FERMION REGGE POLES AND THE COMPTON EFFECT

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The scattering of photons on spin ½ particles for large values of s and finite u is investigated on the basis of the Regge pole hypothesis. The expression for the scattering amplitude is reduced to an explicitly factorized form. The main qualitative results are the same as those of Gribov for πN scattering.[10] Asymptotic expressions for the differential cross section and some polarization effects are calculated for large angle scattering.

RECENTLY, a number of papers appeared in which the Regge pole hypothesis was extended to quantum electrodynamics.[1-3] This is a very natural development of the theory, since quantum electrodynamics has been well confirmed by experiment and a comparison with it might allow one to test the validity of the very idea of the moving poles. Gell-Mann and Goldberger [3] have considered the asymptotic behavior (large s and finite u) of the amplitude for the Compton effect on particles with spin 0 and ½, calculated in second order perturbation theory. In the scalar case the results of perturbation theory are not in disagreement with the Regge pole hypothesis.

In the present paper we consider the Compton effect on a spinor particle for large s and finite u on the basis of the idea of moving poles. Assuming the existence of dominant isolated Regge poles, we determine the asymptotic behavior of the cross section and other experimentally observable quantities for large angle scattering.

1. FERMION REGGE POLES

As is known, the most general expression for the scattering of a photon by a spin ½ particle has the form [4]

$$F = e_{2\nu}^{*}\bar{u}_2(-p_2)\,F_{\mu\nu}u_1(p_1)\,e_{1\mu};$$

$$F_{\mu\nu} = \sum_{i=1}^{6} A_i(s,t,u)\,F^i_{\mu\nu} = A_1\frac{P_\mu P_\nu}{P^2} + A_2\frac{N_\mu N_\nu}{N^2} + A_3\frac{(P_\mu N_\nu - N_\mu P_\nu)\,i\gamma_5}{(P^2N^2)^{1/2}} + A_4\frac{P_\mu P_\nu}{P^2}\frac{i\hat{k}}{m} + A_5\frac{N_\mu N_\nu}{N^2}\frac{i\hat{k}}{m} + A_6\frac{(P_\mu N_\nu - N_\mu P_\nu)\,i\gamma_5 i\hat{k}}{m\,(P^2N^2)^{1/2}}. \quad (1)$$

All particles in the Feynman graph are considered incoming; the following notation is used:

$$P_\mu = P'_\mu - K_\mu(P'K)K^{-2}, \qquad P'_\mu = {}^1/_2(p_{1\mu} - p_{2\mu}),$$
$$K_\mu = {}^1/_2(k_{1\mu} - k_{2\mu}), \quad N_\mu = i\varepsilon_{\mu\nu\rho\sigma}P_\nu K_\rho Q_\sigma, \quad Q_\mu = k_{1\mu} + k_{2\mu}.$$

The Mandelstam variables s, t, and u are in this notation equal to

$$s = -(p_1+k_1)^2,\ t = -(p_1+p_2)^2, \quad u = -(p_1+k_2)^2.$$

We are interested in the asymptotic behavior in the s channel for large positive s and finite negative u. To find the asymptotic amplitude we must follow the usual procedure and go over to the u channel ($u > 0$, $s < 0$), where s has the meaning of a momentum transfer. Using the Legendre expansion of the amplitude, we can obtain the value of the amplitude in the unphysical region of large positive s. We then find the required asymptotic expression by analytic continuation of the amplitude into the region $u < 0$.

The transition to the u channel is most easily accomplished by applying crossing symmetry to formula (1). However, it is convenient to make a partial wave expansion of an amplitude with definite helicity:[5]

$$\langle\lambda_{e_2}\lambda_{\gamma_1}|F|\lambda_{e_1}\lambda_{\gamma_2}\rangle.$$

Following the work of Hearn and Leader,[6] we consider the six helicity amplitudes which completely determine the Compton effect:

$$\Phi_1 = \langle\tfrac{1}{2}1|F|\tfrac{1}{2}1\rangle, \qquad \Phi_2 = \langle-\tfrac{1}{2}-1|F|\tfrac{1}{2}1\rangle,$$
$$\Phi_3 = \langle\tfrac{1}{2}-1|F|\tfrac{1}{2}1\rangle, \qquad \Phi_4 = \langle-\tfrac{1}{2}1|F|\tfrac{1}{2}1\rangle,$$
$$\Phi_5 = \langle-\tfrac{1}{2}1|F|-\tfrac{1}{2}1\rangle, \quad \Phi_6 = \langle\tfrac{1}{2}-1|F|-\tfrac{1}{2}1\rangle. \quad (2)$$

