ASYMPTOTIC EXPRESSION FOR THE TOTAL $\gamma\gamma$ SCATTERING CROSS SECTION AT HIGH ENERGIES

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Submitted to JETP editor April 5, 1963, resubmitted June 10, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 1051-1059 (October, 1963)

It is shown that the requirement of a nonvanishing total cross section for high energy scattering, treated by the complex moment method, entails the necessity of introducing a third-order pole at the point J = 1 with t = 0 for partial amplitudes combining with a P pole. For this case an arrangement of singularities of the indicated amplitudes is suggested which yields a maximal interaction force consistent with unitarity and analyticity [4] and which leads to total and elastic cross sections that increase at high energies.

1. A new trend in the theory of strong interactions, connected with the concept of moving partial-amplitude poles—Regge poles—is being vigorously developed of late. The asymptotic behavior of the elastic scattering of particles at high energies and small momentum transfers is determined in this case by the pole farthest to the right in the complex plane of the total angular momentum J —the principal vacuum pole or the Pomeranchuk pole, which has the quantum numbers of vacuum and a positive signature[1,2]. In order to ensure constancy at high energies of the total scattering cross section, which is determined by the imaginary part of the forward elastic scattering amplitude (t = 0), it is postulated that the trajectory of this P-pole passes at t = 0 through the point J = 1, that is, $\alpha_P(0) = 1$. When t decreases, the pole moves to the left along the real J axis, and this causes a logarithmic fall-off of the elastic cross section at large s.

It is natural to assume that at high energies the P-pole also determines the asymptotic behavior of the photon elastic scattering amplitude[3]. In order not to contradict the unitarity conditions in the s- and t-channels, it was necessary in this case that the residue of the partial amplitude for the transition from a pair of photons to any pair of particles with the quantum numbers of vacuum and positive signature become infinite at the P pole when t = 0.

In the present paper we propose the simplest arrangement of singularities in the complex J plane, which does not cause the asymptotic vanishing of the total cross section for photon scattering. By virtue of this the total cross section becomes smaller than the elastic cross section, thus contradicting unitarity in the s-channel. Namely, in addition to the P-pole there must exist two additional poles which, colliding at the point J = 1 when t = 0, go off to the complex J plane as t decreases. This leads to a constant total cross section for the scattering of photons on photons and to total and elastic cross sections of the strongly-interacting particles which increase as $\ln^2 s$ at large s; this, according to Froissart[4], denotes the maximum possible strength of these interactions permitted by unitarity and analyticity.

2. Let us consider $\gamma\gamma$ scattering. The amplitude of this process can be written in the form[3]

$$F_{\mu\nu,\rho\sigma} = F_1^+(s,\ t,\ u)\,\pi_\mu\pi_\nu\pi_\rho\pi_\sigma + F_2^+\ (s,\ t,\ u)\ n_\mu n_\nu n_\rho n_\sigma + F_3^+(s,\ t,\ u)\ (\pi_\mu\pi_\nu n_\rho n_\sigma + n_\mu n_\nu \pi_\rho\pi_\sigma) + F_4^-(s,\ t,\ u)\ (\pi_\mu n_\nu + n_\mu\pi_\nu)\ (\pi_\rho n_\sigma + n_\rho\pi_\sigma) + F_5^+(s,\ t,\ u)(\pi_\mu n_\nu - n_\mu\pi_\nu)\ (\pi_\rho n_\sigma - n_\rho\pi_\sigma)$$

where

$$\pi_{\mu,\nu} = P_{1\mu,\nu}(-P_1^2)^{-1/2},\quad \pi_{\rho,\sigma} = P_{2\rho,\sigma}(-P_2^2)^{-1/2},$$

$$n_{\mu,\nu} = N_{1\mu,\nu}(-N_1^2)^{-1/2},\quad n_{\rho,\sigma} = N_{2\rho,\sigma}(-N_2^2)^{-1/2},$$

$$P_1 = K_2 - (K_1K_2)\,K_1/K_1^2,\quad P_2 = K_1 - (K_1K_2)\,K_2/K_2^2,$$

$$N_{1\mu} = \varepsilon_{\mu\alpha\beta\gamma}P_{1\alpha}K_{1\beta}Q_\gamma,\quad N_{2\rho} = \varepsilon_{\rho\alpha\beta\gamma}P_{2\alpha}K_{2\beta}Q_\gamma,$$

$$K_1 = k_1 + k_1',\quad K_2 = k_2 + k_2',\quad Q = k_1 - k_1' = k_2' - k_2,$$

$k_{1\nu}$, $k_{2\sigma}$ and $k'_{1\mu}$, $k'_{2\rho}$ are the momenta of the photons before and after the scattering, respectively. The minus and plus signs indicate whether the invariant functions reverse or do not reverse sign following the substitution $s \rightleftarrows u$.

