COHERENT AMPLIFICATION OF SPIN WAVES

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Amplification of spin waves in ferro- and antiferromagnets, based on coherent interaction of a beam of charged particles (electrons) with spin waves is investigated. The amplification is especially appreciable when one of the resonance conditions $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$, $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v} - \omega_B$ is fulfilled, where $\omega_s(\mathbf{k})$ is the frequency of a spin wave with a wave vector **k**, **v** is the particle velocity in the beam and ω_B is the electron cyclotron density. At low particle densities in the beam and at sufficient energy uniformity of the particles, the growth increment is proportional to $n^{1/3}$ if $\omega_s^{(\mathbf{k})} = \mathbf{k}\cdot\mathbf{v}$ and is proportional to $n^{1/2}$ if $\omega_s^{(\mathbf{k})} = \mathbf{k}\cdot\mathbf{v} - \omega_B$.

1. As is well known, oscillations of the magnetic moment in ferromagnets and antiferromagnets propagate at sufficiently low temperatures in the form of weakly attenuating waves (Block spin waves).

The amplitude of these waves is determined under ordinary conditions by the temperature of the body. To investigate the character of the energy spectrum of ferromagnets and antiferromagnets, and also for many applications, it is important to be able to amplify the spin waves.

One of the possibilities of amplifying spin waves is proposed and investigated in the present article. It is based on coherent interaction between streams of charged particles (electrons) and spin waves. This interaction can be realized with the aid of two mechanisms: first, by magnetic forces and, second, by double coupling the electrons with the lattice vibrations and the lattice vibrations with the spin waves. We investigate in the present article only the first interaction mechanism.

The particle streams can be either electron beams from external sources[1)] or currents due to application of an electric field to the specimen.

The strongest growth occurs in spin waves whose frequencies $\omega_s(\mathbf{k})$ (**k** is the wave vector) satisfy one of the resonance conditions $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$ or $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v} - \omega_B$, where **v** is the directed velocity of the beam and ω_B is the electron cyclotron frequency. In order to obtain large amplification coefficients with complete utilization of the coherence effect, it is necessary that the beam particles be highly monoenergetic. In the case of sufficiently small particle density n in the beam, the maximum increment of the spin-wave amplitude is proportional $n^{1/3}$ if $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$ or to $n^{1/2}$ if $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v} - \omega_B$. The relative spread in the velocities $\Delta v/v$ in the beam should be considerably smaller than the ratio of the growth increment to the frequency of the amplified oscillations. If this coherence condition is not satisfied, then the increment will be proportional to n and not to $n^{1/3}$ or $n^{1/2}$. As regards the dependence of the increment on the beam velocity v, it is proportional to $(v/c)^{2/3}$ for a wave with frequency $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$ and to v/c for a wave with frequency $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v} - \omega_B$. When $n \sim 10^{10}$ cm^{-3} and $v/c \sim 10^{-1}$, the relative increment of the amplification of the spin waves can reach values $\sim 10^{-1}$, whereas the relative damping of the spin waves in yttrium iron garnet amounts to $\sim 10^{-2}$—10^{-3}.

2. We start from the linearized Maxwell's equations for the Fourier components ($\sim e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$) of the electric and magnetic fields **E** and **H** and the linearized equations of motion for the Fourier components of the perturbations δn and $\delta\mathbf{v}$ of the density and velocity of the particles in the beam:

$$[\mathbf{kH}] = -\omega\varepsilon c^{-1}\mathbf{E} - 4\pi i e c^{-1}(\delta n\mathbf{v} + n\delta\mathbf{v}),$$

$$[\mathbf{kE}] = \omega c^{-1}\hat{\mu}(\omega, \mathbf{k})\,\mathbf{H}, \qquad (\omega - \mathbf{kv})\,\delta n - n\mathbf{k}\delta\mathbf{v} = 0,$$

$$-i(\omega - \mathbf{kv})\,\delta\mathbf{v} = (e/m)\,\mathbf{E} + (e/mc)\,[\mathbf{v}, \hat{\mu}\mathbf{H}] + (e/mc)\,[\delta\mathbf{v}B_0], \qquad (1)^*$$

where ω—frequency, **k**—wave vector, n and **v**—unperturbed density and velocity of the particles in the beam, B_0—unperturbed magnetic induction,

