MATRIX STRUCTURE OF THE MESON-PHOTON, NUCLEON-PHOTON, AND PHOTON-PHOTON SCATTERING AMPLITUDES AT HIGH ENERGIES

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It is demonstrated by means of the complex moment method that the invariant functions describing $\pi\gamma$, $N\gamma$ and $\gamma\gamma$ scattering can be factorized. The consequences to the observable quantities, ensuing from this factorization, are indicated.

1. Gribov and Pomeranchuk[1] have considered the spin structure of the πN and NN scattering amplitudes at high energies under the assumption that these processes are due essentially to Regge poles. In the present paper we consider under the same assumption the structure of the amplitudes of scattering in which photons participate.

We write down the invariant amplitudes φ, H, $g_{\mu\nu}$, $G_{\mu\nu}$ and $F_{\mu\nu,\rho\sigma}$ for the πN, NN, $\pi\gamma$, $N\gamma$, and $\gamma\gamma$ scattering, respectively, in the form

$$\varphi = A^{+}(s,t,u) + B^{-}(s,t,u)\hat{K}, \qquad K = k + k',$$

$$H = H_1(s,t,u) + H_2(s,t,u)\,[\hat{\mathcal{P}}_2^{(1)} + \hat{\mathcal{P}}_1^{(2)}]$$

$$+ H_3(s,t,u)\,\hat{\mathcal{P}}_2^{(1)}\hat{\mathcal{P}}_1^{(2)} + H_4(s,t,u)\,(i\gamma_5\hat{\mathcal{P}}_2)^{(1)}\,(i\gamma_5\hat{\mathcal{P}}_1)^{(2)}$$

$$+ H_5(s,t,u)\,(i\gamma_5)^{(1)}\,(i\gamma_5)^{(2)},$$

$$\mathcal{P}_1 = p_1 + p_1', \qquad \mathcal{P}_2 = p_2 + p_2';$$

$$g_{\mu\nu} = a^{+}(s,t,u)\,\pi_\mu\pi_\nu + b^{+}(s,t,u)\,n_\mu n_\nu,$$

$$K = k + k', \quad L = l + l', \quad Q = l - l' = k' - k,$$

$$K' = K - \frac{KL}{L^2}L,$$

$$N_\mu = \varepsilon_{\mu\alpha\beta\gamma}K'_\alpha L_\beta Q_\gamma, \quad \pi_\mu = K'_\mu(-K'^2)^{-1/2},$$

$$n_\mu = N_\mu(-N^2)^{-1/2};$$

$$G_{\mu\nu} = G_1^{+}(s,t,u)\,\bar{\pi}_\mu\bar{\pi}_\nu + G_2^{-}(s,t,u)\,\bar{\pi}_\mu\bar{\pi}_\nu\hat{L}$$

$$+ G_3^{+}(s,t,u)\,\bar{n}_\mu\bar{n}_\nu + G_4^{-}(s,t,u)\,\bar{n}_\mu\bar{n}_\nu\hat{L}$$

$$G_5^{+}(s,t,u)\,(\bar{\pi}_\mu\bar{n}_\nu - \bar{\pi}_\nu\bar{n}_\mu)\,i\gamma_5$$

$$+ G_6^{+}(s,t,u)\,(\bar{\pi}_\mu\bar{n}_\nu + \bar{\pi}_\nu\bar{n}_\mu)\,i\gamma_5\hat{L},$$

$$\mathcal{P} = p + p', \qquad L = l + l', \qquad \bar{Q} = l - l' = p' - p,$$

$$\mathcal{P}' = \mathcal{P} - \frac{\mathcal{P}L}{L^2}L, \quad \bar{N}_\mu = \varepsilon_{\mu\alpha\beta\gamma}\mathcal{P}'_\alpha L_\beta\bar{Q}_\gamma,$$

$$\bar{\pi}_\mu = \mathcal{P}'_\mu(-\mathcal{P}'^2)^{-1/2}, \quad \bar{n}_\mu = \bar{N}_\mu(-\bar{N}^2)^{-1/2};$$