To find the asymptotic behavior of the invariant functions it is necessary to represent them in the form of a series in partial waves in the t-channel. In this expansion it is convenient to use the particle states with definite helicity [5]. Two photons can be in the following state with specified total angular momentum J, parity P, signature or parity J, $(-1)^J$, and charge parity C = +1:

$$|J, 0, +\rangle = |J, +1+1\rangle + |J, -1-1\rangle, (-1)^J P = +1, \quad (-1)^J C = +1;$$
$$|J, 2, +\rangle = |J, +1-1\rangle + |J, -1+1\rangle, (-1)^J P = +1, \quad (-1)^J C = +1;$$
$$|J, 0, -\rangle = |J, +1+1\rangle - |J, -1-1\rangle, (-1)^J P = -1, \quad (-1)^J C = +1;$$
$$|J, 2, -\rangle = |J, +1-1\rangle - |J, -1+1\rangle, (-1)^J P = -1, \quad (-1)^J C = -1.$$

The symbols 0 and 2 indicate the minimum value of J in these states.

The $\gamma\gamma$ scattering process is determined by five partial amplitudes of the transitions between the states with identical quantum numbers:

$$\left.\begin{aligned} f_{00}^J(t) &= \langle 0, + | S^J | 0, + \rangle \\ f_{02}^J(t) &= \langle 0, + | S^J | 2, + \rangle \\ f_{22}^J(t) &= \langle 2, + | S^J | 2, + \rangle \end{aligned}\right\} (-1)^J P = +1, (-1)^J C = +1;$$
$$f_0^J(t) = \langle 0, - | S^J | 0, - \rangle, (-1)^J P = -1, (-1)^J C = +1;$$
$$f_2^J(t) = \langle 2, - | S^J | 2, - \rangle, (-1)^J P = -1, (-1)^J C = -1.$$

The first three partial amplitudes have the quantum numbers of vacuum and a positive signature, and therefore can combine with the vacuum pole.

Expansion of the invariant functions in helical partial waves with definite quantum numbers in the t-channel c.m.s. is as follows:

$$F_1^+ + F_2^+ + 2F_3^+ = \frac{1}{\pi}\sum_{J=0}^{\infty}(2J+1) f_{00}^J(t) P_J(z), \tag{1a}$$

$$F_1^+ - F_2^+ = \frac{1}{\pi}\sum_{J=2}^{\infty}(2J+1) f_{02}^J(t) \frac{(1-z^2)}{\sqrt{(J-1)J(J+1)(J+2)}} P_J''(z), \tag{1b}$$

$$F_1^+ + F_2^+ - 2F_3^+ = \frac{1}{\pi}\sum_{J=2}^{\infty}(2J+1) f_{22}^J(t)\, d_+^J(z) + \frac{1}{\pi}\sum_{J=3}^{\infty}(2J+1) f_2^J(t)\, d_-^J(z), \tag{1c}$$

$$4F_4^- = \frac{1}{\pi}\sum_{J=2}^{\infty}(2J+1) f_{22}^J(t)\, d_-^J(z) + \frac{1}{\pi}\sum_{J=3}^{\infty}(2J+1) f_2^J(t)\, d_+^J(z), \tag{1d}$$

$$-4F_5^+ = \frac{1}{\pi}\sum_{J=0}^{\infty}(2J+1) f_0^J(t) P_J(z), \tag{1e}$$

where

$$d_+^J(z) = [(J-1)J(J+1)(J+2)]^{-1}\{2(1+z^2)P_J''(z) + 4z(z^2-1)P_J'''(z) + (z^2-1)^2 P_J''''(z)\},$$
$$d_-^J(z) = [(J-1)J(J+1)(J+2)]^{-1}\{zP_J''(z) + (z^2-1)P_J'''(z)\},$$
$$z = -1 - 2s/t.$$

From the expansions (1) we see that at high energies the poles with the quantum numbers of vacuum and with positive signatures, which include, in particular, the Pomeranchuk pole, will define the functions F_1^+, F_2^+ and F_3^+, and give an asymptotically small contribution to the function F_4^-, since $d_-^J(z) \sim z_{-1} d_+^J(z)$ as $z \to \infty$.

