CONTRIBUTION TO THE STATISTICAL THEORY OF NUCLEAR LEVEL SPACING

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The parameter a, which determines the density of nuclear levels in the Fermi gas model, and also the dependence of the nuclear temperature on the mass number, are derived from experimental data on the mean distance between neutron s-resonances. The effective nuclear excitation energies for an even number of protons and neutrons correspond to the pairing energy calculations made by Nemirovskiĭ and Adamchuk on the basis of experimental nuclear mass values. The parameter σ, which determines the distribution of levels with respect to the total angular momentum, is obtained by employing the mean square magnetic quantum numbers of the nucleons in accordance with the shell model. The calculated values of the parameters a and σ, and the nuclear temperatures for an excitation energy of 7 MeV, are found to agree with other available data. It is also shown that for the excitation energies indicated the moments of inertia of nuclei far away from closed shells are approximately the same as those for rigid bodies.

1. In calculations of the widths of various processes, of the spectra of inelastically scattered neutrons and other particles emitted by the compound nucleus, the distribution of fission fragments with respect to the mass, etc, in the framework of the statistical theory, a knowledge of the energy dependence of the density of nuclear levels is necessary. In the present work, the mass-number dependence of the parameter a, which determines the density of levels in the Fermi gas model, is calculated on the basis of experimental data on the mean distances between the neutron s-resonances. For excitation energies below the Fermi energy, which is the case at hand, the nucleons principally involved in the excitation process are those situated close to the boundary of the distribution, and the parameter a has the meaning of the density of single-nucleon states close to the Fermi surface.

In contrast with a number of similar calculations carried out previously, it is shown here that the values obtained for the parameter a and the parameter σ (which determines the distribution of levels with respect to the total angular momentum), and also the nuclear temperatures at excitation energies of the order of the binding energy of the neutron, are quantitatively consistent with one another if use is made of the mean square magnetic quantum numbers of the nucleons from the shell model.

For an excitation energy of 7 MeV, the computed temperatures are in excellent agreement with the corresponding experimental data obtained from the treatment of the spectra of inelastically scattered neutrons. The dependence computed here of σ on the mass number does not contradict the existent data and for $A \sim 25$ it practically coincides with the direct measurements of Hibdon. The moments of inertia of nuclei far away from closed shells are found here to be of the order of those of rigid bodies, in accordance with the theoretical predictions.

2. We make use here of the well known formula for the density of levels with a given value of the total angular momentum J and given (arbitrary) parity, for the excitation energy E, which is an excellent approximation for not too small a parameter σ (see, for example,[1,2]):

$$\rho(E, J) = \rho(E)\frac{2J+1}{2(2\pi)^{1/2}\sigma^3}\exp\left[-\frac{(J+1/2)^2}{2\sigma^2}\right], \quad (1)$$

where $\rho(E)$ is the total density of states (with account of degeneracy in the total angular momentum):

$$\rho(E) = \sum_J (2J+1)\,\rho(E, J). \quad (2)$$

In the Fermi gas model, which consists of a mixture of the proton and neutron components we have

$$\rho(E) = \frac{\sqrt{\pi}}{12\,a^{1/4}E^{5/4}}\exp\,[2\,(aE)^{1/2}], \quad (3)$$

where

$$a = \pi^2 G/6, \quad (4)$$

and where G is the combined density of proton and neutron single-particle states close to the Fermi surface:

$$G = G_p + G_N. \tag{5}$$

The parameter σ in Eq. (1) is associated with the moment of inertia of the nucleus $\mathcal{J}$ and the temperature T:

$$\sigma^2 = \hbar^{-2} \mathcal{J} T, \tag{6}$$

where in the given case the moment of inertia is a free parameter determined by the properties of the nuclear material and the shape of the nucleus, while the temperature in the Fermi gas model is

$$T = (E/a)^{1/2}. \tag{7}$$

We shall not make any distinction here between the so-called nuclear temperature, which is defined as the reciprocal of the logarithmic derivative of the density of levels, and the thermodynamic temperature, which is equal to the reciprocal of the derivative of the entropy with respect to the energy, since the difference between them is not large for the excitation energies considered here.[3]