$$F_{\mu\nu,\rho\sigma} = F_1^{+}(s,t,u)\,\tilde{\pi}_\mu\tilde{\pi}_\nu\tilde{\pi}_\rho\tilde{\pi}_\sigma + F_2^{+}(s,t,u)\,\tilde{n}_\mu\tilde{n}_\nu\tilde{n}_\rho\tilde{n}_\sigma$$

$$+ F_3^{+}(s,t,u)\,(\tilde{\pi}_\mu\tilde{\pi}_\nu\tilde{n}_\rho\tilde{n}_\sigma + \tilde{\pi}_\rho\tilde{\pi}_\sigma\tilde{n}_\mu\tilde{n}_\nu)$$

$$+ F_4^{-}(s,t,u)\,(\tilde{\pi}_\mu\tilde{n}_\nu + \tilde{n}_\mu\tilde{\pi}_\nu)(\tilde{\pi}_\rho\tilde{n}_\sigma + \tilde{\pi}_\sigma\tilde{n}_\rho)$$

$$+ F_5^{+}(s,t,u)\,(\tilde{\pi}_\mu\tilde{n}_\nu - \tilde{n}_\mu\tilde{\pi}_\nu)(\tilde{\pi}_\rho\tilde{n}_\sigma - \tilde{\pi}_\sigma\tilde{n}_\rho),$$

$$L_1 = l_1 + l_1', \quad L_2 = l_2 + l_2', \quad \tilde{Q} = l_1 - l_1' = l_2' - l_2,$$

$$L_1' = L_1 - \frac{L_1L_2}{L_2^2}L_2, \quad L_2' = L_2 - \frac{L_1L_2}{L_1^2}L_1,$$

$$\tilde{N}_\delta = \varepsilon_{\delta\alpha\beta\gamma}L'_{2\alpha}L_{1\beta}\tilde{Q}_\gamma; \quad \tilde{\pi}_{\mu,\nu} = L'_{1\mu,\nu}(-L_1'^2)^{-1/2},$$

$$\tilde{\pi}_{\rho,\sigma} = L'_{2\rho,\sigma}(-L_2'^2)^{-1/2}, \quad \tilde{n}_\delta = \tilde{N}_\delta(-\tilde{N}^2)^{-1/2}.$$

Here k, k′; p, p_1, p_2, p_1', p_2'; l_ν, $l_{1\nu}$, $l_{2\sigma}$, l_μ', $l_{1\mu}'$, and $l_{2\rho}'$ are the momenta of the mesons, nucleons, and photons, respectively, before and after scattering. The indices + and − indicate whether the covariant functions reverse sign upon making the substitution $s \rightleftarrows u$. The expression for the photon-photon scattering amplitude can be obtained in the given form in analogy with the procedure used in the derivation of the expressions for NN and $N\gamma$ scattering [2,3], taking into account parity conservation and the crossing symmetry for each photon pair.

To obtain asymptotic expressions for the invariant functions, it is necessary to expand the scattering amplitudes of the indicated processes in the t-channel in partial waves. This is conveniently done in terms of the helicity amplitudes. [4] We present a classification of the helicity states of two pions, two photons, and a nucleon-antinucleon pair with specified total momentum J in the channel where t is the energy.

The two pions can be in the sole state $|J, 00\rangle$ with quantum numbers $(-1)^J P = +1$, $(-1)^J C = +1$, and $C = +1$. The nucleon-antinucleon pair can be in the following states[5]:

$$|J, 0, +\rangle_{N\bar{N}} = |J, +{}^1/_2 + {}^1/_2\rangle + |J, -{}^1/_2 - {}^1/_2\rangle \quad (-1)^J P = +1, \quad (-1)^J C = +1,$$

$$|J, 1, +\rangle_{N\bar{N}} = |J, +{}^1/_2 - {}^1/_2\rangle + |J, -{}^1/_2 + {}^1/_2\rangle \quad (-1)^J P = +1, \quad (-1)^J C = +1,$$

$$|J, 0, -\rangle_{N\bar{N}} = |J, +{}^1/_2 + {}^1/_2\rangle - |J, -{}^1/_2 - {}^1/_2\rangle \quad (-1)^J P = -1, \quad (-1)^J C = +1,$$

$$|J, 1, -\rangle_{N\bar{N}} = |J, +{}^1/_2 - {}^1/_2\rangle - |J, -{}^1/_2 + {}^1/_2\rangle \quad (-1)^J P = -1, \quad (-1)^J C = -1.$$

