RELATION OF TOTAL CROSS SECTIONS FOR $\frac{1}{2} + \frac{1}{2} \to 0 + 0$ REACTIONS TO THE INTRINSIC PARTIES OF THE PARTICLES

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We consider reactions of the type $\frac{1}{2} + \frac{1}{2} \to 0 + 0$, where $\frac{1}{2}$ and 0 are the spins of the particles. It is shown that the coefficient of $\mathbf{P}_0 \cdot \mathbf{P}$ in the expression for the total cross section for the reaction with polarized particles (where $\mathbf{P}_0$ and $\mathbf{P}$ are the polarizations of beam and target) is positive if the product of the intrinsic parities is -1, and negative in the opposite case. This property of the total cross sections can be used for determining the parities of strange particles.

1. Recently the first reports have appeared about experiments with targets containing polarized hydrogen.[1,2] The degree of polarization of the hydrogen in these experiments was 20% and the target was a single crystal of lanthanum magnesium nitrate with 24 waters of hydration.

The use of polarized hydrogen targets brings an essential simplification to experiments for measuring polarization effects in elastic processes. For example, the polarization of protons in πp scattering can be measured by observing the left-right asymmetry of the π mesons scattered from a polarized target. If the target is unpolarized, the polarization of the protons is determined by a two-fold experiment which measures the asymmetry of the recoil protons. The triple experiment for determining the polarization correlation in pp scattering is replaced by the measurement of the cross section for scattering of a polarized beam by a polarized target, etc.

The use of a polarized hydrogen target opens up great possibilities for the study of inelastic processes as well. In particular, we have shown[3] that the study of the inelastic reactions

$$\pi + p \to \Sigma(\Lambda) + K, \quad \overline{K} + p \to \Sigma(\Lambda) + \pi \tag{1}$$

on a polarized hydrogen target will allow a unique determination of the relative parities of the strange particles. The proposed method consists in measuring the left-right asymmetry in the reaction with a polarized hydrogen target, and comparing it with the independently determined polarization of the final particle for the case where the target is unpolarized. This method depends only on the requirement of invariance under space inversion, and applies to any reaction of the type $\frac{1}{2} + 0 \to \frac{1}{2} + 0$, where 0 and $\frac{1}{2}$ are the spins of the particles. If the spin of the Ξ is $\frac{1}{2}$, its relative parity can be determined from the reaction $\overline{K} + p \to K + \Xi$.

2. In this note we consider other inelastic reactions with a polarized hydrogen target, and show that measurement of the total cross section for the reactions

$$\overline{\Sigma}(\overline{\Lambda}) + p \to K + \pi, \quad \overline{\Xi} + p \to K + K \tag{2}$$

with polarized protons and antihyperons, and comparing them with the total cross sections for the same reactions with unpolarized particles also will allow a determination of the parity of the strange particles.

It is not difficult to find the general expression for the total reaction cross section, starting from the requirement of invariance under spatial rotations and reflections. We should also consider that the cross section depends linearly on each of the polarizations. Let $\mathbf{P}_0$ and $\mathbf{P}$ be the polarizations of the beam and the target, and $\mathbf{k}$ the unit vector along the direction of the relative momentum of the colliding particles (c.m.s.). From these quantities we can construct the following general expression for the total cross section, satisfying the above requirements:

$$\sigma = \sigma_0 + \alpha \mathbf{P}_0 \mathbf{P} + \beta (\mathbf{P}_0 \mathbf{k})(\mathbf{P}\mathbf{k}). \tag{3}$$

Here σ_0 is the total cross section for the reaction with unpolarized particles.

The coefficients α and β depend on the initial energy, and their values are determined by the dynamics of the process. We shall see, however, that independent of assumptions about dynamics the sign of the coefficient α is uniquely determined by the product of the intrinsic parities of the particles participating in the reaction.

Thus, if we choose the polarization **P** of the target perpendicular to **k**, the comparison of the total cross sections σ and σ_0 enables us to determine the sign of α and, consequently, the relative parity of the particles participating in the reaction.

3. Now let us proceed to prove these assertions. The amplitude for the process (2) we write in the form

$$M_{\sigma\sigma'}(\mathbf{k}', \mathbf{k})\,\varphi(\sigma)\,\chi(\sigma') = \varphi^T M(\mathbf{k}', \mathbf{k})\,\chi, \qquad (4)$$

where $\chi(\sigma')$ and $\varphi(\sigma)$ are the wave functions of the nucleon and antihyperon, **k** and **k′** are unit vectors along the initial and final relative momenta in the cms, while the symbol T denotes the transpose.