On the basis of the data of [3-7] it was assumed in the initial variant of the calculation that $\sigma = 4 \pm 1$ and does not depend on the mass number. If we use the dependence on the mass number of the mean square of the magnetic quantum numbers of the nucleons, assumed in the work of Lang[8] on the basis of the shell model[9]

$$\langle m^2 \rangle \approx 0.146\, A^{2/3} \tag{8}$$

and the expression for the moment of inertia

$$\mathcal{J} = \hbar^2 G \langle m^2 \rangle, \tag{9}$$

we can again compute σ from (6), making use of the values of a and T obtained for $\sigma = 4$, and we can employ the new σ for an iterated calculation of a and T, etc. As a result of several iterations of this nature, the calculations become self consistent and the final values of σ do not contradict the existing data. Therefore, for the final calculation, it was assumed that

$$\sigma^2 = GT \langle m^2 \rangle = 8.89 \cdot 10^{-2} aTA^{2/3}. \tag{10}$$

The observed density of s-resonance for a target nucleus spin $I = 0$ is $\rho(B_n, \frac{1}{2})$, and for $I \neq 0$ it is $\rho(B_n, I-\frac{1}{2}) + \rho(B_n, I+\frac{1}{2})$, where B_n is the binding energy of the neutron in the compound nucleus. Since levels of only one parity are observed in this case (corresponding to the parity of the ground state of the target nucleus), the density of levels computed from experimental data is doubled, inasmuch as the levels can be assumed with great accuracy to have an equilibrium distribution with respect to the parity.[2]

By using (1) and (10), the total density of states was computed for each of the given nuclei, while the transcendental algebraic equation obtained from (3) was solved graphically. The experimental data on the mean distances between neutron s-resonances used here were obtained from [10]. No corrections on the possible missing of levels in the experiment have been introduced. The excitation energies were assumed to be equal to the binding energy of the neutron in the compound nucleus, which was computed from known nuclear masses.[11] The improved accuracy in the region A ~ 100 and for rare earths was made from the data of [12,13].

It was assumed that before the nucleus with even values of Z and N could be regarded as a gas of free nucleons it would be necessary to expend a part of the energy on the breaking of nucleon pairs. The remaining energy, which could be called the effective energy of excitation of the nucleon gas U, is determined in the following way:

$$U = B_n - \left\{ \begin{array}{ll} & \text{Compound nucleus} \\ \delta_p + \delta_n, & \text{even-even (e.e.)} \\ \delta_p, & \text{even-odd (e.o.)} \\ \delta_n, & \text{odd-even (o.e.)} \\ 0 & \text{odd-odd (o.o.)} \end{array} \right. \tag{11}$$

where δ_p and δ_n are the energies of coupling of two protons and two neutrons. For calculation of the coupling energy, we make use of formulas proposed by Nemirovskiĭ and Adamchuk:[14]

$$\delta_p = \varepsilon_p - \frac{1}{9A^{4/3}} \left[17.0 - 0.691 \frac{(3A-1)(3A-2Z)}{A} \right] + 44.5 \frac{2N^2}{A^3}, \tag{12}$$

$$\delta_n = \varepsilon_n - \frac{1}{9A^{4/3}} \left[17.0 - 0.691 \frac{2Z(Z-1)}{A} \right] + 44.5 \frac{2Z^2}{A^3};$$

$$\varepsilon_p = \tfrac{1}{2} [E_{Z,N} - 2E_{Z-1,N} + E_{Z-2,N}],$$

$$\varepsilon_n = \tfrac{1}{2} [E_{Z,N} - 2E_{Z,N-1} + E_{Z,N-2}]. \tag{13}$$

ϵ_p, ϵ_n are the usually determined energies of coupling in a nucleus with even Z or N. The correction terms in (12) are due to the difference in the surface energy, the Coulomb energy and the symmetry energy for neighboring nuclei. The numerical coefficients in these cases are such that δ_p and δ_n are expressed in MeV.

The results obtained for the parameter a can be compared with the equation proposed by Newton:[15]

$$a = 2\alpha (\bar{j}_Z + \bar{j}_N + 1) A^{2/3}, \tag{14}$$

where $\bar{j}_Z$ and $\bar{j}_N$ are the "effective angular momenta" of the protons and neutrons close to the Fermi surface, computed by Newton on the basis of the scheme for shell levels of Klinkenberg.[16] Lang[8], analyzing data of different researches, obtained better agreement with experiment for $\alpha = 0.0374$.