The two photons can be in the following states with C = +1:

$$|J, 0, +\rangle_{\gamma\gamma} = |J, +1+1\rangle + |J, -1-1\rangle \quad (-1)^J P = +1, \quad (-1)^J C = +1.$$

$$|J, 2, +\rangle_{\gamma\gamma} = |J, +1-1\rangle + |J, -1+1\rangle \quad (-1)^J P = +1, \quad (-1)^J C = +1.$$

$$|J, 0, -\rangle_{\gamma\gamma} = |J, +1+1\rangle - |J, -1-1\rangle \quad (-1)^J P = -1, \quad (-1)^J C = +1.$$

$$|J, 2, -\rangle_{\gamma\gamma} = |J, +1-1\rangle - |J, -1+1\rangle \quad (-1)^J P = -1, \quad (-1)^J C = -1.$$

The numbers 0, 1, and 2 indicate the absolute value of the projection of the total spin on the direction of relative motion, and consequently determine the minimum value of the momentum J.

The processes under consideration are determined by the partial amplitudes of the transitions between states with identical quantum numbers. In view of the fact that in the investigated case C has a fixed value +1, three different sets of quantum numbers are possible, corresponding to three types of Regge-pole trajectories: P-trajectories with vacuum quantum numbers $(-1)^J P = +1$, $(-1)^J C = +1$. Q-trajectories with negative signature $(-1)^J P = -1$, $(-1)^J C = -1$, and S-trajectories with negative parity $(-1)^J P = -1$, $(-1)^J C = +1$.

In particular, the $\gamma\gamma$ and $N\gamma$ scattering processes are determined by five and six different partial amplitudes, respectively, corresponding to the number of invariant functions in the indicated amplitudes.

2. Let us consider for the sake of simplicity the $\pi\gamma$ scattering amplitude. The expansion of the spiral amplitudes in partial waves in the c.m.s. in the t-channel is

$$\langle 00 | g | ++\rangle = -{}^1/_2 [a^+(s, t, u) + b^+(s, t, u)] = \frac{2}{\pi}\left(\frac{t}{t-4\mu^2}\right)^{1/4} \sum_{J=0}^{\infty} (2J+1)\, g_{00}^J(t)\, P_J(z), \tag{1}$$

$$\langle 00 | g | +-\rangle = -{}^1/_2 [a^+(s, t, u) - b^+(s, t, u)] = \frac{2}{\pi}\left(\frac{t}{t-4\mu^2}\right)^{1/4} \sum_{J=2}^{\infty} (2J+1)\, g_{02}^J(t) \times \frac{1-z^2}{\sqrt{(J-1)J(J+1)(J+2)}} P_J''(z), \tag{2}$$

where

$$g_{00}^J(t) = \langle 00 | S^J | +1+1\rangle = -\frac{\pi}{2}\left(\frac{t-4\mu^2}{t}\right)^{1/4} \int_{z_0}^{\infty} [a_1^+(s, t) + b_1^+(s, t)]\, Q_J(z)\, dz. \tag{3}$$

$$g_{02}^J(t) = \langle 00 | S^J | +1-1\rangle = -\frac{\pi}{2}\left(\frac{t-4\mu^2}{t}\right)^{1/4} \int_{z_0}^{\infty} [a_1^+(s, t) - b_1^+(s, t)] \times \frac{z^2-1}{\sqrt{(J-1)J(J+1)(J+2)}} Q_J''(z)\, dz, \tag{4}$$

$z = (-2s + 2\mu^2 - t)/\sqrt{t(t-4\mu^2)}$, $a_1^+(s, t)$ and $b_1^+(s, t)$ are the absorptive parts of the invariant functions in the s-channel, and the kinematic factor $[t/(t-4\mu^2)]^{1/4}$ is due to the normalization.