[1)]The beams, naturally, can pass only through openings in the body. Since we are interested in clarifying only the physics of the phenomena and estimating the magnitude of the effect, we disregard the complications brought about by the finite dimensions of the openings.

*$[\mathbf{kH}] = \mathbf{k}\times\mathbf{H}$.

$\hat{\mu}(\omega, \mathbf{k})$—magnetic permeability tensor, and ε—dielectric constant (in the study of spin waves it is possible to assume for simplicity that ε is a scalar quantity).

The unperturbed velocity $\mathbf{v}$ obviously satisfies the equation $(\mathbf{v}\cdot\nabla)\mathbf{v} = (e/mc)\mathbf{v}\times\mathbf{B}_0$. We consider here the case $\mathbf{v} = \text{const}$ and therefore assume that $\mathbf{v}\times\mathbf{B}_0 = 0$.

We start with consideration of antiferromagnets with sublattice magnetic moments oriented along the selected 3-axis (there is no external magnetic field). The tensor $\hat{\mu}(\omega, \mathbf{k})$ has in this case the simplest structure [1]:

$$\hat{\mu} = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}, \quad \mu_1 = \frac{\Omega_2^2(\mathbf{k}) - \omega^2}{\Omega_1^2(\mathbf{k}) - \omega^2}, \quad \mu_3 = 1; \qquad (2)$$

$$\Omega_1^2 = (gM_0)^2\, 2\delta\, [\beta + (\alpha - \alpha_{12})k^2],$$

$$\Omega_2^2 = \Omega_1^2 + 8\pi (gM_0)^2 [\beta + (\alpha - \alpha_{12})k^2],$$

where g—gyromagnetic ratio, M_0—magnetic moment of sublattices, β—magnetic anisotropy constant, and α, α_{12}, and δ—constants characterizing the exchange interaction in the antiferromagnet[2)] (see, for example,[3]).

Using (1) and (2) we obtain the dispersion equation

$$\begin{aligned} &\{((\omega - \mathbf{kv})^2 - \Omega^2/\varepsilon)(\omega^2\varepsilon\,\mu_1/c^2 - k^2 - \mu_1\Omega^2/c^2) \\ &\quad - \mu_1\Omega^2\,[\mathbf{kv}]^2/c^2\} \\ &\quad \times (\omega^2\varepsilon\mu_1/c^2 - k_3^2 - k_\perp^2\,\mu_1/\mu_3 - \mu_1\Omega^2/c^2) \\ &\quad + (\mu_1/\mu_3)(\mu_3 - \mu_1)\,\Omega^2 k^2\,(\mathbf{n}\,[\mathbf{kv}])^2/c^2 = 0, \end{aligned} \qquad (3)$$

where $\mathbf{n}$—unit vector along the 3-axis, $\Omega = \sqrt{4\pi ne^2/m}$ is the plasma frequency of the beam, and $k_\perp^2 = k^2 - k_3^2$.

Putting $\Omega = 0$, in (3) we obtain the equation

$$(\omega - \mathbf{kv})^2\,(\omega^2\varepsilon\mu_1/c^2 - k_3^2 - k_\perp^2\mu_1/\mu_3)\,(\omega^2\varepsilon\mu_1/c^2 - k^2) = 0, \qquad (3')$$

which determines the frequencies of the optical and spin waves, and also the frequency $\omega = \mathbf{k}\cdot\mathbf{v}$ of the oscillations connected with the beam. The spin waves of interest to us correspond to those branches of the dispersion equation (3′), for which $\omega/k \ll c/\sqrt{\varepsilon}$ (the condition $\omega/k \ll c/\sqrt{\varepsilon}$ is not satisfied for optical waves).

