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FLUCTUATIONS OF ELECTRON-PHOTON SHOWERS IN XENON

B. NICHIPORUK and Z. S. STRUGAL'SKIĬ[1)

Joint Institute for Nuclear Research

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Fluctuations in the longitudinal development of electron-photon showers with 4 BeV total energy in xenon are studied with a bubble chamber.

We have investigated in a xenon bubble chamber of 30 liter volume[1] the electron-photon showers produced by γ quanta resulting from the decay of π^0 mesons created in interactions between 9-BeV/c π^- mesons and the Xe nucleus. The xenon has a relatively small radiation length compared with the chamber dimensions, and consequently the γ-quantum registration efficiency is high. This makes it possible to trace in detail the development of the shower over its entire depth and to study the fluctuations in the longitudinal development of the shower.

We present here the results of an investigation of these fluctuations in showers of average energy 4 BeV. In the analysis of the data we use the following constants for the xenon: density $d = 2.2$ g/cm^3, critical energy $\beta = 10.1$ MeV, and radiation unit length $t_0 = 3.8$ cm.

FORMULATION OF THE PROBLEM AND METHOD OF INVESTIGATION

Cascade theory makes it possible to calculate under certain assumptions the average number of electrons $\overline{N}_e(E_0, E, t)$ and photons $\overline{N}_f(E_0, k, t)$ with energy larger than E or k, respectively, at a depth t in a shower produced by a primary particle with energy E_0[2-4]. The question arises of the degree to which the actual number of electrons $N_e(E_0, E, t)$ can deviate from the average number $\overline{N}_e(E_0, E, t)$. The problem of the fluctuations was investigated theoretically by many[5-9], but has not yet been fully solved because of the mathematical difficulties.

In the present paper we determine experimentally the dependence of $\overline{N}$ and $\overline{\delta N^2}$ on the depth t.

Some 50,000 photographs were scanned and 73 electron-photon shower events selected. The following selection criteria were used: 1) the photograph containing the shower must not be cluttered with other tracks which do not pertain to the shower; 2) the development of the shower should take place entirely in the chamber; 3) the shower axes should make angles not exceeding 5° with the chamber axis; 4) the number of particles at the shower maximum should not be less than 15.

The number of electrons and positrons at a distance t from the point of γ-quantum conversion was counted, with the aid of a reprojector, on the left and right photographs of the stereo pair simultaneously. Where the γ-quantum generation points could be observed, the ranges for the conversion γ quanta were measured. They are distributed in the interval 0—63 mm. The number of particles at the maxima of the selected showers ranges from 15 to 23. The average number of particles at the maximum is 16.1.

EXPERIMENTAL DATA

On the basis of the selected events, we plotted the average cascade curve $\overline{N}_e(E_0, E, t)$, the mean-square deviation $\overline{\delta N_e^2}(t)$, and the distribution of the maxima of shower development with depth W(t). The results are listed in Table I.

Nine of the selected events give, it is assumed, the total picture of the decay of a single π^0 meson. In these cases one high-energy γ quantum produces the shower, and a second γ quantum with an energy that is low compared with the first, emitted at a definite angle to the direction of the first γ quantum, produces a single electron-positron pair. Further analysis has shown that only five of these events can be interpreted as decays of a single π^0 meson. They were used for a rough verification of the correctness with which the shower energies were determined by means of the cascade theory. The shower energies in these five cases, determined by the cascade theory and also from the kinematics of the π^0-meson decay, differed from one another by not more than 20%.

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1)Institute of Nuclear Research, Warsaw.

Table I. Experimental data pertaining to the development of showers produced by γ quanta with $E_0 = 4$ BeV

t*	0,82	1,63	2,45	3,26	4.08	4.90	5,71	6.53	7.34	8.16	8.98	9,79	10,61
$\overline{N}_e$	3,09	7,16	11.53	14.68	16,12	15.96	14.12	12.18	9.57	7.85	6,00	4,50	2.7
$\Delta\overline{N}_e$	0.57	0,74	0.75	0,75	0.83	0,86	0.99	0.98	0.90	0.91	0,98	1,1	1,4
$\lg(\delta\overline{N_e^2}/\overline{N}_e)$	1,630	1,866	1,853	1,881	1,981	0.03	0.165	0.210	0.205	0.152	0.166	0.163	0,116
W	0	0	0	0.143	0,393	0.262	0.155	0.048	0	0	0	0	0

*The depth t is measured from the point of conversion of the primary γ quantum.