Using the final values of σ and T, we can estimate the moments of inertia of the nuclei in the excited states by Eq. (6), and compare them with the estimate of the rigid body moment of inertia:[1]

$$\mathcal{J}_r = \frac{2}{5}\frac{M_n R^2 A}{\hbar^2}, \qquad (15)$$

where M_n is the mass of the nucleon and R is the radius of the nucleus. For $r_0 = 1.2$ F,

$$\mathcal{J}/\mathcal{J}_r = 73.1\sigma^2/TA^{5/3}. \qquad (16)$$

3. The results of the calculation of the function a(A) under the assumption that all the excitation energy, which is equal to the binding energy of the neutron B_n, goes over into the excitation energy of single-particle states regardless of the parity of the numbers Z and N in the nucleus, are shown in Fig. 1. The errors indicated are due to the inaccuracy of knowledge of the mean distances between the s-resonances and the excitation energies, and do not include uncertainties in the parameter σ. In Fig. 2 are shown the parameters a computed with account of the coupling energy (12) for nuclei with even Z and N. It is seen that the differences in the parameters a virtually vanish for nuclei of different types, with the exception of a few cases. The general dependence of the parameters on the mass number is approximately linear: $a \approx 0.125A$.

For comparison, Eq. (14) is plotted in Fig. 2 with the numerical coefficient of Lang, and reproduces sufficiently satisfactorily the dependence a(A) obtained here. Figure 3 shows the dependence of the nuclear temperature on the mass number for effective excitation energies U, and Fig. 4 gives the dependence on the mass number of the nuclear temperature normalized according to (7) to the effective excitation energy of 7 MeV.[8,10] In these data, the coupling energy is also taken into account, after which they are normalized to the energy of 7 MeV. From a comparison of Figs. 3 and 4, it is seen that the reduction of the nuclear temperatures to a single excitation energy smooths out the difference in temperatures for neighboring nuclei.

We note that similar experimental data for an initial energy of 14 MeV lead to temperatures close to those shown in Fig. 4., although they should be ~ 40 per cent larger if the Fermi gas model correctly describes the energy dependence of the temperature. There is some evidence[3,4] that the density of levels in the considered energy range of excitation can be described satisfactorily under the assumption of a constant temperature. However, the recent data of Lang,[8] the data of Erba and coworkers,[17] and the results of this research show that the Fermi gas model is applicable for the description of the density of levels in a wide range of energies. The contradiction noted is perhaps connected with the inaccuracy of the methods of analysis of the experimental data.

The values of the parameter σ computed here, together with the existing data[3-7], are shown in Fig. 5.

The ratios of the moments of inertia of nuclei for excitation energy of 7 MeV to the rigid body moment of inertia (for $r_0 = 1.2$ F) are shown as

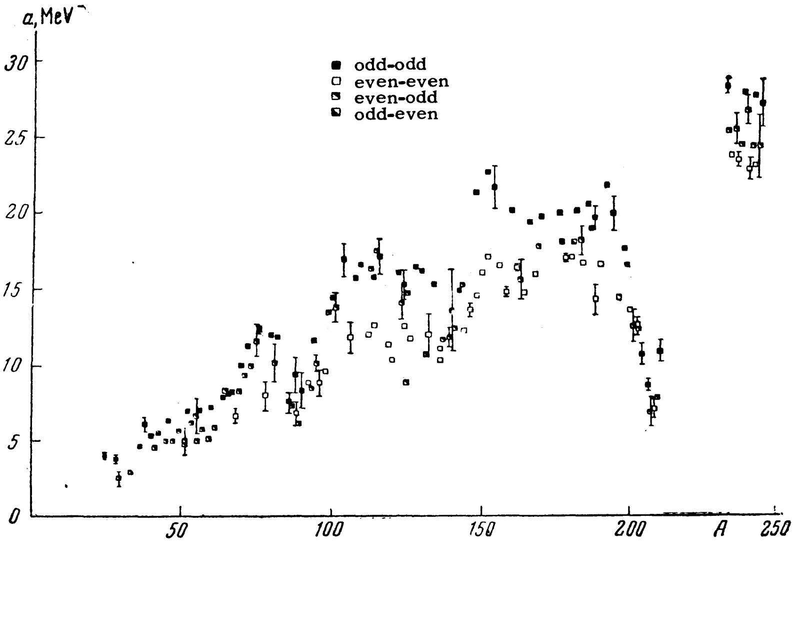

FIG. 1. Dependence of the parameter a on the mass number, obtained under the assumption that the excitation energy is equal to the binding energy of the neutron, B_n.