The poles with negative signature, which combine with the amplitude $f_2^J(t)$, will determine for large s the function F_4^- and make a small contribution to the functions F_1^+, F_2^+, and F_3^+. The poles with negative parity, which combined with $F_0^J(t)$, contribute only to F_5^+.

To the contrary, it is easy to express the partial waves in terms of the s-adsorption parts of the invariant functions. To this end it is necessary to use the orthonormality of the d-functions and to write down the dispersion relations for the functions $F_i(s,t,u)$ in the t-channel with respect to the momentum transfer with no fewer subtractions at the points $z = \pm 1$ than the minimum possible value of J for the sought partial wave, and to use the representation of the Jacobi functions of the second kind in terms of the functions of the first kind[1] (see [6]). Thus, for example,

$$f_{22}^J(t) = 2\int_{z_0}^{\infty} A\,[F_1^+(s,t) + F_2^+(s,t) - 2F_3^+(s,t)]\, q_+^J(z)\, dz + 2\int_{z_0}^{\infty} 4A\,[F_4^-(s,t)]\, q_-^J(z)\, dz;$$

$$q_+^J(z) = [(J-1)J(J+1)(J+2)]^{-1}\{2(1+z^2)Q_J''(z) + 4z(z^2-1)Q_J'''(z) + (z^2-1)^2 Q_J''''(z)\},$$

$$q_-^J(z) = [(J-1)J(J+1)(J+2)]^{-1} \times \{zQ_J''(z) + (z^2-1)Q_J'''(z)\}. \tag{2}$$

We note that by virtue of the optical theorem the total cross section for scattering is determined by the expression

$$A\,[F_1^+(s,t) + F_2^+(s,t) - 2F_3^+(s,t) - 4F_4^-(s,t)]|_{t=0}$$

[1] The d-functions are expressed in terms of the Jacobi functions of the first kind.

in the scattering of quanta with identical circular polarizations, and by the expression

$$A\,[F_1^+(s,t)+F_2^+(s,t)-2F_3^+(s,t)+4F_4^-(s,t)]\,|_{t=0}$$

in the scattering of quanta with different circular polarizations.

At high energies, F_4^- drops out of the expression for the total cross section[3], so that the latter is given by the combination

$$A\,[F_1^+(s,t)+F_2^+(s,t)-2F_3^+(s,t)]\,|_{t=0}.$$

We see thus that the total cross section of $\gamma\gamma$ scattering at high energies is determined by that partial wave in the t-channel, having the total angular momentum projection on the direction of the relative particle motion not equal to zero (in this case the projection is equal to 2), this being a consequence of the photon mass being equal to zero.

This can be readily verified by considering, for simplicity, the scattering of photons and vector mesons with nonzero mass by pions. In the case of $\pi\gamma$ scattering, as in $\gamma\gamma$ scattering, the total cross sections in the case of large s is determined by the partial wave having a total momentum projection on the relative-motion axis equal to 2, while in the case of the scattering of the vector meson by the pion it is determined by the partial wave with zero projection of the total angular momentum. Incidentally, the same situation occurs in the scattering of spin-½ particles. If the fermion mass is finite, then the total cross section is determined at high energies by the partial wave with zero projection of the total momentum, and in the case of zero mass it is determined by the partial wave with unity projection of the total angular momentum on the relative-motion axis.

3. As is well known, to obtain the asymptotic values of the invariant functions it is necessary to represent the sum over the partial amplitudes in the form of a Watson-Sommerfeld integral, and, by shifting the contour of integration to the left, calculate the contribution from the P-pole. In order to obtain the value of the invariant functions in the physical region of the s channel ($t < 0$), it is necessary to be able to move the integration contour to the left of the point $J = 1$, since $\alpha_p(0) = 1$. Obviously, the partial amplitude $f_{02}^J(t)$ should have a root branch point at $J = 1$, in order for the integrand function in the Watson-Sommerfeld integral, obtained from the expansion of (1b) not to have a standing branch point at this point.