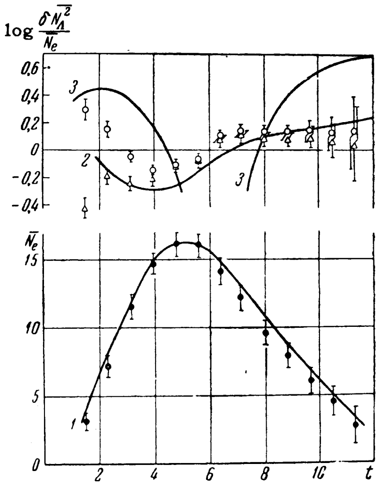
Table II. Data pertaining to the development of a shower produced by a primary γ quantum with energy $E_0 = 4$ BeV in xenon

t	1,46	2.31	3.13	3.94	4.76	5.58	6.39	7,21	8.02	8,84	9,66	10.47	11,29
$\overline{N}_e$	3.09	7.16	11,53	14.68	16,12	15.96	14.12	12.18	9.57	7.85	6.00	4.50	2.7
$\Delta\overline{N}_e$	0,57	0,74	0.75	0,75	0.83	0.86	0,99	0.98	0,90	0.91	0.98	1.10	1.40
lg φ	1,587	1.801	1.745	1.768	1,874	1,914	0,072	0.102	0,078	0.080	0,088	0.056	0.030
Δlg φ	0,038	0,033	0.026	0.024	0.025	0,027	0.032	0.037	0.042	0,053	0.073	0.136	0.262
lg ψ	0.288	0.148	1.947	1,847	1.891	1.926	0.102	0.143	0.130	0.128	0.140	0.119	0.126
Δlg ψ	0.086	0.051	0,034	0.028	0.028	0.029	0.036	0.040	0.047	0.058	0,083	0.127	0,268

Note: $\varphi=(\delta\overline{N_e^2}-\delta\overline{N_{E_0}^2})/\overline{N}_e$, $\psi=(\delta\overline{N_e^2}-\delta\overline{N_{E_0}^2}+\delta\overline{N_\lambda^2})/\overline{N}_e$.

The energy of the low-energy γ quantum was determined by a method described earlier[20].

As mentioned above, the depth of shower development was measured from the point of conversion of the primary quantum. It was our aim to investigate the shower fluctuations due to the γ quanta. It was therefore necessary to take into account a certain shift of the experimentally obtained points of the cascade curve. Inasmuch as the absorption coefficient for photons is $\sigma = 0.77$ at the investigated energies[2], and the absorption of the γ quanta can be given by $ke^{-\sigma t}$, the shift of our averaged cascade curve, corresponding to the distribution of the conversion ranges over the interval 0—63 mm, is $\bar{\lambda} = 0.68 t_0$. We shall henceforth shift all the experimental points by this amount, in order to obtain the result corresponding to the showers due to the γ quanta. Table II lists the corresponding results.



Cascade curve $\overline{N_e}(t)$ and fluctuations $\log(\delta\overline{N^2}/\overline{N_e})$ in an electron-photon shower with energy 4 BeV, produced by a γ quantum in xenon. Δ – fluctuations without account of the scatter in the primary γ-quantum conversion lengths, ○ – fluctuations with account of the scatter in the conversion lengths. 1 – theoretical dependence of $\log(\delta\overline{N^2}/\overline{N})$ on t, calculated for a shower produced by the electron for a ratio $\log(E_0/E) = 2$; 3 – the same for $\log(E_0/E) = 3$[24].

DISCUSSION OF THE RESULTS

A. Cascade curve. The experimental values of the points of the cascade curve $\overline{N}_e(t)$, listed in Table II, were compared with the theoretical values calculated by the method of moments[4,21] for a shower produced by a γ quantum. The minimum energy E of the electrons included in the calculations was assumed equal to the critical value. The average number of particles at the maximum, determined by experiment, was 16.1, corresponding to a $E_0 = 4$ BeV. The cascade curve was calculated for these values of E_0 and E.

