r_D \ll r_0,\ \rho_i), \tag{3.7}$$

$$L_1 = \frac{1}{2}\left(\ln\frac{m_i}{m_e}\right)^2 + \ln\frac{r_D}{\rho_i}\ln\frac{\sqrt{r_D\rho_i}}{r_{min}} \qquad (r_0 \ll \rho_e \ll \rho_i \ll r_D), \tag{3.8}$$

$$L_1 = \ln\frac{m_i}{m_e}\ln\frac{\rho_i}{r_0} + \ln\frac{\sqrt{\rho_e r_0}}{r_{min}}\ln\frac{r_0}{\rho_e} + \ln\frac{\sqrt{r_D\rho_i}}{r_{min}}\ln\frac{r_D}{\rho_i} \qquad (\rho_e \ll r_0 \ll \rho_i \ll r_D), \tag{3.9}$$

$$L_1 = \frac{1}{2}\ln\frac{m_i}{m_e}\ln\frac{\sqrt{r_D\rho_e}}{r_{min}} + \ln\frac{\sqrt{r_D\rho_i}}{r_{min}}\ln\frac{r_D}{\rho_i} \qquad (\rho_e \ll \rho_i \ll r_0,\ r_D). \tag{3.10}$$

Here and below

$$\nu = 4\sqrt{2\pi}\,(ee_i)^2/3T^{3/2}\sqrt{m_e}, \qquad \rho_e = \sqrt{T/2m_e}\,\Omega_e^{-1},$$

$$\rho_i = \sqrt{T/2m_i}\,\Omega_i^{-1}, \qquad r_0 = r_{min}m_i/m_e, \qquad e_i = Ze.$$

We note that analogous results were obtained by Golant[2], with exception of the cases (3.6) and (3.9), which were not considered in that paper. The expression for the coefficient $a_{e,0}^{\mathrm{I}}$, which describes the thermodiffusion particle flux, has the form

$$a_{e,0}^{\mathrm{I}} = (3\nu/2\Omega_e^2)\,L_0 \qquad (r_D \ll \rho_e). \tag{3.11}$$

In the case of stronger fields

$$a_{e,0}^{\mathrm{I}} = (3\nu/2\Omega_e^2)\left(L_0 + \tfrac{1}{4}L_1\right). \tag{3.12}$$

Finally, for the coefficient $a_{e,0}^{\mathrm{II}}$ we obtain

$$a_{e,0}^{\mathrm{II}} = (\nu/\Omega_e^2)\,\eta. \tag{3.13}$$

The form of the function η is shown in the appendix (A.16). We note only that in magnitude this function for $r_D \gg \rho$ is of the order of unity, whereas for $r_D < \rho_e$ we have $\eta \sim r_D/\rho_e$. In the usual theory, which does not take into account the influence of the magnetic field on the collision act, this

1)In the electron-ion collisions with impact parameters ranging from ρ_e to ρ_i, when the influence of the magnetic field on the motion of the ion can be neglected, there is another possible mechanism of particle collision time cutoff, connected with the departure of the ion from the interaction sphere. The time of such a departure is of the order of R/v_i, where v_i is the average ion thermal velocity. When $R = r_0 = r_{min}m_i/m_e$, this time and $\tau_{max}^{(k)}$ turn out to be identical. Thus, in the region of impact parameters from ρ_e to r_0 the interaction of the particles is cut off as the result of the Coulomb acceleration of the electron in the ion field; in the region from r_0 to ρ_i, where the Coulomb field is sufficiently weak, the free ion leaves the interaction sphere earlier.

coefficient is proportional to H^{-3} (see, for example, [10]); in our case for weak fields ($r_D < \rho_e$) we have $a^{II}_{e,0} \sim H^{-1}$. Thus, the dependence of this coefficient on the magnetic field turns out to be entirely different.