Let us determine first the influence of the particle beam on the spin waves whose frequency $\omega_1(\mathbf{k})$ satisfies the equation

$$\Delta_1(\omega, \mathbf{k}) \equiv \omega^2\varepsilon\mu_1(\omega, \mathbf{k})/c^2 - k_3^2 - k_\perp^2\mu_1(\omega, \mathbf{k})/\mu_3 = 0. \qquad (4)$$

Using (2), we obtain

$$\omega_1(\mathbf{k}) = \{\Omega_1^2 + 8\pi (gM_0)^2 [\beta + (\alpha - \alpha_{12})k^2]\sin^2\theta\}^{1/2},$$

where θ—angle between $\mathbf{k}$ and $\mathbf{n}$. Putting $\omega = \omega_1(\mathbf{k}) + \xi$, $\omega_1(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$ and assuming that $|\xi| \ll \omega_1(\mathbf{k})$, we obtain from (3)

$$\xi^3 = \Omega^2 \frac{\mu_1(\omega, \mathbf{k})\,k^2}{(\partial\Delta_1/\partial\omega)_{\omega_1}} \frac{(\mathbf{v}\,[\mathbf{nk}])^2}{c^2\,[\mathbf{nk}]^2}.$$

From this it is to determine the increment of the amplitude of the spin wave with frequency $\omega_1(\mathbf{k})$

$$\eta_1 = |\operatorname{Im}\xi| = \frac{\sqrt{3}}{2}\left(\frac{\Omega}{\omega_1}\right)^{2/3} \frac{(\Omega_2^2 - \Omega_1^2)^{1/3}}{\omega_1^{2/3}}\,\omega_1 \left| \frac{(\mathbf{v}\,[\mathbf{nk}])}{c\,[\mathbf{nk}]}\cos\theta \right|^{2/3}. \qquad (5)$$

(In the derivation of this formula we have assumed that $\Omega^2/\varepsilon \ll \eta^2$.)

From the resonance condition $\omega_1(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$ it follows that the beam velocity v must exceed a certain critical value v_c

$$v_c = \min\,[\omega_1(\mathbf{k})/k],$$

where k_i lies in the interval $(-\pi b_i, \pi b_i)$ and b_i—reciprocal lattice constants. Assuming that (4) is valid up to $k \sim a^{-1}$, we can readily verify that

$$v_c \approx gM_0\,(2\delta\,(\alpha - \alpha_{12}))^{1/2}. \qquad (6)$$

If $v\cos\theta \gg v_c$ (θ—angle between $\mathbf{k}$ and $\mathbf{v}$), then spin waves will be excited with frequency $\omega_0 = gM_0\sqrt{2\beta\delta}$ and wave vector $k = \omega_0/(v\cos\theta)$. The increment of these waves is, according to (6),

$$\eta_1 = \frac{\sqrt{3}}{2}\,\omega_0\left(\frac{\Omega}{\omega_0}\right)^{2/3}(8\pi\beta)^{1/3}\left(\frac{gM_0}{\omega_0}\right)^{2/3}\left|\frac{(\mathbf{v}\,[\mathbf{nk}])^2}{c^2\,[\mathbf{nk}]^2}\cos^2\theta\right|^{1/3}. \qquad (7)$$

Analogously we can obtain the increment of the spin-wave amplitude, with a frequency $\omega_2(\mathbf{k})$ that satisfies the equation

$$\Delta_2(\omega, \mathbf{k}) \equiv \omega^2\varepsilon\mu_1(\omega, \mathbf{k})/c^2 - k^2 = 0. \qquad (8)$$