The partial wave expansion of the amplitudes Φ_i has the form [6]

$$\langle\lambda'|F|\lambda\rangle = \frac{1}{2p}\sum_j (2j+1)\,\Phi^j_{\lambda\lambda'} d^j_{\lambda\lambda'}(\theta). \quad (3)$$

Here j is the total angular momentum, p and θ are the momentum and scattering angle in the c.m.s., $\lambda = \lambda_{e1} - \lambda_{\gamma 2}$; $\lambda' = \lambda_{e2} - \lambda_{\gamma 1}$; the properties of the reduced rotation matrices $d^j_{\lambda\lambda'}(\theta)$ are discussed in detail in the paper of Jacob and Wick.[5]

To determine the contribution of the dominant Regge poles we must express the amplitude as a superposition of partial amplitudes $\Phi^j_{\lambda\lambda'}$, corresponding to transitions between states with definite parity. Here it is convenient to introduce the following combinations of the amplitudes Φ_i, using the explicit form of $d^j_{\lambda\lambda'}(\theta)$:

$$f_1^\pm = \Phi_1/\cos(\theta/2) \pm \Phi_2/\sin(\theta/2);$$
$$f_2^\pm = \Phi_3/\cos(\theta/2) \pm \Phi_4/\sin(\theta/2);$$
$$f_3^\pm = \Phi_5\cos(\theta/2) \pm \Phi_6\sin(\theta/2). \quad (4)$$

Using (3) and the explicit expressions for $d^j_{\lambda\lambda'}(\theta)$, we find

$$f_1^\pm = \frac{1}{p}\sum_j \{P'_{j+1/2}(\Phi_1^j \pm \Phi_2^j) - P'_{j-1/2}(\Phi_1^j \mp \Phi_2^j)\}.$$

$$f_2^\pm = \frac{1}{p}\sum_j \Big\{\left(j+\frac{3}{2}\right)^{1/2}\left(j-\frac{1}{2}\right)^{-1/2}(\Phi_3^j \mp \Phi_4^j)\,P'_{j-1/2} - \left(j-\frac{1}{2}\right)^{1/2}\left(j+\frac{3}{2}\right)^{-1/2}(\Phi_3^j \pm \Phi_4^j)\,P'_{j+1/2}\Big\},$$

$$f_3^\pm = \frac{1}{2p}\sum_j \frac{1}{(j-1/2)(j+3/2)} \times\Big\{\Big[\left(j-\frac{1}{2}\right)\left(j+\frac{7}{2}\right)P'_{j+1/2} - z\left(j+\frac{3}{2}\right)^2 P'_{j-1/2}\Big](\Phi_5^j \mp \Phi_6^j) + \Big[\left(j-\frac{1}{2}\right)^2 zP'_{j+1/2} - \left(j+\frac{3}{2}\right)\left(j-\frac{5}{2}\right)P'_{j-1/2}\Big](\Phi_5^j \pm \Phi_6^j)\Big\}. \quad (5)$$

Here P'_l are the derivatives of the Legendre polynomials with respect to their argument,

$$z = \cos\theta = 1 + 2ut\,(u - m^2)^{-2}.$$

It can be shown that the combinations of partial waves entering in (5) correspond to transitions between states with definite total angular momentum, total spin, and parity P, viz.,

$$\Phi_1^j \pm \Phi_2^j, \quad 1/2 \to 1/2, \quad P = \pm(-1)^{j-1/2},$$
$$\Phi_3^j \pm \Phi_4^j, \quad 1/2 \rightleftarrows 3/2, \quad P = \pm(-1)^{j-1/2},$$
$$\Phi_5^j \pm \Phi_6^j, \quad 3/2 \to 3/2, \quad P = \pm(-1)^{j-1/2}.$$

Using dispersion relations in the momentum transfer t for fixed u we can, in analogy to [7,8], introduce analytic functions of j, denoted by $(\Phi_k^j \pm \Phi_k^j)^\pm$, which coincide with the physical partial amplitudes for even and odd values of $j - 1/2$, respectively. The superscripts ± denote the so-called signature. If we assume that the singularities in j are poles, we have thus two systems of poles for definite signature, corresponding to positive and negative parity. Each of these systems describes in turn three types of transitions.