We note that in accordance with the optical theorem the value of $-[a_1^+(s, t) - b_1^+(s, t)]$ at $t = 0$, i.e., the imaginary part of the combination $-[a^+(s, t, u) - b^+(s, t, u)]$ at $t = 0$, determines the total $\pi\gamma$ scattering cross section, while $-[a^+(s, t, u) + b^+(s, t, u)] = 0$ when $t = 0$, as a consequence of the conservation of the projection of the total momentum on the direction of the relative motion [6].

To obtain asymptotic expressions for the functions $a^+(s, t, u)$ and $b^+(s, t, u)$ it is necessary, in accordance with the general idea, to continue analytically the partial waves to complex values of J, to write the sum over the partial waves in the form of a Watson-Sommerfeld integral, and, stipulating that the moving singularities of the partial waves are only poles, find the contribution from these poles to the invariant functions $a^+(s, t, u)$ and $b^+(s, t, u)$. The partial amplitudes $g_{00}^J(t)$ and $g_{02}^J(t)$ have the quantum numbers of vacuum and can therefore enter in combination with the principal vacuum pole, which gives asymptotically the main contribution to the functions $a^+(s, t, u)$ and $b^+(s, t, u)$. In order to obtain the values of $a^+(s, t, u)$ and $b^+(s, t, u)$ at negative t (the physical region of the s-channel) it is necessary to be able to shift the integration contour in the complex J plane to the left of the line Re J = 1, since the principal vacuum pole passes when t = 0 through the point J = 1 and moves to the left in the J plane as t decreases.

In going through the described procedure, no difficulties arise with expression (1), but expression (2) must be treated with some caution. First, the summation in (2) begins with J = 2, and therefore, according to Mandelstam [7], it is necessary to use the formula

$$-\frac{P_J(z)}{\sin \pi J} = \frac{Q_{-J-1}(z) - Q_J(z)}{\pi \cos \pi J}$$

and retain asymptotically only the contribution from $Q_{-J-1}(z)$, which is equivalent to using the asymptotic expression $P_J(z) \sim z^J$ as $z \to \infty$ for Re $J \le -\frac{1}{2}$ and consequently $P_J''(z) \sim J(J-1)z^{J-2}$ as $z \to \infty$ for Re $J \le \frac{3}{2}$.

Second, in order for the integrand in the Watson-Sommerfeld integral to have no root branch point at J = 1, it is necessary that $g_{02}^J(t)$ have a stationary branch point at J = 1. We have here two possibilities, As $J \to 1$ either $g_{02}^J(t) \sim \sqrt{J-1}\,\varphi(J, t)$ or $g_{02}^J(t) \sim \varphi(J, t)/\sqrt{J-1}$, where the function $\varphi(J, t)$ no longer has a stationary branch point at J = 1.

The choice of the particular variant is determined uniquely by the set of intermediate states in the unitarity conditions in the t-channel. If the lowest intermediate states are considered to be the two-pion states, i.e., if the threshold of the different processes in the t-channel is $t = 4\mu^2$, then we see from (4) that $g_{02}^J(t) \sim \varphi(J, t)/\sqrt{J-1}$, since the integral does not vanish for small t (the combination $-[a_1^+(s, t) - b_1^+(s, t)]$, which determines the total cross section of the $\pi\gamma$ scattering when t = 0, is positive definite). Then we shall have a stationary pole at J – 1 in the analogous partial $\gamma\gamma$ scattering wave, but this will not contradict the unitarity conditions, since two-photon states are not admissible as intermediate states. On the other hand, if the photons are taken into consideration in the intermediate states, i.e., if we assume that the point t = 0 is the threshold, then, in order to avoid contradiction of the unitarity condition, we must choose the opposite variant $g_{02}^J(t) \sim \sqrt{J-1}\,\varphi(J, t)$ as $J \to 1$, which should be ensured by the vanishing of the corresponding integral in (4).