Using (2), we get

$$\omega_2(\mathbf{k}) = [\Omega_1^2 - \Omega_1^2\varepsilon c^{-2}k^{-2}(\Omega_2^2 - \Omega_1^2)]^{1/2} \approx \Omega_1(\mathbf{k}).$$

Putting $\omega = \omega_2(\mathbf{k}) + \xi$ and $\omega_2(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$, and using (3) we get

$$\begin{aligned} \eta_2 &= |\operatorname{Im}\xi| \\ &= \frac{\sqrt{3}}{2}\,\Omega_1\left(\frac{\Omega_2^2 - \Omega_1^2}{2\Omega_1^2}\right)^{1/3}\left(\frac{\Omega^2}{\Omega_1^2}\right)^{1/3}\left[\frac{[\mathbf{kv}]^2}{k^2c^2} - \frac{(\mathbf{n}\,[\mathbf{kv}])^2}{k_\perp^2 c^2}\right]^{1/3}. \end{aligned}$$

It is easy to see that the critical velocity which the beam velocity must exceed in order to be able to excite spin waves with frequency $\omega_2(\mathbf{k})$ coincides with the critical velocity for the excitation of spin waves with frequency $\omega_1(\mathbf{k})$.

Since the frequencies ω_1 and ω_2 differ little from each other (the difference between them is due to the relatively weak dipole interaction), it is possible to distinguish them from the excitation of the waves ω_1 and ω_2 only in the case when $\eta \ll |\omega_1 - \omega_2|$. If $v\cos\theta \gg v_c$, then $\omega_2 \approx \omega_0$ and

[2)]In order of magnitude we have $\alpha \sim \alpha_{12} \sim T_N a^2/\mu M_0$ and $\delta \sim T_N/\mu M_0$, where T_N is the Néel temperature, μ the Bohr magneton and a the lattice constant. We note that $\beta \ll \delta$.

$$\eta_2 = \frac{\sqrt{3}}{2}\omega_0\left(\frac{\Omega}{\omega_0}\right)^{2/3}(4\pi\beta)^{1/3}\left(\frac{gM_0}{\omega_0}\right)^{2/3}\left[\frac{[\mathbf{kv}]^2}{k^2c^2} - \frac{(\mathbf{n}\,[\mathbf{kv}])^2}{k_\perp^2 c^2}\right]^{1/3}. \quad (9)$$

This formula can be used if $\eta \ll |\omega_1 - \omega_2|$, where $|\omega_1 - \omega_2| \sim gM_0\sqrt{\beta/\delta}$.

3. We proceed to investigate the interaction between a beam of charged particles and spin waves in ferromagnets. The tensor of magnetic susceptibility of the ferromagnet has the form [3]

$$\hat{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix},$$

$$\mu_1 = \frac{\Omega_e(\Omega_e + 4\pi g M_0) - \omega^2}{\Omega_e^2 - \omega^2}, \quad \mu_2 = \frac{4\pi g M_0 \omega}{\Omega_e^2 - \omega^2}, \quad (10)$$

where $\Omega_e = gM_0(\alpha k^2 + \beta + H_0/M_0)$ and α and β are the exchange interaction and anisotropy constants [3)]. The external field $\mathbf{H}_0$ is applied along the axis of easiest magnetization (3-axis).

Using (10), we can obtain from (1) the following dispersion equation:

$$\omega'^2(\omega'^2 - \omega_B^2)D(\mathbf{k},\omega) + \Omega^2 G(\mathbf{k},\omega)/\varepsilon + (\Omega^2/\varepsilon)^2 G_1(\mathbf{k},\omega) - (\Omega^2/\varepsilon)^3\omega'^2(\mu_1^2 - \mu_2^2) = 0, \quad (11)$$

