SLOW π^+-MESON CAPTURE BY LIGHT NUCLEI IN THE CORRELATIONAL NUCLEAR MODEL

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Absorption of slow π^+-mesons by light nuclei in the $\pi^+ + A \rightarrow A' + 2p$ reaction is considered. It is shown that an investigation of the proton spectra permits one to study short range pair correlation between nuclear nucleons. Conditions are indicated under which the corresponding experiments should be carried out.

WE consider the reaction of capture of a slow positive pion by a nucleus with emission of two nucleons (for example, protons) of high energy, so that the product nucleus is obtained in the ground state or in weakly excited states:

$$\pi^+ + A \rightarrow A' + 2p. \quad (1)$$

If the kinetic energy of the meson is much smaller than mc^2 (m is the pion mass), then the reaction (1) is accompanied by a large momentum transfer, so that its cross section is directly related to the character of short range pair correlations between the nucleons of the nucleus. Under the conditions indicated above, reaction (1) can be recorded as occurring in accordance with the direct mechanism wherein a pion is captured by a pair of nucleons, and the Born approximation can be used in view of the high kinetic energy of the nucleons.

It will be shown in the article that if the nucleus A is chosen with zero isotopic spin T_A, then a study of the energy distributions in transitions to a state of the nucleus A′ with T′ = 0 and T′ = 1 makes it possible to determine the parameters of the pair correlation functions in triplet and singlet states.

The restriction to reaction (1) is not one of principle, but is connected with the ease of its experimental observation. Further investigation will be based on the reaction

$$C^{12} + \pi^+ \rightarrow B^{10} + 2p \quad (1')$$

as an example, and we shall be interested in final states of B^{10} belonging to the configuration [42] D_I (in terms of LS coupling).

1. In the model that includes short-range pair correlations due to the singular repulsive character of the nucleon-nucleon forces at small distances we can write the wave function of the ground state of the nucleus in the form:[1]

$$\Phi = \Phi_{ipm} + \sum_{i>j} \sum_{s} \{z_s\}^{-1} \hat{q} \chi_{ij}^S \Phi_{ipm}, \quad (2)$$

where Φ_{ipm}—wave function of the ground state in the independent particle model, χ^S—pair correlation functions; the symbol S indicates that χ^S may depend on the total spin S of the correlating nucleon pair. The remaining symbols and the requirements which must be satisfied by the correlation functions χ^S are dealt with in [1].

Only the second term of (2), which contains the correlation functions, makes a contribution to the reaction considered by us.

The positive pion-nucleon interaction operator can be represented in the nonrelativistic approximation in the form[2]

$$H = G\tau\sigma\mathbf{p}_0\varphi(\mathbf{r}), \quad (3)$$

where G—coupling constant, τ—operator transforming the neutron into a proton, σ—Pauli matrices, p_0—relative momentum of the positive pion and nucleon, and $\varphi(\mathbf{r})$—wave function of the positive pion. The term quadratic in φ has been left out of (3), since it does not lead to capture. Summing over all the nucleons of the nucleus and taking the definition of the relative momentum into account, we obtain for the operator of interaction between the π^+ meson and the nucleus:

$$H' = G' \Big[M \sum_i \tau^i \sigma^i \nabla\varphi \Big|_{\mathbf{r}=\mathbf{r}_i} - m \sum_i \varphi(\mathbf{r}_i) \tau^i \sigma^i \nabla_{\mathbf{r}_i} \Big], \quad (4)$$

where $G' = -i\hbar(M + m)^{-1}G$, M—nucleon mass, m—pion mass, $\mathbf{r}_i$—coordinates of nucleons of the nucleus.

It is easy to see that if the initial nucleus has the same parity as the states of the final nucleus of interest to us, then the first term of H′ leads to capture of pions from spatially odd states, whereas the second term leads to capture from spatially even states. Actually, capture from states with orbital angular momenta 0 and 1 predominates.