It is now easy to write the contribution from the poles to the functions $a^+(s, t, u)$ and $b^+(s, t, u)$ for large s:

$$a^+(s, t, u) + b^+(s, t, u) = \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}\, s^{\alpha(t)} \operatorname{res} g_{00}^J(t)\big|_{J=\alpha(t)} \times \left\{-2\left(\frac{t}{t-4\mu^2}\right)^{1/4} \frac{\Gamma[2\alpha(t)+2]}{\Gamma^2[\alpha(t)+1]}\, t^{-\alpha(t)/2}\,(t-4\mu^2)^{\alpha(t)/2}\right\}, \quad (5)$$

$$a^+(s, t, u) - b^+(s, t, u) = \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} \times s^{\alpha(t)} \frac{\alpha(t)[\alpha(t)-1] \operatorname{res} g_{02}^J(t)}{\{[\alpha(t)-1]\,\alpha(t)\,[\alpha(t)+1]\,[\alpha(t)+2]\}^{1/2}}\bigg|_{J=\alpha(t)} \times \left\{-2\left(\frac{t}{t-4\mu^2}\right)^{1/4} \frac{\Gamma[2\alpha(t)+2]}{\Gamma^2[\alpha(t)+1]}\, t^{-\alpha(t)/2}\,(t-4\mu^2)^{-\alpha(t)/2}\right\}. \quad (6)$$

Taking into account the boundary condition for the trajectory of the principal vacuum pole $\alpha_P(0) = 1$, we see that in the case when $g_{02}^J(t) \sim \varphi(J, t)/\sqrt{J-1}$ and for the usual threshold behavior of $g_{02}^J(t)$ as $t \to 0$, the total $\pi\gamma$ scattering cross section tends to a constant value at high energies. On the other hand if $g_{02}(t) \sim \sqrt{J-1} \times \varphi(J, t)$, then for the total $\pi\gamma$ scattering cross section not to vanish it is necessary that the residue of the principal vacuum pole, in addition to exhibiting the usual threshold behavior, become infinite at t = 0 so as to compensate for the factor $[\alpha(t) - 1]$ in (6). At the same time, the residues of the next vacuum poles should behave in the usual manner at t = 0.

3. Proceeding analogously, we can find asymptotic expressions for all the invariant functions of the processes considered. The vacuum poles contribute to all the invariant functions, except $H_5^+(s, t, u)$, $G_5^+(s, t, u)$, and $F_5^+(s, t, u)$, which are determined by S-type poles; however, the contributions to $H_4^-(s, t, u)$[1,8,9], $G_6^+(s, t, u)$, and $F_4^-(s, t, u)$ are asymptotically small (these functions are Q-type poles).

If we take into account the fact that the residues of the partial amplitudes, which combine with a definite Regge pole, factor out [1,10], then it is possible to obtain connections between the invariant functions at large s and small t, which are conveniently written in the form

$$[G_1^+(s,t,u)\,\bar{\pi}_\mu\bar{\pi}_\nu + G_2^-(s,t,u)\,\bar{\pi}_\mu\bar{\pi}_\nu\hat{L} + G_3^+(s,t,u)\,\bar{n}_\mu\bar{n}_\nu$$
$$+ G_4^-(s,t,u)\,\bar{n}_\mu\bar{n}_\nu\hat{L}]\,f_{\pi\pi}^+(s,t,u) = [a^+(s,t,u)\,\bar{\pi}_\mu\bar{\pi}_\nu$$
$$+ b^+(s,t,u)\,\bar{n}_\mu\bar{n}_\nu]$$
$$\times[A^+(s,t,u) + B^-(s,t,u)\,\hat{L}],$$
$$[F_1^+(s,t,u)\,\tilde{\pi}_\mu\tilde{\pi}_\nu\tilde{\pi}_\rho\tilde{\pi}_\sigma + F_2^+(s,t,u)\,\tilde{n}_\mu\tilde{n}_\nu\tilde{n}_\rho\tilde{n}_\sigma$$
$$+ F_3^+(s,t,u)\,(\tilde{\pi}_\mu\tilde{\pi}_\nu\tilde{n}_\rho\tilde{n}_\sigma$$
$$+ \tilde{\pi}_\rho\tilde{\pi}_\sigma\tilde{n}_\mu\tilde{n}_\nu)]\,f_{\pi\pi}^+(s,t,u) = [a^+(s,t,u)\,\tilde{\pi}_\mu\tilde{\pi}_\nu$$
$$+ b^+(s,t,u)\,\tilde{n}_\mu\tilde{n}_\nu]$$
$$\times[a^+(s,t,u)\,\tilde{\pi}_\rho\tilde{\pi}_\sigma + b^+(s,t,u)\,\tilde{n}_\rho\tilde{n}_\sigma], \qquad (7)$$

where $f_{\pi\pi}^+(s,t,u)$ is the invariant $\pi\pi$ scattering amplitude.

