SCATTERING OF ELECTRONS BY NUCLEI ACCORDING TO THE α PARTICLE MODEL OF THE NUCLEUS

V. VADIA, E. INOPIN, and M. YUSEF

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In continuation of previous work, [1] inelastic scattering of electrons on O^{16} and on a nucleus consisting of an α particle core and a weakly bound neutron is considered. Gamma transitions in the O^{16} nucleus are also discussed.

IN this note, we report calculations of the scattering of electrons by nuclei according to the α-particle model for a number of interesting cases which have not been considered in our previous paper [1] devoted to the same problem.

According to the α particle model, the nucleus O^{16} has tetrahedron symmetry. The first two rotational levels will, in accordance with the restrictions on spin and parity imposed by this symmetry, be 3^- and 4^+ and can be identified with the 6.14 and 10.4 MeV levels of the O^{16} nucleus.

The rotational wave function for a state with spin I, projection M, and parity P will in general be of the form

$$\Phi^P_{IM} = \sum_K b_K D^I_{MK}\,. \qquad (1)$$

The symmetry requirements are in this case that this function be invariant under symmetry transformations of the tetrahedron. This gives for the states 3^- and 4^+

$$\Phi^-_{3M} = \frac{\sqrt{7}}{4\pi}[D^3_{M2} - D^3_{M-2}]\,, \qquad (2)$$

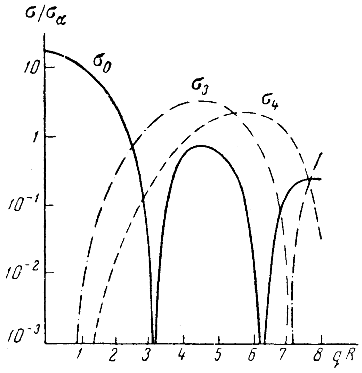
$$\Phi^+_{4M} = \frac{1}{2\pi}\sqrt{\frac{3}{2}}\left[D^4_{M4} + \sqrt{\frac{14}{5}}D^4_{M0} + D^4_{M-4}\right]. \qquad (3)$$

The calculation of the inelastic scattering of electrons with excitation of these rotational states leads in Born approximation to

$$\sigma_3(\theta) = \frac{560}{9}\sigma_\alpha(\theta)\, j_3^2(qR)\,, \qquad \sigma_4(\theta) = \frac{896}{15}\sigma_\alpha(\theta)\, j_4^2(qR)\,, \qquad (4)$$

where $\sigma_\alpha(\theta)$ is the cross section for electron scattering on an α particle, q is the momentum transfer, and R is the distance from the center of the tetrahedron to any one of its vertices, i.e., the distance between the center of gravity of the nucleus and the center of one of the α particles.

The figure shows the ratio of the cross sections (4) over the elastic scattering cross section. We see that in the neighborhood of the first minimum of the elastic cross section, the inelastic cross



sections are comparable in magnitude with the elastic cross section and even exceed it further out.

At sizable scattering angles it will probably be possible to separate experimentally the contributions from the excitation of the 3^- level with energy 6.14 MeV and the 0^+ level with energy 6.06 MeV. Indeed, the transition to the 0^+ level is a monopole transition and, similarly as in elastic scattering at large angles, the corresponding cross section will decrease appreciably more rapidly than the cross section for the octupole transition.

These contributions can no longer be separated in the comparison with experiment at small angles, but equivalent information can be obtained by considering the γ transitions $3^- \to 0^+$, i.e., to the ground state.

A simple computation gives for the reduced transition probability for this transition

$$B(E3) = \frac{80e^2}{9\pi}R^6\,. \qquad (5)$$

In the notation of [1] $R = \sqrt{3/2}\,d$, and by comparing the locations of the minima of the calculated elastic cross section with experiment we obtained d = 1.6 f. From this we find for the lifetime $\tau = 3.3 \times 10^{-11}$ sec. This value lies well above the experimental value[2] $\tau_{\text{exp}} = (1.2 \pm 0.6) \times 10^{-11}$ sec.

This is somewhat surprising, since the collective model which we are considering would be expected to lead to higher transition probabilities and hence to lower lifetimes.

It is interesting to note that our value of τ agrees almost exactly with the value obtained earlier by Kameny[3] who, however, determined R not from the elastic scattering, but from the moment of inertia calculated according to classical mechanics. It appears to us that this agreement is highly accidental, since the calculation of the moment of inertia according to classical mechanics is hardly justified.

The discrepancy between the calculated and experimental values for τ is possibly connected with the neglect of the vibrational degrees of freedom. The inclusion of zero-point vibrations in the elastic scattering leads to an increase in the value of R_0, the equilibrium distance of the center of the α particle determined by the position of the first minimum in the elastic scattering cross section.[1] R_0 enters in (5) as a sixth power, so that a small change in R_0 may lead to a significant decrease in τ. Estimates show that this effect can decrease τ by a factor $2/3$ to $1/2$ and thus lead to good agreement between the experimental and theoretical values of the lifetime.

Let us now consider the scattering of electrons on a nucleus consisting of an α particle core and an external neutron. We shall assume that the nonsphericity of the potential has a small effect on the motion of the neutron (small coupling approximation). In this approximation the nuclear wave function can be written in the form

$$\Psi_{J\mathfrak{M}} = \sum_m (IjMm \mid J\mathfrak{M}) \Psi_{IM} \chi_{jm}, \tag{6}$$

where Ψ_{IM} and χ_{jm} are the wave functions of the α particle core and the external neutron.

Let us consider the cross section for a scattering process in which the nucleus goes from a state with angular momentum J_0 to a state with angular momentum J by rotational excitation of the α particle core from the state $I_0 = 0$ to the state I. The neutron remains in its initial state. Simple calculations show that

$$\sigma_{JJ_0}(\theta) = \frac{2J+1}{(2j+1)(2I+1)} \sigma_{I0}(\theta), \tag{7}$$

where σ_{I0} is the cross section for inelastic scattering on the α particle core. Thus the presence of the neutron leads only to the appearance of the kinematical factor $(2J+1)/(2j+1)(2I+1)$, which gives the probability for the occurrence of the state with angular momentum J in the addition of j and I. It is natural that the sum of the cross sections (7) over all admissible J gives simply the cross section for scattering on the α particle core:

$$\sum_J \sigma_{JJ_0}(\theta) = \sigma_{I0}(\theta). \tag{8}$$

These results may possibly be useful in the study of the scattering of electrons by C^{13} and O^{17}.

[1] E. V. Inopin and B. I. Tishchenko, JETP **38**, 1160 (1960), Soviet Phys. JETP **11**, 840 (1960).

[2] D. Kohler and H. H. Hilton, Phys. Rev. **110**, 1094 (1958).

[3] S. L. Kameny, Phys. Rev. **103**, 358 (1956).

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189