2. Let us proceed to calculate the matrix element of the first term of the operator (4), which we denote by H_1. For the pion wave function it is necessary to choose the part of the scattering-problem wave function corresponding to orbital angular momentum $l = 1$:

$$\varphi_1(\mathbf{r}) = \frac{i}{k_0}\sqrt{\frac{3}{4\pi}}\, e^{i\delta_1} \sum_m D^1_{m0}(\alpha, \beta)\, \varphi_1(r)\, Y_{1m}(\theta, \varphi), \quad (5)$$

where k_0—momentum of π^+ meson and δ_1—phase shift in the p state; we have transformed to a coordinate system with z axis directed along the total momentum **K** of the final-state protons.

To calculate $\nabla\varphi_1(r)$ we can use the formula

$$\nabla_\mu \Phi_L(r)\, Y_{LM}(\mathbf{r}_0) = -\sqrt{(L+1)\,(2L+1)}\, D_L^+ \Phi_L\, (\mathbf{T}^M_{L,L+1})_\mu + \sqrt{L/(2L+1)}\, D_L^- \Phi_L\, (\mathbf{T}^M_{L,L-1})_\mu,$$

$$D_L^+ \equiv d/dr - L/r, \quad D_L^- \equiv d/dr + (L+1)/r,$$

$$\mathbf{T}^M_{L,\lambda} = \sum_\mu G^{LM}_{1-\mu\lambda M+\mu} Y_{\lambda M+\mu}\, \boldsymbol{\xi}(\mathbf{r}_0)_{-\mu}, \quad (6)$$

where the components of the unit vector ξ are determined in accordance with (2.36) of [3]. The scalar product is defined here as follows:

$$\boldsymbol{\sigma}\,\nabla = \Sigma\,(-)^\mu \sigma_{-\mu}\nabla_\mu,$$

where

$$\sigma_1 = -\sqrt{1/2}\,(\sigma_x + i\sigma_y),\ \sigma_{-1} = \sqrt{1/2}\,(\sigma_x - i\sigma_y) \text{ and } \sigma_0 = \sigma_z.$$

At small distances we have $\Phi_L \sim r^L$, so that as $r \to 0$ we have $D_L^\pm \Phi_L \to 0$, and for $L > 1$, $D_1^+\Phi_1 \to 0$ and $D_1^-\Phi_1 \to$ const. Since the radii of the light nuclei are small compared with the Bohr radius $\hbar^2/mZe^2$, then we retain in (6) only the term with $D_1^-\varphi_1(r)$ and then replace its magnitude by its effective value in the region of the nucleus, $\overline{D_1^-\varphi_1(r)}$. Questions connected with the determination of $\overline{D_1^-\varphi_1(r)}$ and the concomitant errors will be discussed at the end of the article.

We thus have

$$H_1 \to A_1 \overline{D_1^-\varphi_1(r)} \sum_i \sum_m D^1_{m0}(\alpha, \beta)\, \tau^i \sigma^i_m,$$

$$A_1 = \frac{i}{4\pi k_0} G' M e^{i\delta_1}. \quad (7)$$

We now must calculate the matrix elements $\langle \tau^i \sigma^i_m \rangle$ of the operators $\tau^i \sigma^i_m$ with respect to spin and iso-spin functions. Since H_1 does not contain operators acting on the orbital wave functions of the nucleons, and the correlation function χ^S acts on nucleon pairs which are in a relative s state, the two protons will have in the final state a zero orbital angular momentum of relative motion, so that their total spin should be $S_0 = 0$. Using the matrix elements given in the Appendix, we obtain $\langle \tau^i \sigma^i_m \rangle$ in the form

$$2\delta_{S'1} C^{T'0}_{1/2\,-1/2\,1/2\,1/2} \begin{cases} \delta_{\Sigma'0}\delta_{T'1} & m = 0 \\ -\delta_{\Sigma',-m}\delta_{T'0} & m = \pm 1 \end{cases},$$

where the primed quantities pertain to the initial state of the nucleon pair absorbing the pion. If $S' = 1$, then from the requirement that the wave functions be antisymmetrical we get that $T' = 0$. Therefore

$$\langle H_1 \rangle = -\sqrt{2} A_1 \overline{D_1^- \varphi_1} \delta_{S'1}\delta_{T'0} \sum_{m=\pm 1} D^1_{m0}(\alpha, \beta)\, \delta_{\Sigma',-m}. \quad (8)$$