Before going into a detailed discussion of these poles, we note a few results which are independent of the character of the singularities in j. To this end, we use the formulas of [6] for the transformation from the helicity amplitudes $\Phi_i(2)$ to the invariants $A_i(1)$, keeping only the leading terms in z, and express the f_i^+ in terms of the A_i:

$$\varphi_1^+ + \varphi_1^- = \frac{p}{\pi}\left[\frac{2mA_1}{m^2-u} - A_4\right];$$

$$\varphi_1^+ - \varphi_1^- = \frac{p}{\pi\sqrt{u}}\left[\frac{(m^2+u)A_1}{m^2-u} - mA_4\right];$$

$$\varphi_3^+ + \varphi_3^- = -pA_6/\pi; \qquad \varphi_3^+ - \varphi_3^- = pA_3/\pi\sqrt{u};$$

$$\varphi_2^+ + \varphi_2^- = \frac{p}{\pi}\left[A_5 - \frac{2mA_2}{m^2-u}\right];$$

$$\varphi_2^+ - \varphi_2^- = \frac{p}{\pi\sqrt{u}}\left[\frac{(m^2+u)A_2}{m^2-u} - mA_5\right]. \quad (6)$$

For reasons of economy we have here introduced the notation

$$\varphi_1^\pm = f_1^\pm - 2f_2^\pm + 2f_3^\pm/z;$$
$$\varphi_2^\pm = f_1^\pm + 2f_2^\pm + 2f_3^\pm/z; \quad \varphi_3^\pm = f_1^\pm - 2f_3^\pm/z. \quad (7)$$

The invariant amplitudes A_i have no singularities at u = 0,[9] in particular, none of the square root type. We see therefore, that, for arbitrary j, the partial waves corresponding to $\varphi_1^+ - \varphi_1^-$ go to infinity or zero for $u \to 0$, whereas the partial waves corresponding to $\varphi_1^+ + \varphi_1^-$ remain finite. This is possible only if the singularities of the partial waves φ_1^+ and φ_1^-, as functions of j, coincide for u = 0. Moreover, it follows from the expressions for φ_1^+ and φ_1^-, which we shall not write down, that they transform one into the other under the interchange $\sqrt{u} \to -\sqrt{u}$. Both of these results have been obtained by Gribov for the case of pion-nucleon scattering.[10]

If we assume that the closest singularities for large j are poles we can, in the usual way,[7] replace the sum in (5) by an integral, extend the integration contour along the imaginary axis, and take only into account the contribution from the dominant pole in the sum over residues. Let us, for example, consider a pole with parity $(-1)^{j-1/2}$. This pole receives contributions from the partial amplitudes $(\Phi_i^j + \Phi_k^j)^\pm$ whose residues we shall denote by $r_{11}^\pm$, $r_{13}^\pm$, and $r_{33}^\pm$, in correspondence with the transitions to which they refer. However, these residues are not independent. It can be shown that the unitarity condition in the u channel leads to the following relation:[1)]

$$r_{11}^{\pm} r_{33}^{\pm} = (r_{13}^{\pm})^2. \qquad (8)$$

Calculating the contribution from the dominant pole to $f_i^{\pm}$ and finding the corresponding contributions to the invariant amplitudes A_i with the help of (6) and (7), we obtain the following expression for the total amplitude (1):

$$F_{\mu\nu} = \frac{C\,[s^{j-1/2} \pm (-s)^{j-1/2}]}{\cos \pi j} \sum_{i=1}^{6} A_i F_{\mu\nu}^i;$$
$$C = \pi^{3/2} 2^{2j} \gamma^{j+1/2} \Gamma(j+1)/\Gamma(j+1/2), \qquad \gamma = u(u-m^2)^{-2}. \qquad (9)$$

The quantities A_i have the following values:

$$A_1 = -(m+\sqrt{u})\, r_{\pm}^2, \qquad A_2 = (m-\sqrt{u})\, r_{\pm}'^2,$$
$$A_3 = -\sqrt{u}\, r_{\pm} r_{\pm}', \qquad A_6 = m r_{\pm} r_{\pm}',$$
$$A_4 = -m r_{\pm}^2, \qquad A_5 = m r_{\pm}'^2;$$
$$r_{\pm} = (m-\sqrt{u})^{1/2} (m+\sqrt{u})^{-1/2} \times \{(r_{11}^{\pm})^{1/2} + [(j-1/2)\, r_{33}^{\pm}/(j+3/2)]^{1/2}\},$$
$$r_{\pm}' = (m+\sqrt{u})^{1/2} (m-\sqrt{u})^{-1/2} \times \{-(r_{11}^{\pm})^{1/2} + [(j-1/2)\, r_{33}^{\pm}/(j+3/2)]^{1/2}\}. \qquad (10)$$

The contribution from the pole with parity $-(-)^{j-1/2}$ differs from (9) by the replacement $\sqrt{u} \to -\sqrt{u}$.