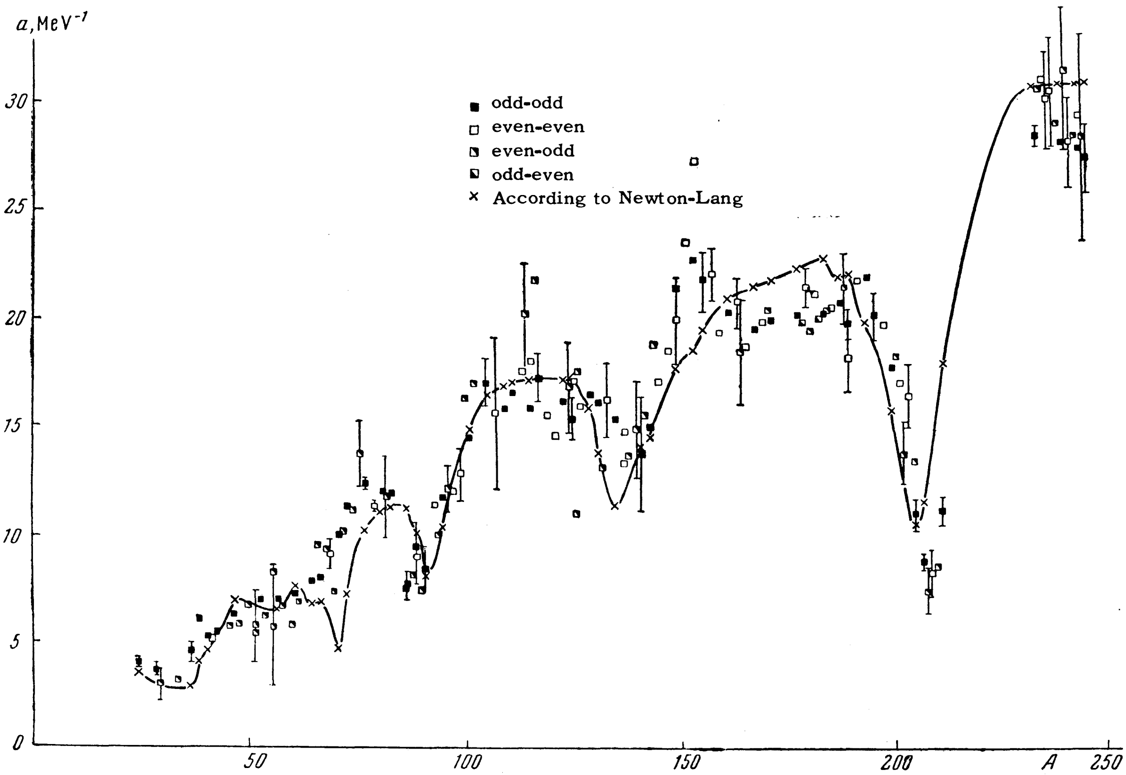

FIG. 2. Parameter a computed under the assumption that the excitation energy is equal to the effective excitation energy U, in accord with the relation (11).