We now proceed to calculate the matrix element of $\langle H_1 \rangle$ with respect to the orbital functions. The wave function of the C^{12} nucleus can be represented in the following form 1):

$$\Phi_{ipm}(C^{12}) = \sum_{\xi'} R_{[\lambda']L'S'T'} (\hat{L}'\hat{S}'\hat{T}')^{-1} (-)^{L'+S'+T'} \sum (-)^{\Lambda'+\Sigma'+M'_T} \times \Phi(p^6\, [\lambda']\, L'S'T';\, -\Lambda' - \Sigma' - M'_T)\, |\, \xi'\rangle,$$

$$\hat{x} \equiv \sqrt{2x+1}, \quad (9)$$

where $R_{[\lambda']L'S'T'}$ is the fractional parentage coefficient for the separation of two nucleons in the state $\xi' \equiv L'S'T'$, $\Phi(p^6[\lambda']\, L'S'T';\, -\Lambda' - \Sigma' - M'_T)$—wave function of A-2 nucleons, and $|\xi'\rangle$—wave function of the separated pair.

Applying next the Talmi transformation 2) (see [1]) and recognizing that we are interested in final states of the nucleus of the type [42] D_I, we obtain

$$\Phi_{ipm}(C^{12}) \to -\sqrt{\frac{1}{30}} \sum_{S'T'} R_{[42]2S'T'} \sum_{\Lambda'\Sigma'} (-)^{\Lambda'+\Sigma'} \times \Phi(p^6\, [42]\, 2S'T';\, -\Lambda' - \Sigma' 0)\, \varphi_0(\mathbf{r})\, \Phi_{2\Lambda'}(\mathbf{R})\, X_{S'\Sigma'} \Omega_{T'0}. \quad (10)$$

The wave function of the final state should correspond to a state with total angular momentum $J = 1$. Therefore

$$\Psi^{1M}_j = \sum_{\mu\Lambda'} C^{1M}_{j-\mu 2\Lambda'} C^{j\mu}_{1\Sigma' 2\Lambda'}\, \Phi(p^6\, [42]\, 2S'T';\, -\Lambda' - \Sigma' 0) \times \Psi_{2\Lambda'}(\mathbf{R})\, \psi_0(\mathbf{r})\, X_{00}\Omega_{11}, \quad (11)$$

where j—total angular momentum of the final state of the product nucleus, $\psi_{2\Lambda'}(\mathbf{R})$ and $\psi_0(\mathbf{r})$—plane-

1) We use here the equality $C^{00}_{j\mu j-\mu} = \hat{j}^{-1}(-)^{j-\mu}$.

2) The one-particle wave functions are chosen in the form of oscillator functions.

wave components corresponding to the orbital angular momenta 2 and 0.

Gathering formulas (2) and (8)—(11) together, we obtain the transition matrix element for capture of a positive pion from the p state in the reaction $C^{12}+\pi^+ \rightarrow B^{10}+2p$ in the following form:

$$T_j^{1M} = \sqrt{\frac{1}{45}}A_1\overline{D_1^-\varphi_1}\,\hat{j}R_{[42]210}D^1_{M0}(\alpha,\beta)\,\delta_{M,\pm1}f_1^1(\varepsilon)\,\mathfrak{M}(E), \tag{12}$$

where $\mathfrak{M}(E)$ is the radial matrix element in the coordinate $\mathbf{R}$ (E—kinetic energy of the center of gravity of the two protons) and

$$f_1^{S'}(\varepsilon) = \sqrt{4\pi}\int_0^\infty j_0(kr)\,\hat{q}\chi^{S'}(r)\,\varphi_0(r)\,r^2dr,\quad S'=0,1 \tag{12'}$$

(ϵ—relative kinetic energy: $k=\hbar^{-1}\sqrt{M\epsilon}$).

We note that, as follows from the derivation of (12), mesons are captured in the p state only by triplet-correlated nucleon pairs.