2. FACTORIZATION

Formula (9) solves our problem and permits the calculation of experimentally observable quantities. However, it becomes much more lucid if we reduce it to an explicitly factorized form, i.e., express it as a product of two similar expressions with some propagation function in between. It is rather natural to make the following ansatz for this factorization:

$$\sum_{i=1}^{6} A_i F_{\mu\nu}^i = D \Gamma_\nu\, [(i\hat{f} - m) + \beta m]\, \Gamma_\mu;$$
$$\Gamma_\nu = \gamma_\nu + (i\mu/4m)(\gamma_\nu \hat{k}_1 - \hat{k}_1 \gamma_\nu),$$
$$\Gamma_\mu = \gamma_\mu + (i\mu/4m)(\gamma_\mu \hat{k}_2 - \hat{k}_2 \gamma_\mu),$$
$$f = p_1 + k_2. \qquad (11)$$

Here $\mu(u)$ is the anomalous magnetic moment of the fermion.

Expanding the right-hand side of (11) in terms of the tensors $F_{\mu\nu}^i$, we find

$$A_1 = -2 - \mu(m^2-u)/m^2 + \beta\,[1 - \mu^2 (m^2-u)/4m^2],$$
$$A_2 = \mu(\mu+2)(m^2-u)/2m^2 + \beta\,[1 - \mu^2 (m^2-u)/4m^2],$$
$$A_3 = -(1-\beta)\,[1 + \mu + \mu^2 (m^2-u)/4m^2],$$
$$A_6 = 1 + \mu + \mu^2 (m^2-u)/4m^2,$$
$$A_4 = -1 + \mu^2 (m^2-u)/4m^2 - \beta\mu(\mu+2)/2,$$
$$A_5 = 1 + \mu(\mu+2) - \mu^2 (m^2-u)/4m^2 - \beta\mu(\mu+2)/2. \qquad (12)$$

Comparing (12) with (10), we obtain a system of six equations for the three unknown quantities D, β, and μ. The solution of this system is therefore at the same time a verification of the ansatz (11). It turns out that the system of equations is consistent and has the following solution:

$$\beta = (m - \sqrt{u})/m,$$
$$\mu = 2m\,(r_{\pm} - r_{\pm}')\,[(m-\sqrt{u})\, r_{\pm}' - (m+\sqrt{u})\, r_{\pm}]^{-1},$$
$$D = [(m+\sqrt{u})\, r_{\pm} - (m-\sqrt{u})\, r_{\pm}']^2/4u. \qquad (13)$$

Combining C and D into a single factor G and taking account of the poles with both parities, we obtain finally

$$F_{\mu\nu} = G^{(1)} \Gamma_\nu^{(1)} (i\hat{f} - \sqrt{u})\, \Gamma_\mu^{(1)}\, [s^{j_1 - 1/2} \pm (-s)^{j_1-1/2}]/\cos \pi j_1 + G^{(2)} \Gamma_\nu^{(2)} (i\hat{f} + \sqrt{u})\, \Gamma_\mu^{(2)}\, [s^{j_2-1/2} \pm (-s)^{j_2-1/2}]/\cos \pi j_2. \quad (14)$$

The quantities with the superscript (2) are obtained from those with the superscript (1) by the replacement $\sqrt{u} \to -\sqrt{u}$. It is seen from (14) that the spinor structure of the amplitudes of the Compton effect derived on the basis of the Regge pole hypothesis is similar to the one obtained by Gell-Mann and Goldberger by perturbation theory.

3. ASYMPTOTIC FORM FOR LARGE ANGLE SCATTERING

Let us consider the scattering for large values of s and finite u in the channel where s is the energy. In this case u is negative. It can be shown that for $u < 0$ the poles of amplitudes with opposite parity as well as the residues in these poles are complex conjugates of one another. To this end it suffices to determine the absorptive parts of the expressions standing on the right-hand side of (6) and repeat word for word the argumentation of the paper of Gribov.[10] For $u < 0$ we must therefore include the contribution from both poles.