We proceed to consider the particle heat flux $\mathbf{q}_\alpha$:

$$\mathbf{q}_\alpha = \int f_\alpha \mathbf{v}_\alpha \frac{m_\alpha v_\alpha^2}{2} d\mathbf{v}_\alpha$$
$$= \tfrac{5}{2} n_\alpha T \bar{\mathbf{v}}_\alpha - \tfrac{5}{2} T^2 (n_\alpha/m_\alpha)(b^{I}_{\alpha,1}\mathbf{d} + a^{I}_{\alpha,1}\,\partial \ln T/\partial \mathbf{r})$$
$$- \tfrac{5}{2} T^2 (n_\alpha/m_\alpha)(b^{II}_{\alpha,1}[\mathbf{hd}] + a^{II}_{\alpha,1}[\mathbf{h}\partial \ln T/\partial \mathbf{r}]). \quad (3.14)$$

The next three coefficients are related with the equations given above by:

$$b^{I}_{e,1} = -(2n/5n_e)\, a^{I}_{e,0}, \qquad b^{I}_{i,1} = -(2n/5n_i Z)\, a^{I}_{i,0},$$
$$b^{II}_{e,1} = -(2n/5n_e)\, a^{II}_{e,0}, \qquad b^{II}_{i,1} \sim (m_e/m_i)\, b^{II}_{e,1}. \quad (3.15)$$

The coefficient $a^{I}_{i,1}$ is of the form

$$a^{I}_{i,1} = (2\nu/5\Omega_e^2)\sqrt{2}\,(m_i/m_e)^{3/2} \ln (r_D/r_{min}) \qquad (r_D \ll \rho_i), \quad (3.16)$$

$$a^{I}_{i,1} = \frac{2\nu}{5\Omega_e^2}\sqrt{2}\left(\frac{m_i}{m_e}\right)^{3/2}\left[\ln\frac{\rho_i}{r_{min}} + \frac{3}{4}\ln\frac{r_D}{\rho_i}\ln\frac{\sqrt{r_D\rho_i}}{r_{min}}\right] \qquad (r_D \gg \rho_i). \quad (3.17)$$

For the coefficient $a^{I}_{e,1}$ we obtain the following expressions:

$$a^{I}_{e,1} = (2\nu/5\Omega_e^2)\left(\tfrac{13}{4} + n_e\sqrt{2}/n_i Z^2\right) L_0 \qquad (r_D \ll \rho_e). \quad (3.18)$$

For stronger fields

$$a^{I}_{e,1} = (2\nu/5\Omega_e^2)$$
$$\times \left(\tfrac{13}{4}L_0 + \tfrac{51}{16}L_1 + (L_0 + L_2)\, n_e\sqrt{2}/n_i Z^2\right), \quad (3.19)$$

where

$$L_2 = \tfrac{3}{4}\ln (r_D/\rho_e)\ln\left(\sqrt{r_D\rho_e}/r_{min}\right).$$

Thus, we see from the foregoing formulas that an account of the influence of the strong magnetic field on the particle collision act leads to an essential modification of the heat fluxes and the particle fluxes transverse to the field.

Let us stop to compare our results with the data of the preceding investigations. What is to be compared, usually, is only the coefficient $b^{I}_{\alpha,0}$, which describes the process of diffusion in the plasma, inasmuch as the remaining coefficients have not yet been investigated for the presence of a strong magnetic field that affects particle collision. The first consistent analysis of the diffusion process under such conditions was undertaken by Belyaev [1]. By averaging over the rapidly changing variables, he obtained a kinetic equation for the distribution function of the centers of the Larmor circles, with the aid of which the diffusion flux due to the difference of the density gradient was investigated.

In that work, however, a different cutoff method was used to eliminate the logarithmic divergences. Namely, the quantity $v_{\parallel\,min}$ of (2.6) is replaced by some minimum relative velocity v_c of the colliding particles, due to their interaction with the third particle:

$$v_c \cong \sqrt{R v_e/\tau}, \qquad \tau \sim 1/\nu \ln (r_D/r_{min}). \quad (3.20)$$

It can be shown (see, for example, [2]), that this velocity is usually smaller than $v_{\parallel}$ min determined from (2.5), so that it is necessary to choose $\tau^{(k)}_{max}$ as the maximum interaction time. Thus, only case (3.5) is in agreement with Belyaev [2]).

Gurevich and Firsov [3] obtained with the aid of a diagram technique an expression for the coefficient of transverse diffusion of the plasma. The influence of the magnetic field on the particle collisions in this work was accounted for in case (3.5) only.