Therefore, as $J \to 1$ the function $f_{02}^J(t)$ can behave in two alternative ways: either

$$f_{02}^J(t)=\sqrt{J-1}\;\varphi_{02}^J(t)/(J-\alpha_P(t))$$

or

$$f_{02}^J(t)=\varphi_{02}^J(t)\,/\,\sqrt{J-1}\,(J-\alpha_P(t)),$$

where $\varphi_{02}^J(t)$ no longer has any standing singularities near the point $J = 1$. In view of the fact that the partial waves $f_{00}^J(t)$, $f_{22}^J(t)$, and $f_{02}^J(t)$ are combined with the vacuum pole, their residues at this pole factor out:

$$\operatorname{res} f_{00}^J(t)\cdot\operatorname{res} f_{22}^J(t)=(\operatorname{res} f_{02}^J(t))^2.$$

Consequently, in order for the partial amplitudes $f_{00}^J(t)$ or $f_{22}^J(t)$ not to have a standing pole at the point $J = 1$, which would contradict the unitarity conditions in the t-channel, it is necessary to choose for $f_{02}^J(t)$ near $J = 1$ a behavior of the first type, namely

$$f_{02}^J(t)=\sqrt{J-1}\;\varphi_{02}^J(t)/(J-\alpha_P(t)).$$

By virtue of the factorization relation between the residues of the considered amplitudes and the form of $f_{02}^J(t)$ near $J = 1$, one of the functions $f_{00}^J(t)$ or $f_{22}^J(t)$ should have near the point $J = 1$ the form

$$f_{00}^J(t)=(J-1)\,\varphi_{00}^J(t)/(J-\alpha_P(t))$$

or

$$f_{22}^J(t)=(J-1)\,\varphi_{22}^J(t)/(J-\alpha_P(t)),$$

where one of the functions $\varphi_{00}^J(t)$ or $\varphi_{02}^J(t)$ does not have any standing singularities near $J = 1$.

A unique choice of the variant can be made by bringing into consideration $\pi\gamma$ scattering. For the same reasons that have led to the choice of the behavior of $f_{02}^J(t)$ near $J = 1$, the partial amplitude $g_{02}^J(t)$, which determines at high energies the total cross section of the $\pi\gamma$ scattering[3], should have near $J = 1$ the form

$$g_{02}^J(t)=\sqrt{J-1}\;\psi_{02}^J(t)/(J-\alpha_P(t)),$$

where the function $\psi_{02}^J(t)$ does not have any standing singularities near this point.

The residue $g_{02}^J(t)$ on the trajectory of the principal vacuum pole satisfies the relation

$$\operatorname{res} f_{\pi\pi}^J(t)\cdot\operatorname{res} f_{22}^J(t)=(\operatorname{res} g_{02}^J(t))^2,$$

where $f_{\pi\pi}^J(t)$ is the partial wave which determines the asymptotic behavior of the $\pi\pi$ scattering at high energies. It is seen from this relation that

$$f_{22}^J(t)=(J-1)\,\varphi_{22}^J(t)/(J-\alpha_P(t))$$

near $J = 1$, for otherwise a factor $\alpha_P(t) - 1$ would appear in the $\pi\pi$ scattering amplitude at high energies, and would cause the elastic scattering cross section to be larger than the total cross section, which in turn would vanish.

Thus, an account of the unitarity relations in the s- and t-channels yields for $f_{22}^J(t)$ near the point $J = 1$ a behavior of the type

$$f_{22}^J(t) = (J-1)\,\varphi_{22}^J(t)/(J-\alpha_P(t)).$$

Taking into account such a behavior of $f_{22}^J(t)$, we can readily find the asymptotic behavior of the expression $F_1^+ + F_2^+ - 2F_3^+$, the imaginary part of which for $t = 0$ determines $\sigma_{tot}^{\gamma\gamma}$.

Using the explicit form of $d_+^J(z)$, we find that for large s there appears in the expression for the total cross section a factor

$$(\alpha_P(t)-1)^2\varphi_{22}^J(t)\,|_{t=0}.$$

If the function $\varphi_{22}^J(t)$ has no singularity capable of compensating for the factor $(\alpha p(t) - 1)^2$ at $t = 0$, then the total cross section for $\gamma\gamma$ scattering would vanish at high energies, whereas the elastic cross section would differ from zero, since the differential cross section for elastic scattering would differ from zero at any rate in some interval $t < 0$. This contradicts the obvious inequality $\sigma_{tot}^{\gamma\gamma} \geq \sigma_{el}^{\gamma\gamma}$, which is the consequence of the unitarity relation in the s-channel. It is therefore necessary that the function $\varphi_{22}^J(t)$ have a second-order pole at $t = 0$.