On the other hand, using formula (7) of the paper by Gribov and Pomeranchuk [1] we can readily obtain the relation

$$[F_1^+(s,t,u)\,\tilde{\pi}_\mu\tilde{\pi}_\nu\tilde{\pi}_\rho\tilde{\pi}_\sigma + F_2^+(s,t,u)\,\tilde{n}_\mu\tilde{n}_\nu\tilde{n}_\rho\tilde{n}_\sigma$$
$$+ F_3^+(s,t,u)\,(\tilde{\pi}_\mu\tilde{\pi}_\nu\tilde{n}_\rho\tilde{n}_\sigma + \tilde{\pi}_\rho\tilde{\pi}_\sigma\tilde{n}_\mu\tilde{n}_\nu)]\,[H_1^+(s,t,u)$$
$$+ H_2^-(s,t,u)\,(\hat{\mathscr{P}}_2^{(1)} + \hat{\mathscr{P}}_1^{(2)})$$
$$+ H_3^+(s,t,u)\,\hat{\mathscr{P}}_2^{(1)}\hat{\mathscr{P}}_1^{(2)}] = [G_1^+]\,(s,t,u)\,\tilde{\pi}_\mu\tilde{\pi}_\nu$$
$$+ G_2^-(s,t,u)\,\tilde{\pi}_\mu\tilde{\pi}_\nu\hat{\mathscr{P}}_2^{(1)}$$
$$+ G_3^+(s,t,u)\,\tilde{n}_\mu\tilde{n}_\nu + G_4^-(s,t,u)\,\tilde{n}_\mu\tilde{n}_\nu\hat{\mathscr{P}}_2^{(1)}]$$
$$\times[G_1^+(s,t,u)\,\tilde{\pi}_\rho\tilde{\pi}_\sigma + G_2^-(s,t,u)\,\tilde{\pi}_\rho\tilde{\pi}_\sigma\hat{\mathscr{P}}_1^{(2)}$$
$$+ G_3^+(s,t,u)\,\tilde{n}_\rho\tilde{n}_\sigma + G_4^-(s,t,u)\,\tilde{n}_\rho\tilde{n}_\sigma\hat{\mathscr{P}}_1^{(2)}]. \qquad (8)$$

For the $\pi\gamma$, $N\gamma$, and $\gamma\gamma$ scattering amplitudes at large s and small t we can write down an explicitly factorized expression, analogous to that proposed by Okun' for NN scattering [5]:

$$g_{\mu\nu} = D(s)\,s^{-2}\left\{\left[\Gamma_\gamma^{(1)}\left(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\alpha}\frac{L_\nu L_\beta}{t}\right.\right.\right.$$
$$\left.+ \delta_{\nu\beta}\frac{L_\mu L_\alpha}{t} + \frac{L_\mu L_\nu L_\alpha L_\beta}{t^2}\right)$$
$$\left.\left.+ \Gamma_\gamma^{(2)}\,\varepsilon_{\mu\alpha\alpha'\alpha''}\varepsilon_{\nu\beta\beta'\beta''}\frac{L_{\alpha'}L_{\beta'}}{t}\,Q_{\alpha''}Q_{\beta''}\right]\Gamma_\pi K_\alpha K_\beta\right\},$$

$$G_{\mu\nu} = D(s)\,s^{-3}\left\{\left(\Gamma_N^{(1)}\frac{\mathscr{P}_\delta}{m} + \Gamma_N^{(2)}\gamma_\delta\right)\mathscr{P}_\alpha\mathscr{P}_\beta\right.$$
$$\times\left[\Gamma_\gamma^{(1)}\left(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\alpha}\frac{L_\nu L_\beta}{t} + \delta_{\nu\beta}\frac{L_\mu L_\alpha}{t} + \frac{L_\mu L_\nu L_\alpha L_\beta}{t^2}\right)\right.$$
$$\left.\left.+ \Gamma_\gamma^{(2)}\varepsilon_{\mu\alpha\alpha'\alpha''}\varepsilon_{\nu\beta\beta'\beta''}\frac{L_{\alpha'}L_{\beta'}}{t}\,\bar{Q}_{\alpha''}\bar{Q}_{\beta''}\right]L_\delta\right\},$$

