THE $\pi + N \rightarrow \pi + \gamma + N$ REACTION AND THE CONSTANT FOR PION-PION PHOTOPRODUCTION

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The determination of the pion-pion photoproduction constant through the analysis of the $\pi + N \rightarrow \pi + \gamma + N$ reaction is considered. The differential cross sections for this reaction are calculated in the pole approximation.

THE emission of γ rays in interactions of pions with matter was first detected [1] in a study of π^+-meson scattering on light nuclei at kinetic energies in the interval from 80 to 300 MeV. In most cases the radiation was produced by mesons of energy greater than 200 MeV.

Ermolov and Moskalev, [2] in an analysis of π^--meson scattering on hydrogen ($E_{\pi^-} = 128$ and 162 MeV), did not observe any interactions in which a γ ray was emitted, although the technique employed permitted the recording of photons of extremely low energy ($E_\gamma \approx 15$ MeV). The emission of γ rays was subsequently detected in interactions of π^- mesons with hydrogen at 225 MeV [3] and 340 MeV [4]. In the latter case it was shown, in particular, that the cross section for the reaction

$$\pi^- + p \rightarrow \pi^- + \gamma + p \qquad (1)$$

for γ rays of energy exceeding 100 MeV is $0.09^{+0.03}_{-0.06}$ mb.

As a possible mechanism giving rise to energetic photons we can consider the process described by the Feynman diagram shown in Fig. 1. The usefulness of such a diagram to describe reaction (1) is suggested by the fact that a similar approach to the reactions $\pi + N \rightarrow \pi + \pi + N$ proposed by Goebel and Schnitzer [5] made it possible to give a satisfactory explanation of the experimental data for these reactions. As will be shown below, the diagram of Fig. 1 contributes a characteristic singularity to the distribution of the total energy in the $\pi\gamma$ c.m.s., which makes it possible to estimate the contribution of this diagram to reaction (1) and to determine the unknown parameters of the amplitude for the photoproduction of a pion on a pion.

To calculate the amplitude for the process $\pi + N \rightarrow \pi + \gamma + N$, we introduce the following notation: q and q_1 are the 4-momenta of the incident and scattered pions, p and p_1 are the 4-momenta of the nucleon before and after the interaction, k is the 4-momentum of the photon, M is the nucleon mass in units of pion mass (in the system $\hbar = \mu = c = 1$), and $q_2 = p - p_1$.

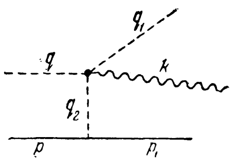

FIG. 1

The cross section for the process can be written in terms of the F matrix:

$$\sigma = (2\pi)^2 \int \frac{p^0 q^0}{B} |\bar{F}|^2 \delta^4 (p + q - k - q_1 - p_1)\, d^3\mathbf{k}\, d^3\mathbf{q}_1 d^3\mathbf{p}_1;$$
$$B = \{(p^0\mathbf{q} - q^0\mathbf{p})^2 - [\mathbf{qp}]^2\}^{1/2},$$
$$F = -\frac{i}{(2\pi)^{7/2}} \frac{M}{\sqrt{p^0 p_1^0}} \frac{1}{\sqrt{8 q^0 q_1^0 k^0}} \frac{\varepsilon_{\lambda\mu\nu\rho} q^\mu q_1^\nu k^\rho e_f^\lambda}{q_2^2 - 1} f(\nu)\, g \bar{u}(p_1) \gamma_5 u(p), \qquad (2)^*$$

where ω is the total energy in the $\pi\gamma$ c.m.s.; $\nu = \omega^2/4 - 1$; e_f^λ is the photon polarization vector; g is the pion-nucleon coupling constant ($g^2/4\pi \approx 15$); $f(\nu)$ is a scalar function which is a solution of the dispersion equation for the process $\gamma + \pi \rightarrow \pi + \pi$ and depends on the $\pi\pi$ scattering phase shift in the P state. [6-10] Moreover, the form of the function $f(\nu)$ is determined by the approximations and assumptions made in the derivation and solution of the dispersion equation.

In the present work, we use the following expression [8] for $f(\nu)$:

$$f(\nu) = \{\Lambda A^{-1} (\nu + 9/16)^2 + C\} e^{\rho(\nu) + i\delta(\nu)}; \qquad (3)$$

$$\rho(\nu) + i\delta(\nu) = \frac{\nu + 9/16}{\pi} \int_0^\infty \frac{\delta(x)}{x + 9/16} \left(\frac{1}{x - \nu - i\varepsilon} - \frac{1}{x + \nu + 9/8}\right) dx,$$
$$A = [\nu^2 e^{\rho + i\delta}]_{\nu \to \infty}, \qquad (4)$$

*[qp] = q × p.