3. Let us consider the calculation of the matrix elements of the second term of (4), which we denote by H_2, for the capture of a pion from an s state. Introducing the effective value of the pion wave function in the region of the nucleus, $\overline{\varphi_0(r)}$, we obtain

$$H_2 \rightarrow -A_0\overline{\varphi_0(r)}\sum_i\tau^i\sigma^i\nabla_{r_i},\quad A_0=\frac{1}{4\pi k_0}mG'e^{i\delta_0}, \tag{13}$$

where δ_0—phase shift in the s state and $\varphi_0(r)$—corresponding radial function. Going over to new coordinates

$$\rho=\mathbf{r}_i-\mathbf{r}_j,\quad \rho_\alpha=\mathbf{r}_\alpha-\frac{1}{A-2}\sum\mathbf{r}_\alpha,$$
$$\mathbf{R}=\frac{1}{2}(\mathbf{r}_i+\mathbf{r}_j)-\frac{1}{A-2}\sum\mathbf{r}_\alpha,\quad \alpha\neq i,j,$$

we represent (13) in the form

$$H_2(ij) = -A_0\overline{\varphi_1(r)}\,[(\tau^i\sigma^i-\tau^j\sigma^j)\nabla_\rho+\tfrac{1}{2}(\tau^i\sigma^i+\tau^j\sigma^j)\nabla_{\mathbf{R}}]. \tag{13'}$$

We calculate first the matrix element of the first term of (13′). In view of the presence of the del operator, two protons in the final state will be in a relative-motion state, and therefore $S_0=1$. As can be seen from the formulas in the Appendix, the possible values are $S'=0$ and $S'=1$. Let us consider the case $S'=0$, and consequently $T'=1$. Then the matrix element with respect to spin and isospin functions will be equal to

$$-\sqrt{2}\sum_{\mu=\pm1}\delta_{\Sigma_0,-\mu}\nabla_\mu. \tag{14}$$

The wave function of the final state should correspond now to the total angular momentum of the system $J=0$:

$$\Psi_{J=0}=\sqrt{\frac{1}{10}}(-)^{j'}\sum_{\Lambda' m}(-)^{\Lambda'}C^{2\Lambda'}_{2\Lambda' j'0}C^{j'0}_{1\Sigma_0\,1m}D^1_{m0}(\alpha',\beta')$$
$$\times\,\Phi(p^6\,[42]\,201;\,-\Lambda'00)\,\psi_{2\Lambda'}(\mathbf{R})\,\phi_{1m}(\mathbf{r})\,X_{1\Sigma_0}\Omega_{11}, \tag{15}$$

where $D^1_{m0}(\alpha',\beta')$ arise because of the rotation of the relative-motion wave function to a coordinate system having a z axis directed along the total momentum $\mathbf{K}$ of the protons.

Using (2), (6), (10), and (13)—(15) we obtain the matrix element for the capture of a positive pion from the s-state by an np pair with $S'=0$ and $T'=1$:

$$T_{\rm I}=\sqrt{\frac{1}{54}}A_0\overline{\varphi_0(r)}\,R_{[42]201}\sum_{m=\pm1}D^1_{m0}(\alpha',\beta')\,f_2^0(\varepsilon)\,\mathfrak{M}(E), \tag{16}$$

where

$$f_2^{S'}(\varepsilon)=i\sqrt{12\pi}\int j_1(kr)\frac{d}{dr}\hat{q}\chi^{S'}(r)\,\varphi_0(r)\,r^2dr. \tag{16'}$$

We note that by integrating (16′) by parts and using the relation $(x^2j_1(x))'=x^2j_0(x)$, we can obtain the connection between $f_1^{S'}(\epsilon)$ and $f_2^{S'}(\epsilon)$:

$$f_1^{S'}(\varepsilon)=i\hbar\,(3M\varepsilon)^{-1/2}f_2^{S'}(\varepsilon), \tag{17}$$

where M—mass of the nucleon.