The coefficients in formulas (10) for A_i contain only m and $\sqrt{u}$. Therefore, the expression for the amplitude will be composed of combinations of the type

$$H = m\{r_{\pm}^2\, [s^{j-1/2} \pm (-s)^{j-1/2}]/\cos \pi j + r_{\pm}^{*2}\, [s^{j^*-1/2} \pm (-s)^{j^*-1/2}]/\cos \pi j^*\},$$
$$B = \sqrt{u}\,\{r_{\pm}^2\, [s^{j-1/2} \pm (-s)^{j-1/2}]/\cos \pi j - r_{\pm}^{*2}\, [s^{j^*-1/2} \pm (-s)^{j^*-1/2}]/\cos \pi j^*\}. \qquad (15)$$

[1)]We are grateful to V. B. Berestetskiĭ for explanations on this point.

Writing the residues in the poles r with parity $\pm(-1)^{j-1/2}$ as $2^{-1/2}\rho e^{\pm i\varphi}$, we obtain for the imaginary parts of expressions of the type H and B

$$H_1^{(i)}(u, s) = \pm mR_i^{\mp}(u)\, s^{j'-1/2} \cos(j''\xi + \psi_i),$$
$$B_1^{(i)}(u, s) = \pm \sqrt{-u}\, R_i^{\mp}(u)\, s^{j'-1/2} \sin(j''\xi + \psi_i), \quad (16)$$

where $\xi = \ln s$; j′ and j″ are the real and imaginary parts of the function j(u) defining the position of the pole;

$$R_1 = \rho^2, \quad R_2 = \rho\rho', \quad R_3 = \rho'^2;$$
$$\psi_1 = 2\varphi, \quad \psi_2 = \varphi + \varphi', \quad \psi_3 = 2\varphi'.$$

The real parts of the expressions H and B have the form

$$\operatorname{Re} H^{(i)}(u, s) = m\alpha_{\pm}R_i^{\mp}(u)\, s^{j'-1/2} \cos(j''\xi + \psi_i \mp \beta),$$
$$\operatorname{Re} B^{(i)}(u, s) = \sqrt{-u}\,\alpha_{\pm}R_i^{\mp}(u)\, s^{j'-1/2} \sin(j''\xi + \psi_i \mp \beta);$$
$$\alpha_{\pm}^2 = (\mathrm{ch}\,\pi j'' \mp \sin\pi j')/(\mathrm{ch}\,\pi j'' \pm \sin\pi j'),$$
$$\mathrm{tg}\,\beta = \mathrm{sh}\,\pi j''/\cos\pi j'. \quad (17)^*$$

Using the formulas of [11], one can now determine the asymptotic behavior of the differential cross section for the Compton effect and of various polarization coefficients in the region of large angles. We have

$$d\sigma/d\Omega = -u(\rho_{\mp}^2 + \rho_{\mp}'^2)^2(1 + \alpha_{\pm}^2)\, s^{2j'-1},$$
$$C^3 = -u(\rho_{\mp}'^4 - \rho_{\mp}^4)(1 + \alpha_{\pm}^2)\, s^{2j'-1},$$
$$b_{\mu} = \pm 4N_{\mu}\alpha_{\pm}(\rho_{\mp}^4 - \rho_{\mp}'^4)\sqrt{-u}\sin(\pm\beta)\, s^{2j'-2},$$
$$h^{11} = 0, \quad h^{33} = -u(\rho_{\mp}^2 - \rho_{\mp}'^2)^2(1 + \alpha_{\pm}^2)\, s^{2j'-1},$$
$$h^{22} = -4u\rho_{\mp}^2\rho_{\mp}'^2[\alpha_{\pm}^2\cos 2(j''\xi + \varphi + \varphi' \mp \beta) + \cos 2(j''\xi + \varphi + \varphi')]\, s^{2j'-1}. \quad (18)$$

*ch = cosh, sh = sinh, tg = tan.

The constant factors are included in the residues; the notation is taken from [11].

It is seen from (18) that the cross section and certain simple polarization coefficients depend on the energy monotonically, despite the oscillatory character of the energy dependence of the scattering amplitude. On the other hand, the polarization coefficient h^{22}, which describes the change of the circular polarization of the photon in the scattering process, oscillates as the energy varies.

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48