The diffusion process was investigated in greatest detail by Golant [2], who obtained results analogous to ours [he did not consider cases (3.6) and (3.9)]. Finally, Kihara et al [4] obtained an entirely different magnetic-field dependence for the addition to the diffusion coefficient, due to the account of the influence of the magnetic field on the collisions. The main reason for this is that no account was taken of the contribution of the particles with relative velocity smaller than the thermal velocity of the electrons. This has made it possible to neglect the motion of the ions, so that the final expressions in [4] contain likewise no dependence on m_i/m_e. Our formulas show that the main effect of the magnetic field is on the collisions of particles with small relative velocities along the field, that is, in fact on those collisions which are not considered in [4].

We note the following with respect to the character of our results. The corrections to the kinetic coefficients, due to an account of the influence of the magnetic field on the particle collisions, turn out to be most significant when the Larmor radii of both colliding particles are smaller than the Debye radius. It is easiest to realize such a case in electron-electron collisions. Consequently, the greatest modification is experienced by the electron heat flux due to such collisions.

2)In this case there is a small factor preceding the logarithmically diverging expressions, and the result is therefore insensitive to the cutoff method.

Going over to a discussion of the numerical values of the quantities, we turn to (3.5)—(3.10). We note here that formulas (3.5)—(3.7) hold for the case when $\rho_e \ll r_D \ll \rho_i$, whereas (3.8)—(3.10) hold for $\rho_e \ll \rho_i \ll r_D$. Therefore, in order for (3.5)—(3.7) to hold, it is necessary to satisfy in the case of a hydrogen plasma the following conditions:

$$1 \ll 4.1 \cdot 10^2 H/\sqrt{n} \ll 43, \tag{3.21}$$

where H is the magnetic field in Gausses and n is the density. To satisfy (3.8)—(3.10) it is necessary to have

$$H/\sqrt{n} \gg 0.1. \tag{3.22}$$

The value of the temperature determines the parameter $r_0 = 3 \times 10^{-4}\ T^{-1}$, where T is the temperature in electron volts, and thus picks out different particular cases from the groups (3.5)—(3.7) and (3.8)—(3.10).

On the other hand, since our calculation is of logarithmic accuracy, it is necessary that the logarithm $\ln(\rho_e/r_{min})$ be large compared with unity. This leads to an upper limit on the magnetic fields: $H \ll 1.2 \times 10^7 T^{3/2}$. As can be seen from (3.22), formulas (3.8)—(3.10), which pertain to sufficiently strong fields, can be made valid under laboratory conditions, in particular, for example, for $n \sim 10^8\ cm^{-3}$ and $H \sim 10^4$ G. We present an example illustrating the magnitude of the corrections to the transport coefficients made necessary by the influence of the magnetic field on the particle collision act.

Let us consider a plasma with $\rho_i \gg r_D \gg \rho_e \gg r_{min}$. This occurs for a hydrogen plasma of density $n \sim 10^{10}\ cm^{-3}$ and temperature 1 eV, situated in a magnetic field of intensity $H \sim 10^4$ G (these conditions are close to those of the experiment described by Rostas et al [9]). In this case we have $r_D \cong 0.7 \times 10^{-2}$ cm, $\rho_e \cong 0.17 \times 10^{-3}$, $r_{min} \cong 0.74 \times 10^{-6}$ cm and $L_0 = 10.6$, $L_1 \cong 28.0$, $L_2 \cong 24.5$. We see therefore that the correction is due to the magnetic field are more than double the Coulomb logarithm L_0, obtained from the ordinary theory.

In conclusion we take the opportunity to thank V. P. Silin for suggesting the topic and for numerous useful discussions.

APPENDIX

1. By way of illustration of the method of calculation, we consider here in detail the coefficient $b^I_{e,0}$. From (1.9) we obtain with the aid of (2.1)—(2.3), after integrating over all the velocities, the following expression for $b^I_{e,0}$:

$$b^I_{e,0} = -(n\nu/n_e\Omega_e^2)\, J, \tag{A.1}$$

$$J = \frac{3}{2\pi\sqrt{\pi}} \int \frac{d\varkappa}{\varkappa^4} \varkappa_\perp^2 \int_0^\infty d\xi \exp\left[-t_e - t_i\right]. \tag{A.2}$$

In (A.1) $\kappa = \rho_e \mathbf{k}$ is the dimensionless wave vector, $\xi = \Omega_e/2$ is the dimensionless time of interaction between two particles,

$$t_e = 4\varkappa_\parallel^2 \xi^2 + 4\varkappa_\perp^2 \sin^2 \xi,$$

$$t_i = 4\varkappa_\parallel^2 \xi^2 m_e^2/m_i^2 + 4\varkappa_\perp^2 (m_i/m_e) \sin^2 (\xi m_e/m_i),$$

$\kappa_\parallel$ and $\kappa_\perp$ are the components of the vector κ parallel and perpendicular to the magnetic field.