From the integral representation for $f_{22}^J(t)$ we see that as $t \to 0$ the function $\varphi_{22}^J(t)$ has the usual threshold behavior t^J, which in the asymptotic expression for the invariant functions cancels out the factor t^{-J} due to the d-functions. Such a behavior agrees also with the threshold behavior of $g_{02}^J(t)$, for as $t \to 0$ we have $\psi_{02}^J(t) \sim t^{J/2}$. Therefore, $\varphi_{22}^J(t)$ can become infinite at $t = 0$ only as a result of an expression that contains both J and t, that is, as a result of a moving singularity, if we exclude the presence of an essential singularity at $J = 1$, for example, the accumulation of poles. The character of this singularity, generally speaking, can be quite complicated, but if we wish to remain within the framework of the moving-pole picture, we must assume the existence, in addition to the P-pole, of a pair of moving poles which pass through the point $J = 1$ at $t = 0$.

Let us see whether the foregoing behavior of $f_{22}^J(t)$ near $J = 1$ contradicts the integral representation for $f_{22}^J(t)$. From (2) and from the expression

$$f_{22}^J(t) = (J-1)\,\varphi_{22}^J(t)/(J-\alpha_P(t))$$

we see that the function $\varphi_{22}^J(t)$, which one might think should have no standing poles, has a second-order pole at $J = 1$. From this point of view, the first term in (2) is particularly dangerous since the integrand contains a combination of invariant functions which has a definite sign at $t = 0$.

In order to answer the question, we put $J = 1$ in (2). At this point, if we use the Regge asymptotics, the representation certainly exists for $t < 0$. However, in this region, the function under the integral sign is, generally speaking, not positive-definite, so that the integral can vanish and there may be no standing pole. On the other hand, at the point $t = 0$, when the integral sign contains the half-sum of the total cross section for the scattering of protons with like and unlike polarizations (accurate to a multiplier), that is, a positive-definite quantity, the integral diverges logarithmically and the representation is meaningless. When $t = 0$ the integrand contains, generally speaking, a sign-definite function, so that with respect to unitarity in the t channel the complete system comprises a state which can contain only photons, and all the invariant functions will have a singularity at $t = 0$. Consequently there can again be no standing pole.

It is sensible to expect this second-order pole to be in fact moving and to ensure the self consistency of the theory—the total cross section does not vanish and does not become smaller than the elastic cross section. We note that if we do not admit intermediate states that contain only photons in the unitarity conditions of the t-channel, then $\varphi_{22}^J(t)$ will most likely have a standing pole at $J = 1$, since the expression

$$A\,[F_1^+(s,t) + F_2^+(s,t) - 2F_3^+(s,t)$$

will be positive definite in the region $0 \leq t < t_0$ up to the first Karplus singularity. But in this case the pole will no longer contradict the conditions of unitarity in the t-channel, and the analysis of photon scattering at high energies will not differ in principle from the analysis of the scattering of other particles with spin, for example nucleons.

It is natural to regard these two additional poles, the introduction of which is dictated by the need of avoiding contradiction in the analysis of $\gamma\gamma$ scattering from the point of view of the Regge poles, not simply as singularities of the partial wave of $\gamma\gamma$ scattering in the t channel, but singularities in the complex J plane of the partial amplitudes of all possible processes which combine with the vacuum poles, including the partial waves of strongly interacting particles.

It must be noted that the need for introducing, in addition to the P pole, two other poles that pass through $J = 1$ when $t = 0$ has arisen in the analysis of photons—particles with spin 1 and mass 0. If we consider the scattering of vector mesons, that is, particles having the same spin 1 but with nonzero mass, then there would be no need for introducing these two additional poles within the framework of the theory proper, since the factor $(\alpha_p(t) - 1)^2$, which causes the total scattering cross section of the photons to vanish if no additional poles are introduced, would occur for such a combination of invariant functions, which would not determine for $t = 0$ the total cross section (the total cross section is determined by the partial wave with zero spin projection on the direction of the relative motion of the particle), and the question of introducing these poles could be solved only by experiment.