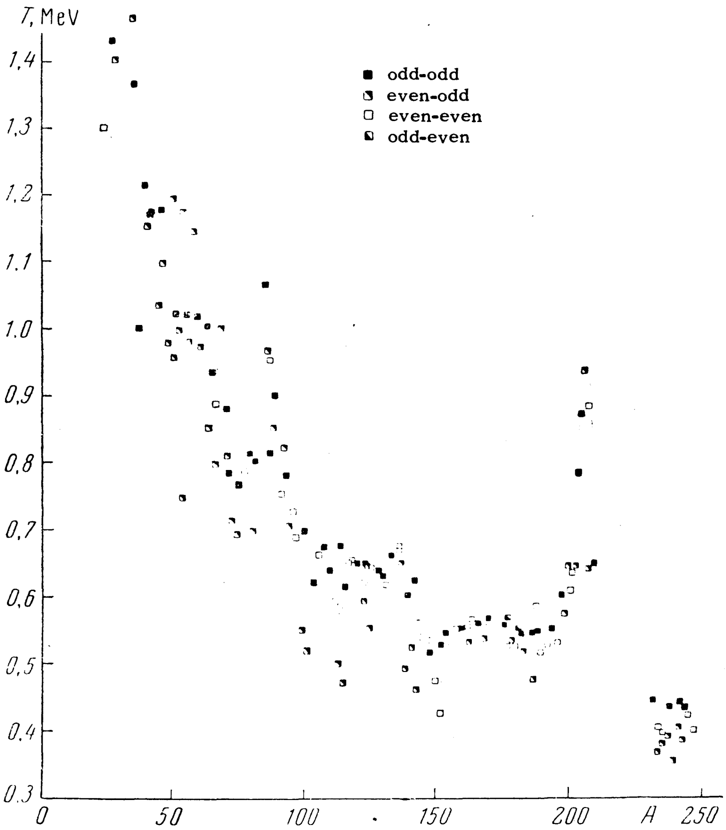

FIG. 3. Nuclear temperatures for effective excitation energies U.

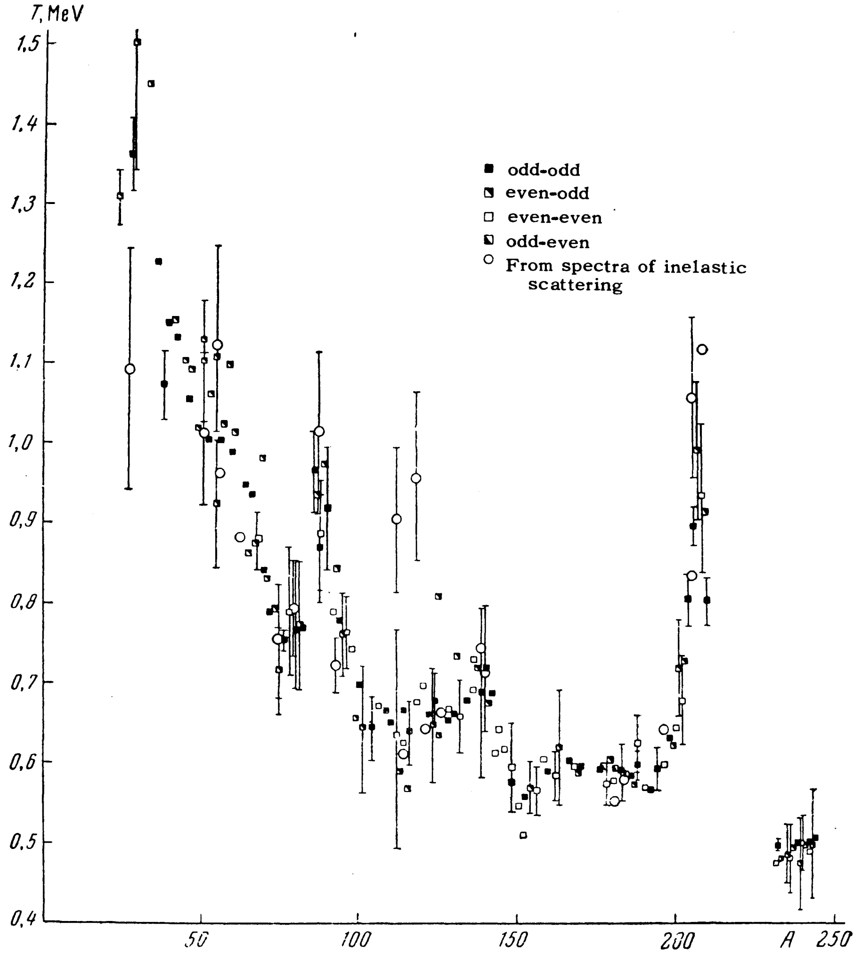

FIG. 4. Nuclear temperatures normalized to an excitation energy of 7 MeV. Data on temperatures obtained from the spectra of inelastic scattering were taken from Refs. 8, 10.