where $\omega' = \omega - \mathbf{k}\cdot\mathbf{v}$, $\omega_B = eB_0/mc$ and

$$D(\mathbf{k},\omega) = (c^2k^2/\varepsilon - \mu_1\omega^2)\,[c^2k_3^2/\varepsilon - \mu_1(\omega^2 - c^2k_\perp^2/\varepsilon\mu_3)] + \mu_2^2\omega^2(c^2k_\perp^2/\varepsilon\mu_3 - \omega^2),$$

$$\begin{aligned} G(\mathbf{k},\omega) = {} & (\mu_1^2 - \mu_2^2)\{\omega^2(\omega_B^2 - \omega'^2)(\omega^2 - c^2k_\perp^2/\varepsilon\mu_3) \\ & - \omega'^4(2\omega^2 - c^2k_\perp^2/\varepsilon\mu_3) - \omega'^2(\omega^2 - c^2k_\perp^2/\varepsilon\mu_3)[\mathbf{kv}]^2\} \\ & + \mu_1\{(\omega'^2 - \omega_B^2)(2\omega^2 - c^2k_\perp^2/\varepsilon\mu_3)c^2k_3^2/\varepsilon \\ & + \omega'^2(\omega^2 - c^2k_\perp^2/\varepsilon\mu_3)c^2k_\perp^2/\varepsilon + \omega'^4c^2(k^2 + k_3^2)/\varepsilon \\ & + \omega'^2[\mathbf{kv}]^2c^2k_3^2/\varepsilon\} + 2\omega_B\omega'^2c^2\mu_2(k_\perp^2\omega - k^2\omega')/\varepsilon \\ & + c^4k_3^2\varepsilon^{-2}\{(\omega_B^2 - \omega'^2)k_3^2 - \omega'^2k_\perp^2\}, \end{aligned}$$

$$\begin{aligned} G_1(\mathbf{k},\omega) = {} & (\mu_1^2 - \mu_2^2)\omega'^2\{\omega'^2 + [\mathbf{k},\mathbf{v}]^2 + 2\omega^2 - c^2k_\perp^2/\varepsilon\mu_3\} \\ & - \omega'^2c^2(k^2 + k_3^2)\mu_1/\varepsilon + 2\omega'\omega_Bc^2k_3^2\mu_2/\varepsilon \end{aligned} \quad (12)$$

(we recall that $\mathbf{v} \parallel \mathbf{B}_0$).

When $\Omega = 0$ Eq. (11) determines the frequencies of the optical and spin waves. Spin waves in ferromagnets correspond to that branch of the dispersion equation $D(\mathbf{k}, \omega) = 0$, for which $\omega^2\varepsilon/c^2k^2 \ll 1$. Using (10) and (12) we readily obtain the spin-wave frequencies, accurate to terms of order $\omega^2\varepsilon/c^2k^2$:

$$\omega_s^2 = \Omega_e(\mathbf{k})(\Omega_e(\mathbf{k}) + 4\pi g M_0\sin^2\theta). \quad (13)$$

Let us now take into account the influence of the particle beam on the spin waves. Assuming the density of the particles in the beam to be sufficiently small, we neglect the last two terms of (11). Putting $\omega = \omega_s(\mathbf{k}) + \xi$ and $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$, and assuming that $|\xi| \ll \omega_s$, we get

$$-\xi^3\omega_B^2(\partial D/\partial\omega)_{\omega_s} + \Omega^2 G(\mathbf{k},\omega_s)/\varepsilon = 0.$$

From this we readily obtain the increment of the amplitude of the spin waves:

$$\eta = \frac{\sqrt{3}}{2}\left(\frac{\Omega}{\omega_B}\right)^{2/3}\left[\frac{G(\mathbf{k},\omega_s)}{\varepsilon(\partial D/\partial\omega)_{\omega_s}}\right]^{1/3}.$$

In the derivation of this formula it is assumed that the inequality $\Omega^2/\varepsilon \ll \eta\omega_B$ is satisfied.