The comparison of the experimental data with the theoretical ones is shown in the figure.

The experimental points are plotted together with the statistical errors estimated with the aid of confidence intervals[22]. The reliability of the confidence interval was assumed equal to 0.95. In addition to the statistical errors, there are also others connected with the inaccuracy in the determination of the t_0-unit referred to the reprojector scale and the inaccuracy in the calculation of the number of particles at a given depth t. These, however, can be neglected compared with the statistical errors. It can be seen that the experimental points fit the theoretical curve within the limits of errors.

B. Fluctuations. The function $\delta\overline{N_e^2(t)}$ which we obtained by experiment (see the figure and Table I) contains fluctuations due to the scatter in the energies of the primary photons $\delta\overline{N_{E_0}^2}$ but does not take into account the fluctuations $\delta\overline{N_\lambda^2}$ connected with the scatter of conversion ranges of the γ quanta producing the shower. Thus, the fluctuations $\delta\overline{N_\Lambda^2(t)}$ in a shower produced by a γ quantum will be given by the formula 2)

$$\delta\overline{N_\Lambda^2(t)} = \delta\overline{N^2(t)} - \delta\overline{N_{E_0}^2} + \delta\overline{N_\lambda^2(t)}. \qquad (1)$$

In order to estimate the value of $\delta\overline{N_{E_0}^2}$, we assume that the energy spectrum of the primary γ quanta has the form $kE_0^{-2.15}$[23] 3). The energy interval containing the selected showers has as its limits 3.4 and 5.5 BeV. This corresponds, for the assumed form of the spectrum, to a variance $\delta E_0^2 = 0.37\ \mathrm{BeV}^2$. From this it is easy to estimate the value of $\delta\overline{N_{E_0}^2}$. The value of $\delta\overline{N_\lambda^2(t)}$ at different depths is determined by suitably shifting the cascade curve through $\pm 0.68\, t_0$.

The values of $\delta\overline{N_\Lambda^2(t)}$ obtained by the foregoing method are listed in Table II. The figure shows the function $f(t) = \log(\delta\overline{N_\Lambda^2}/\overline{N}_e)$; we see that at small depths the main contribution to the fluctuations is made by the scatter of the ranges for the conversion of the primary γ quanta. Starting with the maximum of shower development, this contribution amounts to a relatively small fraction of the fluctuations. The minimum of the fluctuations occurs, as expected, at the maximum shower development. The form of the curve $\log[\delta\overline{N_\Lambda^2(t)}/N_e(t)]$ for an energy ratio $\log(E_0/E) = 2.6$ is similar to the form of the same curve but obtained theoretically for a value $\log(E_0/E) = 2$[24] 4).

CONCLUSION

The results obtained reduce to the following.

1) The cascade curve, calculated for xenon with the assumed values $\beta = 10.1$ MeV, $E = \beta$, $E_0 = 4$ BeV, and $t_0 = 3.8$ cm agrees well, within the limits of experimental errors, with the experimental data.

2) The positions of the maxima of shower development fluctuate from $t = 4t_0$ to $t = 7t_0$ with an average value $\bar{t} = 5t_0$. The fluctuations in the position of $t_{\max}$, connected with the scatter of the energy, should range accordingly from $t = 4.8\,t_0$ to $t = 5.3\,t_0$.

3) The fluctuations in showers with energy $E_0 = 4$ BeV in xenon are minimal near the maximum of shower development.

In the first t-units before the maximum, and far beyond the maximum, the fluctuations obtained by experiment in the showers investigated by us are larger than the Poisson fluctuations. In the region of maximum shower development, the fluctuations are smaller than the Poisson fluctuations.

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2) We assume that in our energy range the quantities $\delta\overline{N^2}$, $\delta\overline{N_{E_0}^2}$, and $\delta\overline{N_\lambda^2}$ are independent.

3) This form of spectrum does not differ, within the limits of errors, from the spectrum of the number of particles at the maximum for the 73 showers selected by us.

4) The theoretical curve is for showers produced by a primary electron.

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