In a spherical coordinate system with polar axis along the magnetic field, we can rewrite (A.2) in the form

$$J = \frac{6}{\sqrt{\pi}} \int_{\varkappa_{min}}^{\varkappa_{max}} d\varkappa \int_0^1 (1 - x^2)\, dx \int_0^\infty d\xi \exp\left[-4\varkappa^2 (\xi^2 x^2 + (1 - x^2)\Psi)\right], \tag{A.3}$$

$$\Psi = \sin^2 \xi + (m_i/m_e) \sin^2 (\xi m_e/m_i), \tag{A.4}$$

where x is the cosine of the angle between κ and **H**.

The function Ψ can be written for different regions of ξ in the following form:

$$\begin{aligned} &\Psi \cong \xi^2 && (0 \leqslant \xi \leqslant 1), \\ &\Psi \cong \sin^2 \xi && (1 \leqslant \xi \leqslant \sqrt{m_i/m_e}), \\ &\Psi \cong m_e\xi^2/m_i && (\sqrt{m_i/m_e} \leqslant \xi \leqslant m_i/m_e), \\ &\Psi \cong (m_i/m_e) \sin^2 (m_e\xi/m_i) && (m_i/m_e \leqslant \xi < \infty). \end{aligned} \tag{A.5}$$

It is convenient to break down the region of integration with respect to ξ into $0 \le \xi \le 1$ and $\xi > 1$. Then J is rewritten in the form

$$J = J_0 + J_1;$$

$$J_0 = \frac{6}{\sqrt{\pi}} \int_{\varkappa_{min}}^{\varkappa_{max}} d\varkappa \int_0^1 dx\, (1 - x^2) \int_0^1 d\xi \exp\left[-4\varkappa^2\xi^2\right], \tag{A.6}$$

$$J_1 = \frac{6}{\sqrt{\pi}} \int_{\varkappa_{min}}^{\varkappa_{max}} d\varkappa \int_0^1 dx\, (1 - x^2) \int_1^\infty d\xi \exp\left[-4\varkappa^2 (\xi^2 x^2 + \Psi)\right]. \tag{A.7}$$

We have neglected in the exponential of J_1 the quantity Ψ compared with ξ^2, which is valid if $\xi > 1$. Expression (A.6) is easy to integrate and as a result it gives, accurate to terms of order of unity

$$J_0 = \ln (r_D/r_{min}) \qquad (\rho_e \gg r_D),$$

$$J_0 = \ln (\rho_e/r_{min}) \qquad (\rho_e \ll r_D). \tag{A.8}$$

In the calculation of (A.7) we shall neglect the single logarithms and retain the double logarithmic terms, so that we neglect x^2 compared with unity in the integration with respect to x. We get

$$J_1 = \frac{3}{2}\int_{\varkappa_{min}}^{\varkappa_{max}} \frac{d\varkappa}{\varkappa}\int_1^{\infty} \frac{d\xi}{\xi}\,\Phi(2\varkappa\xi)\,e^{-4\varkappa^2\Psi}, \qquad (A.9)$$

where

$$\Phi(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-y^2}\,dy$$

is the probability integral.

From (A.9) we see that the double logarithmic contribution is made by the region

$$\varkappa\xi > 1, \qquad \varkappa^2\Psi < 1, \qquad (A.10)$$

in which J_1 is of the form

$$J_1 = \frac{3}{2}\int \frac{d\varkappa}{\varkappa}\int \frac{d\xi}{\xi}. \qquad (A.11)$$

The integration in (A.11) is over the region

$$\varkappa\xi > 1, \qquad \varkappa^2\Psi < 1, \qquad \varkappa_{min} < \varkappa < \varkappa_{max}, \qquad 1 < \xi < \xi_{max}, \qquad (A.12)$$

where

$$\xi_{max} = \Omega_e\tau_{max}^{(k)}/2 \cong \varkappa^{-3/2}\sqrt{\rho_e/r_{min}}$$

is the maximum particle interaction time, determined by the applicability of perturbation theory (see the main text).