Using (10), (12), (13), and the equation $D(\mathbf{k}, \omega_s) = 0$, we obtain

$$G(\mathbf{k},\omega_s) = \frac{c^2k^2}{\varepsilon}\omega_B^2\omega_s^2\left(\frac{\omega_s}{\Omega_e}\right)^2,$$

$$\left(\frac{\partial D}{\partial\omega}\right)_{\omega_s} = \left(\frac{c^2k^2}{\varepsilon}\right)^2\frac{\omega_s}{\Omega_e}\frac{1}{2\pi g M_0\sin^2\theta}.$$

Therefore the relative increment of the spin wave amplitude can be represented finally in the form

$$\frac{\eta}{\omega_s} = \frac{\sqrt{3}}{2}\left(\frac{2\pi g M_0}{\Omega_e}\frac{\Omega^2}{c^2k^2}\sin^2\theta\right)^{1/3}. \quad (14)$$

This formula shows that the relative increment is particularly large for small k.

From the resonance condition $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$ we can readily obtain the critical velocity of the beam:

$$v_c = 2gM_0\sqrt{\alpha(\beta + H_0M_0)}. \quad (15)$$

We note that for specified v and θ ($v > v_c$) the resonance condition can, generally speaking, be satisfied for several values of the wave vector (the number of these values is determined by the character of the function $\omega_s(\mathbf{k})$ for $k \sim a^{-1}$).

At sufficiently large velocities v and at angles θ not too close to $\pi/2$, spin waves will be excited with frequency

$$\omega_0 = gM_0\sqrt{(\beta + H_0/M_0)(\beta + H_0/M_0 + 4\pi\sin^2\theta)},$$

corresponding to small wave vectors ($\alpha k \ll \sqrt{\mu M_0/\Theta_C}$). According to (14) the relative increment of the amplitude of these waves is equal to

$$\frac{\eta}{\omega_0} = \frac{\sqrt{3}}{2}\left(\frac{\pi g M_0}{2\omega_0}\right)^{1/3}\left(\frac{\Omega v}{\omega_0 c}\right)^{2/3} \times \left(\frac{\sin^2 2\theta}{[1 + (2\pi g M_0/\omega_0)^2\sin^4\theta]^{1/2} - 2\pi g M_0\sin^2\theta/\omega_0}\right)^{1/3}. \quad (16)$$

We note that if the length of the excited spin waves becomes comparable with the dimensions of the body, then ω_0 in (16) should be meant as the frequency of one of the Walker modes [14].

[3)]The value of α is best represented in the form $\alpha = \Theta_C a^2/\mu M_0$ where Θ_C is of the order of the Curie temperature.

Putting in (16) $n = 10^{10}\ \text{cm}^{-3}$, $v/c \approx 10^{-1}$, $M_0 \approx 10^3$ G and $\sin\theta \sim \sin 2\theta \sim 1$, we get $\eta/\omega_0 \approx 10^{-1}$. Since the relative decrement of the spin waves in sufficiently pure yttrium iron garnets is 10^2—10^{-3}, they can apparently be used to observe the effect of coherent amplification of spin waves by a beam of particles. This beam should be sufficiently monoenergetic, namely: the relative spread in the particle velocity should be less than η/ω_0.

If the particle velocity lies in the interval $\Theta_C a/\hbar \gtrsim v\cos\theta \gg v_c$, then along with the excitation of the "long-wave spin waves" with wave vector $k = \omega_0/v\cos\theta$, there can be excited in principle "short wave spin waves" with wave vector $k = \hbar v\cos\theta/\Theta_C a^2$. The relative increment of these waves is determined by the formula

$$\frac{\eta}{\omega_s} = \frac{\sqrt{3}}{2}(2\pi)^{1/3}\frac{\Theta_C}{\mu M_0}\left(\frac{\Omega}{gM_0}\right)^{2/3}\left(\frac{v}{c}\right)^{2/3}\left(\frac{agM_0}{v}\right)^2\left(\frac{\sin\theta}{\cos^2\theta}\right)^{2/3}. \quad (17)$$