Averaging the square modulus of (4) over the initial spin states, we get the following expression for the differential cross section:

$$d\sigma/d\omega = \mathrm{Sp}\, M(\mathbf{k}', \mathbf{k})\,\rho M^+(\mathbf{k}', \mathbf{k})\,\rho_0^T. \qquad (5)$$

Here $\rho_0 = \frac{1}{2}(I + \boldsymbol{\sigma}\cdot\mathbf{P}_0)$ and $\rho = \frac{1}{2}(I + \boldsymbol{\sigma}\cdot\mathbf{P})$ are the density matrices for the beam and the target.

From invariance under space rotations and inversion we get

$$M(\mathbf{k}', \mathbf{k}) = R^T M(\mathbf{k}_R', \mathbf{k}_R)\,R, \qquad (6)$$

$$M(\mathbf{k}', \mathbf{k}) = IM(-\mathbf{k}', -\mathbf{k}), \qquad (7)$$

where R is the spin rotation operator, $I = I_p I_{\bar{Y}} I_K I_\pi$ is the product of the intrinsic parities of all four particles, and $\mathbf{k}_R'$ and $\mathbf{k}_R$ are the vectors into which **k′** and **k** are transformed by the rotation.

If we introduce the matrix[1)] $N(\mathbf{k}', \mathbf{k}) = \sigma_2 M(\mathbf{k}', \mathbf{k})$, then from the relation $\sigma_2 R^T \sigma_2 = R^+$, we find from (6) and (7),

$$N(\mathbf{k}', \mathbf{k}) = R^+ N(\mathbf{k}', \mathbf{k})\,R, \qquad (8)$$

$$N(\mathbf{k}', \mathbf{k}) = IN(-\mathbf{k}', -\mathbf{k}). \qquad (9)$$

It then follows that for I = 1 the matrix $N(\mathbf{k}', \mathbf{k})$ is a scalar:

$$N(\mathbf{k}', \mathbf{k}) = a + b\boldsymbol{\sigma}\mathbf{n}, \qquad \mathbf{n} = [\mathbf{k}\mathbf{k}']/|[\mathbf{k}\mathbf{k}']|. \qquad (10)^*$$

For the case I = −1, the matrix $N(\mathbf{k}', \mathbf{k})$ is a pseudoscalar:

$$N(\mathbf{k}', \mathbf{k}) = c\boldsymbol{\sigma}\mathbf{k} + d\boldsymbol{\sigma}\boldsymbol{\varkappa},$$

$$\boldsymbol{\varkappa} = [\mathbf{k}' - (\mathbf{k}'\mathbf{k})\,\mathbf{k}]/(1 - (\mathbf{k}'\mathbf{k})^2)^{1/2}. \qquad (11)$$

The differential cross section (5), expressed in terms of the matrix $N(\mathbf{k}', \mathbf{k})$ has the form

$$d\sigma/d\omega = \mathrm{Sp}\, N(\mathbf{k}', \mathbf{k})\,\rho N^+(\mathbf{k}', \mathbf{k})\,\bar{\rho}_0;$$

$$\bar{\rho}_0 = \sigma_2\rho_0^T\sigma_2 = \tfrac{1}{2}(I - \boldsymbol{\sigma}\mathbf{P}_0). \qquad (12)$$

By using formulas (10), (11), and (12) we easily find the following expressions for the differential cross sections for the reaction in the two cases:

1) for I = 1

$$d\sigma/d\omega = \tfrac{1}{2}\{|a|^2 + |b|^2 + 2\,\mathrm{Re}\,ab^*(\mathbf{P}\mathbf{n} - \mathbf{P}_0\mathbf{n}) - |a|^2\mathbf{P}_0\mathbf{P} - |b|^2[2(\mathbf{P}\mathbf{n})(\mathbf{P}_0\mathbf{n}) - \mathbf{P}_0\mathbf{P}] - 2\,\mathrm{Im}\,ab^*([\mathbf{P}\mathbf{P}_0]\,\mathbf{n})\}; \qquad (13)$$

2) for I = −1

$$d\sigma/d\omega = \tfrac{1}{2}\{(|c|^2 + |d|^2)[1 + \mathbf{P}_0\mathbf{P}] + 2\,\mathrm{Im}\,cd^*[\mathbf{P}\mathbf{n} + \mathbf{P}_0\mathbf{n}] - 2|c|^2(\mathbf{k}\mathbf{P})(\mathbf{k}\mathbf{P}_0) - 2|d|^2(\boldsymbol{\varkappa}\mathbf{P})(\boldsymbol{\varkappa}\mathbf{P}_0) - 2\,\mathrm{Re}\,cd^*[(\mathbf{k}\mathbf{P})(\boldsymbol{\varkappa}\mathbf{P}_0) + (\boldsymbol{\varkappa}\mathbf{P})(\mathbf{k}\mathbf{P}_0)]\}. \qquad (14)$$

Integrating (13) and (14) over the directions **k′**, we get the following expressions for the total cross sections for reaction (2):