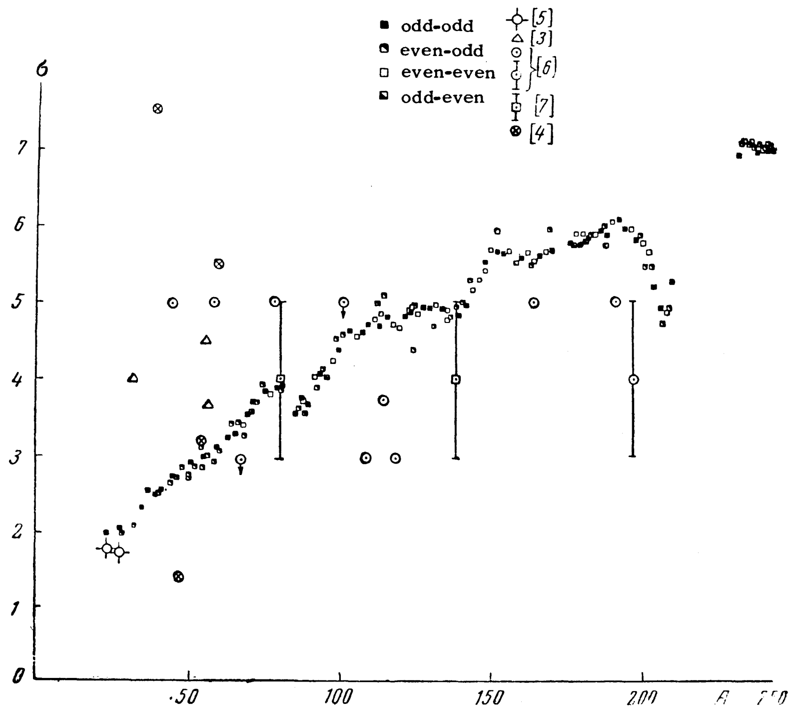

FIG. 5. The parameter σ for excitation energy of 7 MeV [according to Eq. (10)]. Other data taken from [3-7].

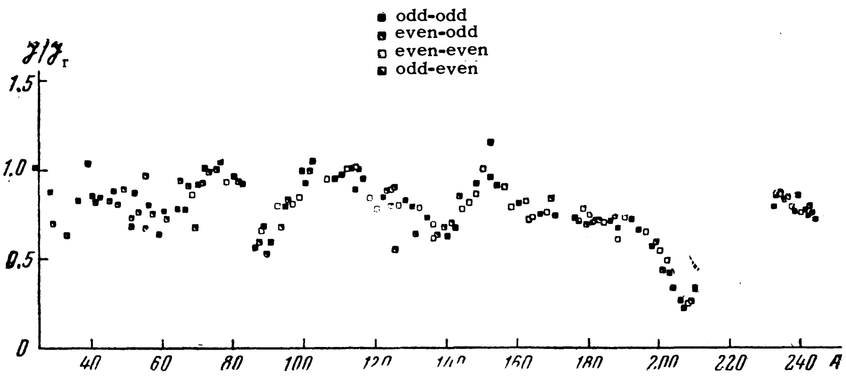

FIG. 6. Ratio of the moments of inertia of nuclei for effective excitation energy of 7 MeV to the rigid-body moments of inertia.

a function of the mass number in Fig. 6.

It was pointed out earlier[18] that for sufficiently large excited moments, the inertia of the nuclei should be close to the rigid-body value, since then the quasi-classical consideration is valid.[1,19] However, in nearly closed shells, the moments of inertia decrease, especially for Z = 82 and N = 126, where they amount to ~ 25 per cent of the rigid-body value. Another consequence of the quasiclassical model[1]—the approximately linear dependence of the parameter a on the mass number—is also confirmed by the results of the present research. It should be noted that the mean energy of the Fermi level for protons and neutrons, corresponding in the quasiclassical model to the proportionality constant 0.125, is equal to ~ 20 MeV, which is significantly less than the expected value if one starts out from the depth of the actual potential of the optical model (around 50 MeV). Discussion of the reasons for this discrepancy, however, lie outside the framework of the present research.

An explanation of the effects associated with the parity of the numbers Z and N by the effect of coupling of only the very "uppermost" nucleons, and shell effects—the multiplicity of states of nucleons close to the Fermi surface, points up the fact that in the range of excitation energies considered (4—11 MeV) essentially only the nucleons in the upper filled states take part in the excitation process; this also corresponds to the assumed model.

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