Comparison of this expression with (16) shows that the amplitude increment of a spin wave with large k ($k = \hbar v\cos\theta/\Theta_C a^2$) is much smaller than the increment of a spin wave with small k ($k = \omega_0/v\cos\theta$), but under certain conditions the increment (17) can exceed the damping decrement of the spin waves. For this it is necessary that the beam velocity not exceed the velocity of sound s and that the temperature satisfy the inequality $T \ll \Theta_D^2/\Theta_C$, where Θ_D is the Debye temperature (we assume that the Debye temperature is lower than the Curie temperature Θ_C). Under these conditions the spin waves interact essentially with one another, and not with the lattice vibrations[5].

The spin-wave damping due to spin-spin interaction is determined by the formula (see [6])

$$\gamma \approx 10^{-2}(T/\Theta_C)^{5/2}(ak)^3\,\Theta_C/\hbar. \quad (18)$$

Substituting here $ak = \hbar s/a\Theta_C \approx \Theta_D/\Theta_C$ and comparing (17) with (18) we get the condition for the build-up of spin waves:

$$(\Omega/gM_0)^{2/3} > 10^{-2}(T/\Theta_C)^{5/2}(c/s)^{2/3}\,\Theta_D^3/\mu M_0\Theta_C^2.$$

Substituting for T the maximum permissible value of $\Theta^2{}_D/\Theta_C$, we get

$$(\Omega/gM_0)^{2/3} > 10^{-2}(\Theta_D/\Theta_C)^8(c/s)^{2/3}\,\Theta_C/\mu M_0. \quad (19)$$

Assuming that $\Theta_D \approx 10^2$, $\Theta_C \approx 10^3$, $S \approx 3\times 10^5$ and $\mu M_0 \approx 10^{-1}$ we obtain $\Omega/gM_0 > 10^{-4}$ and $gM_0 \sim 10^{10}\ \text{sec}^{-1}$.

4. Let us examine, finally, the interaction between the particle beam and spin waves for which the resonance condition[4)] $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v} - \omega_B$ is satisfied. Putting $\omega_s = \omega_3(k) + \xi$, we obtain in accordance with (11)

$$\xi^2 = -\eta_B^2 = \frac{1}{2}\frac{\Omega^2}{\varepsilon}\frac{G(\mathbf{k},\omega_s)}{\omega_B^3(\partial D/\partial\omega)_{\omega_s}}.$$

Using further (10) and (12) we obtain the relative increment of the spin wave amplitude:

$$\frac{\eta_B}{\omega_s} = \sqrt{\pi}\,\frac{\Omega}{\omega_s}\left(\frac{gM_0\Omega_e\omega_B}{\omega_s(kc)^2}\right)^{1/2}\left|\left(1-\frac{\omega_s}{\Omega_e}\right)\cos\theta - \frac{kv}{\omega_B}\frac{\omega_s}{\Omega_e}\sin^2\theta\right|. \quad (20)$$

The critical velocity of the beam is determined in this case by the formula

$$v_c = 2gM_0\sqrt{\alpha(\beta + H_0/M_0 + \omega_B/gM_0)}.$$

If $v\cos\theta \gg v_c$, then spin waves with frequency $\omega_0 + \omega_B$ and with wave vector $k = (\omega_0 + \omega_B)/v \times \cos\theta$ will be excited. The relative increment of these waves has an order of magnitude $\eta_B/\omega_0 \sim v\Omega/cgM_0$. Putting $\omega_0 \approx 10^{10}\ \text{sec}^{-1}$, $n \approx 10^{10}\ \text{cm}^{-3}$ and $v/c \approx 10^{-1}$ we get $\eta_B/\omega_0 \sim 10^{-1}$.

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[4)] Spin waves for which the resonance condition $\omega_s(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v} + \omega_B$ is satisfied are not amplified by the beam at low densities.

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