In the case when $\rho_i \gg \rho_e \gg r_D$, the region (A.12) makes no contribution and $J = J_0$. When $\rho_i \gg r_D \gg \rho_e$, depending on the value of the magnetic field intensity and the value of m_1/m_e, we must distinguish three cases (see (A.12)):

1) $\rho_e > r_0 = r_{min}m_i/m_e$ (only this case is considered in [1]). In this region we have

$$J_1 = \frac{3}{2}\int_{\varkappa_{min}}^{1} \frac{dx}{\varkappa}\int_{\varkappa^{-1}}^{\varkappa^{-1}(m_i/m_e)^{1/2}} \frac{d\xi}{\xi} = \frac{3}{4}\ln\frac{m_i}{m_e}\ln\frac{r_D}{\rho_e}, \qquad (A.13)$$

2) $r_D > r_0 > \rho_e$ (this case is not considered in [2])

$$J_1 = \frac{3}{2}\left[\int_{\varkappa_{min}}^{\rho_e/r_0} \frac{d\varkappa}{\varkappa}\int_{\varkappa^{-1}}^{\varkappa^{-1}(m_i/m_e)^{1/2}} \frac{d\xi}{\xi} + \int_{\rho_e/r_0}^{1} \frac{d\varkappa}{\varkappa}\int_{\varkappa^{-1}}^{\varkappa^{-3/2}(\rho_e/r_{min})^{1/2}} \frac{d\xi}{\xi}\right]$$

$$= \frac{3}{4}\ln\frac{r_D}{r_0}\ln\frac{m_i}{m_e} + \frac{3}{4}\ln\frac{r_0}{\rho_e}\ln\frac{\sqrt{\rho_e r_0}}{r_{min}}. \qquad (A.14)$$

3) For $r_0 > r_D$ we have

$$J_1 = \frac{3}{2}\int_{\varkappa_{min}}^{1} \frac{d\varkappa}{\varkappa}\int_{\varkappa^{-1}}^{\varkappa^{-3/2}(\rho_e/r_{min})^{1/2}} \frac{d\xi}{\xi} = \frac{3}{4}\ln\frac{r_D}{\rho_e}\ln\frac{\sqrt{\rho_e r_D}}{r_{min}}. \qquad (A.15)$$

The case when $\rho_e \ll \rho_i \ll r_D$ is treated analogously.

2. We present below the form of the function η which enters in (3.13):

$$\eta = \frac{3}{2\pi\sqrt{\pi}}\int \frac{d\varkappa}{\varkappa^4}\varkappa_\perp^2\int_0^{\xi_{max}} d\xi e^{-t_e - t_i}\frac{\partial t_e}{\partial\xi}\sin^2\xi. \qquad (A.16)$$

[1] S. T. Belyaev, Fizika plazmy i problema upravlyaemykh termoyadernykh reaktsiĭ (Plasma Physics and Problem of Controlled Thermonuclear Reactions), **3**, AN SSSR, 1958, p. 66.

[2] V. E. Golant, ZhTF **33**, 3 (1963), Soviet Phys. Tech. Phys. 8, 1 (1963).

[3] V. L. Gurevich and Yu. A. Firsov, JETP **41**, 1151 (1961), Soviet Phys. JETP **14**, 822 (1962).

[4] Kihara, Midzuno, and Kaneko, J. Phys. Soc. Japan **15**, 1101 (1960).

[5] V. P. Silin, JETP **38**, 1771 (1961), Soviet Phys. JETP **11**, 1277 (1960).

[6] V. P. Silin, JETP **41**, 861 (1961), Soviet Phys. JETP **14**, 617 (1962).

[7] E. E. Lovetskii, Izvestiya VUZ'ov, Radiofizika **4**, 813 (1962).

[8] V. P. Silin, JETP **43**, 1813 (1962), Soviet Phys. JETP **16**, 1281 (1963).

[9] Rostas, Bhattacharya, and Cahn, Phys. Rev. **129**, 495 (1963).

[10] W. Marshall, AERE, T/R 2419, Harwell (1960).

Translated by J. G. Adashko
246