We shall not present the detailed calculations for the case when $S'=1$ and $T'=0$. The end result has the following form:

$$T^j_{\rm II}=A_0\overline{\varphi_0(r)}\,[T_j^{(1)}+T_j^{(2)}], \tag{18}$$

$$T_j^{(1)}=\frac{2}{9\sqrt{10}}\hat{j}R_{[42]210}D^1_{00}(\alpha',\beta')\,f_2^1(\varepsilon)\,\mathfrak{M}(E), \tag{18'}$$

$$T_j^{(2)}=\frac{2}{9\sqrt{10}}\hat{j}R_{[42]210}\sum_m mD^1_{m0}(\alpha',\beta')\,f_2^1(\varepsilon)\,\mathfrak{M}(E). \tag{18''}$$

Using analogous derivations, we can readily verify that the matrix element of the second term of (13′) is equal to zero when $j=2$; we shall henceforth be interested in precisely such transitions.

4. Using (2), (16), and (18) and integrating over the angles of the vectors $\mathbf{k}$ and $\mathbf{K}$, we obtain the following expressions for the π^+-meson capture cross sections $d\sigma_{S'T'j}$ (for $j=2$):

$$d\sigma_{102}=\frac{28\,(4\pi)^3}{27\hbar^2k_0}mG'^2\,|R_{[42]210}\mathfrak{M}(E)|^2\Big\{|A_1'\overline{D_1^-\varphi_1}f_1^1(\varepsilon)|^2$$
$$+\frac{1}{3}|A_0'\overline{\varphi_0(r)}\,f_2^1(\varepsilon)|^2\Big\}\frac{(M\mu)^{3/2}}{\hbar^6\sqrt{2}}\sqrt{E\varepsilon}\,dE\,d\varepsilon, \tag{19}$$

$$d\sigma_{012}=\frac{28\,(4\pi)^3}{162\,\hbar^2k_0}mG'^2\,|R_{[42]201}\mathfrak{M}(E)|^2\,|A_0'\overline{\varphi_0(r)}\,f_2^0(\varepsilon)|^2$$
$$\frac{(M\mu)^{3/2}}{\hbar^6\sqrt{2}}\sqrt{E\varepsilon}\,dE\,d\varepsilon, \tag{20}$$

where

$$A_1'\equiv G'^{-1}A_0,\quad A_0'\equiv G'^{-1}A_0\ \text{and}\ \mu^{-1}=M^{-1}_{A-2}+(2M)^{-1}$$

(M_{A-2}—mass of the reaction product nucleus).

Let us see now what information concerning the correlation functions $\chi^{S'}$ can be gained from the experimental data on the values of $d\sigma_{102}$ and $d\sigma_{012}$.

In view of the fact that the pair-correlation radii are assumed small, the quantities $f_2^{S'}(\epsilon)$ can be represented in the form

$$f_2^{S'}(\varepsilon) \approx i\sqrt{12\pi}\,\varphi_0(0)\int j_1(kr)\frac{d}{dr}\hat{q}\chi^{S'}(r)\,r^2dr. \qquad (21)$$

Expanding $j_1(kr)$ in powers of kr, we obtain [3)]

$$f_2^{S'}(\varepsilon) \approx i\sqrt{12\pi}\,\varphi_0(0)\,[k\xi_1^{S'} + k^3\xi_2^{S'} + k^5\xi_3^{S'} + \dots],$$

$$\xi_1^{S'} = \frac{1}{3}\int\frac{d}{dr}\chi^{S'}r^3dr, \quad \xi_2^{S'} = -\frac{1}{30}\int\frac{d}{dr}\chi^{S}r^5dr \dots \qquad (22)$$

It is obvious that the set of quantities $\xi_n^{S'}$ yields complete information on the pair correlation function $\chi^{S'}(r)$. If $kr_{cor}^{S'} \lesssim 2$ (where $r_{cor}^{S'}$ is the correlation radius; when $r_{cor}^{S'} \lesssim 1.3$ F, the condition above corresponds to $\epsilon \lesssim 100$ MeV), then it is sufficient to retain in (22) the first two terms, and consequently it is possible to extract from the experimental data two parameters characterizing the function $\chi^{S'}(r)$. In addition, we know the boundary conditions [1], namely $\chi^{S'} \to -1$ as $r \to 0$ and $\chi^{S'} \to 0$ as $r \to \infty$.