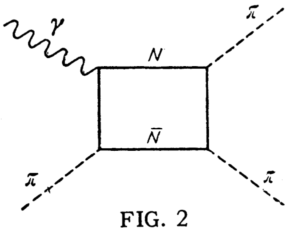

FIG. 2

where $\delta(\nu)$ is the phase shift for $\pi\pi$ scattering in the state T = J = 1. The expression for $f(\nu)$ is the exact complete solution of the dispersion equation and is given in [8]. The constant Λ introduced into this olution has a simple physical meaning; it approximates the contribution to the process $\gamma + \pi \rightarrow \pi + \pi$ from intermediate states with a heavy mass. As was shown in [8], from the viewpoint of perturbation theory, such intermediate states should include first of all the $N\bar{N}$ states. An estimate of the quantity Λ made on the basis of perturbation theory with allowance for the diagrams shown in Fig. 2 yields the value $\Lambda = 32 \times f^3\sqrt{\alpha} \approx 0.062$.

From the viewpoint of the solution of the dispersion equations, the constant C, which is equal to the photoproduction amplitude at the point $\nu = -9/16$, is, roughly speaking, arbitrary. However, in considering the process $\gamma + \pi \rightarrow \pi + \pi$ in perturbation theory, we do not have to introduce new constants,[8] i.e., from the viewpoint of perturbation theory the parameters Λ and C should be related to each other. In [8] a requirement was introduced which made it possible to select a unique solution of the dispersion equation, i.e., to express the constant C in terms of Λ. This was the requirement that for phase shifts $\delta(\nu)$ tending to π as $\nu \rightarrow \infty$, the function $f(\nu)$ should tend to Λ for large ν faster than the decrease in the solution to the homogeneous equation. We note that this requirement is valid only for phase shifts having the behavior $\delta(\nu) = \pi - \epsilon\nu^{-\beta}$ (where $\beta \geq 3$) as $\nu \rightarrow \infty$. For phase shifts tending to π more slowly, it can be formulated in the following way: the function $f(\nu)$ should not contain terms of the type ν^{-2} for large ν.

We calculated the function

$$f(\nu)/C = [\alpha(\nu + 9/16)^2 + A]\,A^{-1}e^{\rho + i\delta}$$

for different real values of the parameter $\alpha = \Lambda/C$ in the interval of ω from 1 to 3 (ν changed from $-3/4$ to $5/4$). In the calculations the phase shifts $\delta(\nu)$ were selected in two different forms and the integral (4) was calculated for both explicitly. As the first form we took the Breit-Wigner phase shift:

$$\operatorname{tg}\delta(\nu) = \gamma\sqrt{\nu}/(\nu_r - \nu), \tag{5}*$$

where ν_r is the square of the pion momentum at the resonance energy and γ is a parameter connected with the position and width of the ω-resonance in the pion scattering cross section. The amplitude corresponding to such a choice of the phase shifts is[11]

$$\frac{f(\nu)}{C} = -\frac{[\alpha(\nu + 9/16)^2 + A]\,A^{-1}(9/16 + 3/4\gamma + \nu_r)^2}{(\nu + \gamma\sqrt{\nu + 9/8} + \nu_r + 9/8)(\nu + i\gamma\sqrt{\nu} - \nu_r)}; \tag{6}$$

the branches of the roots in (6) are given by the condition $\sqrt{\nu} = i\sqrt{|\nu|}$ for $\nu < 0$.

Since the behavior of $\delta(\nu)$ in (5) for small ν corresponds to an S wave and not a P wave, we took account of the influence of the specific behavior of the phase shift for small and large ν on the form of the function $f(\nu)$, by carrying out the calculation also for the phase shift $\delta(\nu)$ taken from [8] [formula (45)]. For low energies the values of the P-wave phase shift are contained between the values of the phase shifts corresponding to the chosen form. At high energies the phase shifts approach π in different ways. The calculations showed that for $1 \leq \omega \leq 3$ the values of $f(\nu)$ obtained with both phase shifts practically coincide.[1)] Here we took the following values of the parameters:[12]

$$\omega_r = 5.5, \qquad \Gamma = 0.929$$

(Γ is the energy width of the $\pi\pi$ resonance). We then have $\gamma = 0.5$, b = 2.55 [b is the parameter from formula (45) in [8]].