4. Thus, is it possible to introduce in non-contradictory fashion three vacuum poles that cross at the point $J = 1$ when $t = 0$ in a system of strongly interacting particles? It is clear that a third-order pole moving along the P-trajectory cannot be introduced, for although the total cross section is found to increase like $\ln^2 s$, which does not contradict the Froissart conditions, the elastic cross section increases like $\ln^3 s$ and exceeds the total cross section at high energies. Three poles crossing at the point $J = 1$ when $t = 0$, having for $t < 0$ real trajectories, are not suitable for the same reason. The point is that, although for large s and for $t < 0$ the amplitude is determined by a single P-pole, all three poles contribute to the elastic cross section, expressed in terms of the square of the amplitude, for effective $t \sim 1/\ln s$. Therefore, in order not to arrive at the aforementioned contradiction, we must decrease the effective momentum transfer; it is sufficient, in particular, to have $t_{\mathrm{eff}} \sim \ln^{-2} s$.

For example, the following picture is possible: one pole has the usual P-trajectory; the others collide at the point $J = 1$ for positive t tending to zero, and become complex conjugate for $t < 0$. If the real part of their trajectories is to the left of the third pole for $t < 0$, then this P-pole remains the principal pole for large s and for small non-zero momentum transfer.

If we disregard the imaginary parts of the trajectories for $t > 0$, which generally speaking are due to the fact that the threshold of the different processes in the t-channel is equal to zero, the partial amplitude in the t-channel near the point $J = 1$ can be represented in the form

$$f^J(t) = \varphi^J(t)/(J - 1 - \gamma t) \times (J - 1 - \gamma_1 t - \sqrt{\beta t})\,(J - 1 - \gamma_1 t + \sqrt{\beta t}),$$

where $\gamma_1 > \gamma > 0$, $\beta > 0$, and the function $\varphi^J(t)$ has no singularities near $J = 1$.

The invariant amplitude determining the total cross section will have, for large s and for negative near-zero t, the form

$$A(s, t) \sim \frac{s}{-t[\beta - t(\gamma_1 - \gamma)^2]}\Big\{e^{t\gamma \ln s} - e^{t\gamma_1 \ln s}\cos\sqrt{-t\beta \ln^2 s} + \frac{(\gamma_1 - \gamma)t}{\sqrt{-t\beta}} e^{t\gamma_1 \ln s}\sin\sqrt{-t\beta\ln^2 s}\Big\}. \quad (3)$$

From (3) we see, first, that $t_{\mathrm{eff}} \sim \ln^{-2} s$ and, second, for any non-zero momentum transfer there are such s for which the scattering amplitude is determined by only one principal P-pole.

Putting for simplicity $\gamma_1 = \gamma$, we can readily obtain by direct calculation that $\sigma_{\mathrm{el}} \sim \beta^{-1}\ln^2 s$ whereas $\sigma_{\mathrm{tot}} \sim \ln^2 s$. This means that for sufficiently large β (the poles diverge sufficiently rapidly in the complex plane after collision) the theory becomes noncontradictory: $\sigma_{\mathrm{tot}} > \sigma_{\mathrm{el}}$.

Since for $t < 0$ and large s only one pole makes a contribution to the elastic amplitudes, all formulas that follow from the factorization of the residues of the different processes remain in effect and only for $t = 0$, that is, for the total cross sections, do the factorization formulas acquire an inessential factor, namely

$$\sigma_{NN} = \tfrac{1}{2} a^2 \operatorname{res} f^J_{NN}(t)|_{t=0} \ln^2 s,$$

$$\sigma_{N\gamma} = ab \operatorname{res} f^J_{N\gamma}(t)|_{t=0} \ln s,$$

$$\sigma_{\gamma\gamma} = b^2 \operatorname{res} f^J_{\gamma\gamma}(t)|_{t=0}.$$

Consequently $\sigma_{NN}\sigma_{\gamma\gamma} = \tfrac{1}{2}\sigma^2_{N\gamma}$, which differs by a factor $\tfrac{1}{2}$ from the corresponding formula obtained earlier (see [2,3]).

Analogously, for the total cross sections of the scattering of an arbitrary particle A and a photon at high values of s, the following factorization formula is valid

$$\sigma_{AA}\sigma_{\gamma\gamma} = \tfrac{1}{2}\sigma^2_{A\gamma}.$$

Thus, an examination of the scattering of photons at high energies, from the point of view of the Regge poles, without contradicting the conditions of unitarity in the s- and t-channels and under the condition that the total system admit of states containing at least only two photons, leads to a singularity in the complex J plane which ensures the maximum interaction strength allowed by analyticity and unitarity for strongly interacting particles.

In conclusion, I am deeply grateful to I. Ya. Pomeranchuk for guidance of this work and to V. B. Berestetskiĭ, V. N. Gribov, and L. B. Okun' for numerous valuable discussions.

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Translated by J. G. Adashko
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