1) for I = 1

$$\sigma = \tfrac{1}{2}\int[|a|^2 + |b|^2]\,d\omega - \tfrac{1}{2}\mathbf{P}_0\mathbf{P}\int|a|^2\,d\omega + \tfrac{1}{2}(\mathbf{k}\mathbf{P})(\mathbf{k}\mathbf{P}_0)\int|b|^2\,d\omega; \qquad (15)$$

2) for I = −1

$$\sigma = \tfrac{1}{2}\int[|c|^2 + |d|^2]\,d\omega + \tfrac{1}{2}\mathbf{P}_0\mathbf{P}\int|c|^2 d\omega + \tfrac{1}{2}(\mathbf{k}\mathbf{P})(\mathbf{k}\mathbf{P}_0)\int[|d|^2 - 2|c|^2]\,d\omega. \qquad (16)$$

The coefficients of $\mathbf{P}_0\cdot\mathbf{P}$ and $(\mathbf{P}\cdot\mathbf{k})(\mathbf{P}_0\cdot\mathbf{k})$ in the expressions for the cross sections have a simple meaning. If the quantization axis is taken along **k**, then when I = 1 the amplitude M_0^t for the reaction from the triplet state with projection zero vanishes, while the other amplitudes are

$$M_{\pm 1}^t = be^{\pm i\varphi}, \qquad M^s = -i\sqrt{2}a. \qquad (17)$$

For the case of I = −1, the reaction from the singlet state is forbidden and the reaction amplitudes from the triplet states are

$$M_{\pm 1}^t = \mp\, ide^{\pm i\varphi}, \quad M_0^t = i\sqrt{2}c. \qquad (18)$$

If the polarization of the target is directed perpendicular to **k**, then in accordance with (17) and (18) the expressions for the total cross sections can be written in the form

1) for I = 1

$$\sigma = \sigma_0 - \tfrac{1}{4}\sigma^s\mathbf{P}_0\mathbf{P}; \qquad (19)$$

2) for I = −1

$$\sigma = \sigma_0 + \tfrac{1}{4}\sigma_0^t\mathbf{P}_0\mathbf{P}, \qquad (20)$$

where σ^s and σ_0^t are the respective cross sections for the reaction from the singlet state and the triplet state with zero projection.

[1)]We use the standard representation for the Pauli matrices.

*$[\mathbf{k}\mathbf{k}'] = \mathbf{k}\times\mathbf{k}'$, $\boldsymbol{\sigma}\mathbf{n} = \boldsymbol{\sigma}\cdot\mathbf{n}$.

Thus the coefficient of $\mathbf{P}_0 \cdot \mathbf{P}$ in the expression for the total cross section is negative if the product of the intrinsic parities of the particles is +1, and positive if I = −1. Correspondingly, when $\mathbf{P}_0 \cdot \mathbf{P} > 0$, the total cross section σ for the reaction is less than that with unpolarized particles when I = + 1, and σ is greater than σ_0 if I = −1. If the polarizations $\mathbf{P}$ and $\mathbf{P}_0$ are known, this property of the cross sections can be used for determining the intrinsic parities of the strange particles.

In the general case of a reaction with two spin ½ particles in the initial state, it is not difficult to obtain ($\mathbf{P}$ is perpendicular to $\mathbf{k}$) the following relation between the total cross sections:

$$\sigma = \sigma_0 + \frac{1}{4}(\sigma_0^t - \sigma^s)\,\mathbf{P}_0\mathbf{P}.$$

From this relation we see that in the general case the situation where the relation between the sign of the coefficient of $\mathbf{P}_0 \cdot \mathbf{P}$ and the intrinsic parities is independent of the dynamics is no longer the case.

We note that the relative parities can also be determined from studying the differential cross sections of reaction (2). As we see from (13), for I = 1 the ratio of the left-right asymmetry ϵ_{P_0} in the reaction using a polarized beam and an unpolarized target to the asymmetry ϵ_P in the reaction with an unpolarized beam and a polarized target is

$$\varepsilon_{P_0}/\varepsilon_P = -P_0/P. \qquad (21)$$

For the case where I = −1 this ratio is

$$\varepsilon_{P_0}/\varepsilon_P = P_0/P. \qquad (22)$$

We have assumed that the polarizations $\mathbf{P}_0$ and $\mathbf{P}$ are along the normal to the reaction plane.

4. We note that our conclusion about the behavior of total cross sections applies to any reaction of the type ½ + ½ → 0 + 0, for example to the reaction

$$p + \mathrm{He}^3 \to \mathrm{He}^4 + \pi^+, \qquad \bar{p} + p \to K + \bar{K}.$$

The product of the intrinsic parities can, in particular, be determined by comparing the total cross sections of the reactions

$$p + \mathrm{He}^3 \to \mathrm{He}^4_\Lambda + K^+, \qquad \Sigma^- + \mathrm{He}^3 \to \mathrm{He}^4 + K^-$$

with polarized and unpolarized particles.

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