Introducing the F matrix in quadratic form, averaging it over the spins of the initial nucleon and summing over the spins of the final nucleon and over the polarization of the γ ray, we have

$$\overline{|F|^2} = \frac{g^2[(kq)^2 + (kq_1)^2 - 2(qq_1)(kq)(kq_1)]\,|f(\nu)|^2\,[M^2 - pp_1]}{8(2\pi)^7\cdot(q_2^2 - 1)^2}.$$

Substituting $\overline{|F|^2}$ in formula (2), we obtain

$$\sigma = \frac{g^2}{8(2\pi)^5}\int\frac{(M^2 - pp_1)\,d^3\mathbf{p}_1\,d^4q_2\,\delta^4(p - p_1 - q_2)}{Bp_1^0(q_2^2 - 1)^2}$$
$$\times\int\frac{[(kq)^2 + (kq_1)^2 - 2(qq_1)(kq)(kq_1)]}{q_1^0k^0}$$
$$\times|f(\nu)|^2\delta^4(q + q_2 - q_1 - k)\,d^3\mathbf{k}\,d^3\mathbf{q}_1. \tag{7}$$

The second integral in the product of (7) was calculated in the $\pi\gamma$ c.m.s.[9] and is equal to

*tg = tan

1)The values of α obtained with the aid of the requirements formulated above are close to one another and are equal to 1.21 for the case of the Breit-Wigner phase shift and 0.96 for the phase shift from [8].

$$I_2 = 8\pi\tilde{k}^3 \left[\tfrac{1}{3}\left(2(\tilde{q}^0)^2\omega + \omega\right) - \tilde{q}^0 (1 + qq_2)\right],$$

$$\omega^2 = (k + q_1)^2$$

($\tilde{k}$ is the photon momentum in the $\pi\gamma$ c.m.s.). After introduction of $\Delta^2 = (p - p_1)^2$, the second integral can be expressed in simple form in terms of invariant quantities

$$I_2 = -\tfrac{1}{12}\pi (\omega^2 - 1)^3\omega^{-4} [(\omega^2 - 1)^2 - \Delta^2] [(\omega + 1)^2 - \Delta^2]. \quad (8)$$

Substituting (8) into (7) and expressing d^3p_1 in terms of the differentials of the invariant quantities, we obtain the final expression for the differential cross section:

$$\frac{d^2\sigma}{d\omega^2 d\Delta^2} = \frac{g^2 (\omega^2 - 1)^3 |f(\nu)|^2 [(\omega - 1)^2 - \Delta^2] [(\omega + 1)^2 - \Delta^2] \Delta^2}{3328(2\pi)^3 \cdot M^2 q_L^2 \omega^4 (\Delta^2 - 1)^2} \quad (9)$$

(the subscript L denotes the laboratory system.

The distribution of ω obtained from (5) by integration over Δ^2 is shifted toward the maximum allowable values of the total energy (see Fig. 3), which is characteristic for the mechanism of γ-ray emission considered here. Comparison of the differential cross sections calculated by means of the diagram of Fig. 1 with statistical theory shows that the separation of the contribution from other diagrams[2)] is difficult, especially at large values of α (≈ 10) (Fig. 3). If it is assumed that the analyzed process is described by the diagram we have been considering, then the constants C and Λ can be determined experimentally. We note that the rapid increase in the differential cross section with increasing ω does not contradict the experimental results[1,2] in which cases of reaction (1) were observed primarily for pion energies above the pion production threshold.

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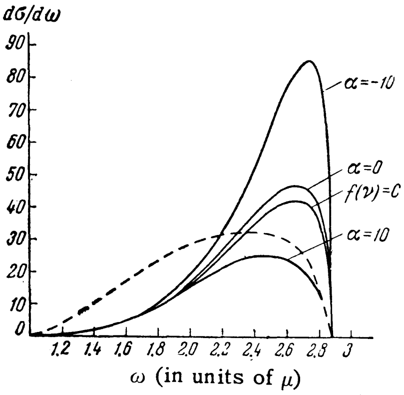

FIG. 3. Dependence of $d\sigma/d\omega$ on ω for different values of the parameter α. The dashed curve is the same distribution calculated from statistical theory and normalized to the same area as the curve with $\alpha = 0$.

[2)]Qualitative considerations indicate that the contribution from other diagrams to the differential cross section is close to the statistical one.

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