Further, using oscillator wave functions, we can readily find that $|\varphi_0(0)|^2 = \nu^{3/2}\sqrt{2/\pi}$ and

$$|\mathfrak{M}(E)|^2 = \frac{8}{15\sqrt{2\pi}}\nu^{-3/2}e^{-2E/\hbar\omega}\left(\frac{2E}{\hbar\omega}\right)^2, \qquad (23)$$

where $\nu = M\omega/\hbar \approx R_0^{-2}$ (R_0 is the nuclear radius). Substituting (22) and (23) in (19) and (20), and comparing $d\sigma/d\epsilon$ with the experimental data, we obtain the values of $\xi_1^{S'}$ and $\xi_2^{S'}$. Choosing for $\chi^{S'}$ some arbitrary function with two independent parameters $\beta_1^{S'}$ and $\beta_2^{S'}$, we can determine these parameters from the already known quantities $\xi_1^{S'}$ and $\xi_2^{S'}$. Of course, the arbitrariness in the choice of the form of $\chi^{S'}(r)$ is limited by the boundary conditions indicated above.

We note that in order to determine the parameters $\beta_1^{S'}$ and $\beta_2^{S'}$ it is sufficient to know just the form of the spectrum $d\sigma/d\epsilon$, the absolute value of the cross section not being needed. Indeed, expression (20) can be represented in the form (we are leaving out the symbol S')

$$d\sigma_{012}/d\varepsilon = c\,[\varepsilon\varkappa\xi_1^2 + 2\varepsilon^2\varkappa^2\xi_1\xi_2 + \varepsilon^3\varkappa^3(\xi_2^2 + 2\xi_1\xi_3)]\sqrt{\varepsilon E}\,|\mathfrak{M}(E)|^2,$$

where $\kappa = \hbar^{-2}M$ and $\epsilon_m = E + \epsilon$—maximum value of ϵ allowed by energy and momentum conservation laws; c is some unknown constant. From comparison with the experimental curve we obtain the system of equations

$$c\varkappa\xi_2^2(\beta_1\beta_2) = a_1, \quad 2c\varkappa^2\xi_1(\beta_1\beta_2)\,\xi_2(\beta_1\beta_2) = a_2,$$

$$c\varkappa^3[\xi_2^2(\beta_1\beta_2) + 2\xi_1(\beta_1\beta_2)\,\xi_3(\beta_1\beta_2)] = a_3, \qquad (24)$$

from which we can determine the parameters β_1 and β_2 by eliminating the quantity c. An analogous procedure can be applied also to (19), after which we obtain a system of four equations for β_1, β_2, and the two unknown constants. It follows therefore that to determine the parameters of the correlation functions it is not necessary to know the values of $\overline{\varphi_0(r)}$ and $\overline{D_1^{-}\varphi_1(r)}$. The foregoing procedure calls for high experimental accuracy, since it is based essentially on a comparison of the quantities ξ_2^2 and $2\xi_1\xi_3$.

5. Let us proceed to an examination of the effective values of $\overline{\varphi_0(r)}$ and $\overline{D_1^{-}\varphi_1(r)}$. From the experimental data on the shift of the s level of the π-mesic atom it is known [4] that the interaction between the pion and the nucleus is repulsive and the scattering length is appreciably shorter than the nuclear dimensions. If we approximate the potential in the s-state by a rectangular well, we obtain from the data previously obtained [4] for the C^{12} nucleus that $V_0 \approx 5$ ($\pm 20\%$) MeV. The total potential acting on the pion in the region of the nucleus will be $U_0 \approx V_0 + Ze^2/R_0 \approx 8$ MeV. Expanding the wave function in the region $r \leq R_0$ in powers of $k'r$ ($k' = \hbar^{-1}\sqrt{2m|E_0 - U_0|}$) we can easily see that when $E_0 \lesssim 15$ MeV (E_0 is the pion energy) the wave functions for $r = 0$ and $r = R_0$ differ by not more than 1%. Therefore under the limitation imposed above on the pion energy we can put $\overline{\varphi_0(r)} = \varphi_0(R_0)$. The value of the wave function at the boundary of the nucleus can be calculated if the phase shift in the s state is known.