$$F_{\mu\nu,\rho\sigma} = D(s)\,s^{-4}\left\{\left[\Gamma_\gamma^{(1)}\left(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\alpha}\frac{L_{1\nu}L_{1\beta}}{t}\right.\right.\right.$$
$$\left.+ \delta_{\nu\beta}\frac{L_{1\mu}L_{1\alpha}}{t} + \frac{L_{1\mu}L_{1\nu}L_{1\alpha}L_{1\beta}}{t^2}\right)$$
$$\left.\left.+ \Gamma_\gamma^{(2)}\varepsilon_{\mu\alpha\alpha'\alpha''}\varepsilon_{\nu\beta\beta'\beta''}\frac{L_{1\alpha'}L_{1\beta'}}{t}\,\tilde{Q}_{\alpha''}\tilde{Q}_{\beta''}\right]L_{1\tau}L_{1\varkappa}\right\}$$
$$\times\left\{\left[\Gamma_\gamma^{(1)}\left(\delta_{\rho\tau}\delta_{\sigma\varkappa} + \delta_{\rho\tau}\frac{L_{2\sigma}L_{2\varkappa}}{t} + \delta_{\sigma\varkappa}\frac{L_{1\rho}L_{2\tau}}{t} + \frac{L_{2\rho}L_{2\sigma}L_{2\tau}L_{2\varkappa}}{t^2}\right)\right.\right.$$
$$\left.\left.+ \Gamma_\gamma^{(2)}\varepsilon_{\rho\tau\tau'\tau''}\varepsilon_{\sigma\varkappa\varkappa'\varkappa''}\frac{L_{2\tau'}L_{2\varkappa'}}{t}\,\tilde{Q}_{\tau''}\tilde{Q}_{\varkappa''}\right]L_{2\alpha}L_{2\beta}\right\}, \qquad (9)$$

where

$$D(s) = (1 - e^{-i\pi\alpha(t)})\,s^{\alpha(t)}/\sin\pi\alpha(t).$$

4. Let us note some consequences of (7), (8), and (9).

a) When t = 0 the optical theorem leads to relations between the total cross sections of the corresponding reactions, valid without neglecting $H_4^-(s,t,u)$[8], $G_6^+(s,t,u)$, and $F_4^-(s,t,u)$, viz.: $\sigma_{\pi\pi}\sigma_{\gamma\gamma} = (\sigma_{\pi\gamma})^2$, $\sigma_{\pi\pi}\sigma_{N\gamma} = \sigma_{\pi N}\sigma_{\pi\gamma}$, and $\sigma_{\gamma\gamma}\sigma_{NN} = (\sigma_{\gamma N})^2$ [10].

b) Analogous relations hold for the differential elastic scattering cross sections averaged over the polarizations, but now with neglect of $H_4^-(s,t,u)$, $G_6^+(s,t,u)$, and $F_4^-(s,t,u)$.

c) Taking into consideration the fact that the products $\Gamma_i(t)\Gamma_\gamma(t)$, which are determined by the imaginary parts of the invariant functions in the physical region, are real [1], we can show that there is no nucleon polarization in N scattering, and that the photons are plane-polarized with a polarization that is the same for all three reactions ($\pi\gamma$, $N\gamma$, and $\gamma\gamma$ scattering).

d) There is no correlation of the particle polarization in $N\gamma$ and $\gamma\gamma$ scattering.

e) The nucleon spin flip in $N\gamma$ scattering is the same as in πN and NN scattering [1]. The polarization state of the scattered photons is the same for all three reactions, ($\pi\gamma$, $N\gamma$, and $\gamma\gamma$), if the incoming photons have the same polarization state. If the incoming photons are completely polarized, than the rotation of the Stokes vector is the same in the c.m.s. of these reactions.

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