Before we proceed to determine $\overline{D_1^{-}\varphi_1(r)}$, we note that the most essential region of the nucleus is the one in which the density of pion-absorbing nucleons has a maximum. We are interested in the capture by nucleons in the p shell of the nucleus. It is easy to show that in this case the maximum of the density lies at $r_{max} \approx 3R_0/r$. We can therefore define $\overline{D_1^{-}\varphi_1(r)}$ as being equal to $D_1^{-1}\varphi(3R_0/4)$; the resultant errors in the radial integrals do not exceed 1—3%, as shown by calculation.

The potential of interaction between a pion and a nucleus in the p state is not known with any degree of reliability. However, since no bound state of the positive pion is observed even in the heaviest nuclei (this would manifest itself in radiative capture of the π^+ meson by the nucleus), the interaction potential in the p state is bounded by the inequality $|U_1| \leq \pi^2\hbar^2/2mR_0^2$. For $R_0 = 8$ F we obtain from this $U_1 \geq 20$ MeV. Since the intensity of interaction of the meson with the nuclear matter should be proportional to the density of the latter, the estimate obtained for U_1 should remain in force for light nuclei, too.

3) $j_1(x) = \frac{1}{3}x - \frac{1}{30}x^3 + \frac{1}{840}x^5 - \frac{1}{45360}x^7 + \dots$

Expanding the pion wave function in the region $r < R_0$ in powers of $k'r$ ($k' = \hbar^{-1}\sqrt{2M(E_0 - U_1)}$), we can show that by calculating the value of $D_1^- \varphi_1(r)$ for $r \geq R_0$ (for which we must know the phase shift in the p state) and by extrapolating linearly in the region $r < R_0$ to the point r_{max}, we obtain an error not exceeding 3%, if we confine ourselves to energies $E_0 \leq 10$ MeV.

We also estimated the contributions of the matrix elements discarded in Sections 2 and 3 above. When $E_0 \leq 10$ MeV, their total contribution does not exceed 2–3% of the calculated value; the estimate presented here has been obtained for a purely Coulomb nuclear field.

We can therefore conclude that the approximations made during the course of deriving (19) and (20) are sufficiently good if we confine ourselves to $E_0 \lesssim 10$ MeV.

6. There is presently under development a theory of direct nuclear reactions, based on the idea that the direct processes are connected with the presence of singularities in the amplitude as a function of momentum transfer [5]. It is assumed here that the role of the particular Feynman diagram used to represent the corresponding amplitudes depends on how close the singularity lies to the physical region of variation of the invariant.

From this point of view it is of interest to investigate the positions of the singularities of various very simple Feynman diagrams which represent the reaction considered in the present article.

APPENDIX

Let us write out the matrix elements of the operators σ_m^i and τ^i $\langle S_0\Sigma_0|\sigma_m^i|S'\Sigma'\rangle$ and $\langle T_0M_{T0}|\tau^i|T'M_T'\rangle$, used in the text:

$$\langle 00|\sigma_m^i|00\rangle = 0, \qquad \langle 1\Sigma|\sigma_m^1|00\rangle = \delta_{\Sigma m},$$

$$\langle 1\Sigma|\sigma_m^2|00\rangle = \begin{cases} \delta_{\Sigma 0} & m = 0 \\ -\delta_{\Sigma m} & m = \pm 1 \end{cases},$$

$$\langle 00|\sigma_m^1|1\Sigma\rangle = \begin{cases} \delta_{\Sigma 0} & m = 0 \\ -\delta_{\Sigma_1, -m} & m = \pm 1 \end{cases},$$

$$\langle 00|\sigma_m^2|1\Sigma\rangle = \delta_{\Sigma_1, -m},$$

$$\langle 1\Sigma|\sigma_m^i|1\Sigma'\rangle = \begin{cases} \Sigma\delta_{\Sigma\Sigma'}, & m = 0 \\ -m\delta_{\Sigma-\Sigma', m}, & m = \pm 1 \end{cases},$$

$$\langle 11|\tau^1|T'0\rangle = C^{T'0}_{1/2\, -1/2\, 1/2\, 1/2},$$

$$\langle 11|\tau^2|T'0\rangle = (-)^{1-T'}\langle 11|\tau^1|T'0\rangle.$$

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116