THE THEORY OF THE ACOUSTOELECTRIC EFFECT

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The acoustoelectric effect which consists of the production of a direct current $\mathbf{j}^{ac}$ under the action of a sound wave propagating in a conductor is studied theoretically. A phenomenological theory of the effect is developed which considers the effect as one of second order in the deformation and which is valid in the limiting case of long waves. The frequency dependence and the tensor characteristics are determined for this case. The concept of an odd or an even acoustoelectric effect is introduced depending on whether the sign of the current $\mathbf{j}^{ac}$ remains the same or is reversed when the direction of propagation of the sound wave is reversed. It is shown that the even effect can exist only in crystals without a symmetry center. The general considerations are illustrated by a number of cases such as, for example, those of a piezoelectric conductor, a semiconductor with many energy minima, and a conductor having electrons and holes. In the last two cases sound absorption is also evaluated. The Mandel'shtam-Leontovich theory has been utilized for the calculation of the absorption coefficient.

1. GENERAL THEORY

A traveling sound wave propagated in a conductor drags the current carriers and creates a direct acoustoelectric current $\mathbf{j}^{ac}$ or in the case of an open circuit a constant acoustoelectric field $\mathbf{E}^{ac}$. This effect was first predicted by Parmenter[1], but the physical picture which he utilized for the description of the effect has called forth a number of objections[2]. The effect was observed for the first time by Weinreich et al[3] in the case of n-germanium. They also developed a theory of the effect in this case which agrees well with the experimental data.

The object of the present paper is to develop for the acoustoelectric effect a general phenomenological theory which is valid in the limiting case of low sound frequencies. In this section we obtain the frequency dependence of the effect and determine its tensor characteristics. In subsequent sections the general theory is illustrated on a number of examples.

The acoustoelectric effect is an effect of the second order in the deformation. The deformation produced by a sound wave is a periodic function of the coordinates and the time and, therefore, in the lowest order the direct current $\mathbf{j}^{ac}$ must be expressed as an average (with respect to the coordinates and the time) of a quadratic combination of the components of the displacement vector $\mathbf{u}$ or of its derivatives. It is evident that the displacement vector can appear in this combination only in the form of components of the deformation tensor or of its derivatives, since the current cannot be proportional to quantities which characterize a translation or a rotation of the crystal as a whole.[1)] Further, it is evident that at least one of the factors in this quadratic combination must contain the time derivative of the deformation tensor since a static deformation cannot give rise to a current.

The acoustoelectric effect can be odd or even with respect to a reversal of the direction of propagation of a traveling sound wave. We begin by considering the odd effect. In the lowest order with respect to the sound frequency ω and the propagation vector $\mathbf{q}$ the following expression (summation is implied over repeated indices) satisfies the requirements enumerated above:

$$j_i^{ac} = Y_{ik,abcd}\left\langle \frac{\partial u_{ab}}{\partial x_k}\dot{u}_{cd}\right\rangle, \tag{1.1}$$

where the symbol $\langle \ldots \rangle$ denotes averaging over time intervals considerably longer than the period of oscillation, and over a volume greater than $1/q^3$.

[1)]Strictly speaking, the last assertion is not valid if we take into account the Stewart-Tolman effect, which consists of the appearance of a current as a result of the crystal being accelerated as a whole. We shall neglect this effect because of its smallness (cf. [4a]).

We note that in fact (1.1) takes into account all the possible combinations of the given type of derivatives of the tensor u_{ab} since the following relations hold

$$\left\langle \frac{\partial u_{ab}}{\partial x_k} \dot{u}_{cd} \right\rangle = \left\langle \dot{u}_{ab} \frac{\partial u_{cd}}{\partial x_k} \right\rangle = -\left\langle u_{ab} \frac{\partial \dot{u}_{cd}}{\partial x_k} \right\rangle, \quad (1.2)$$

which are obtained as a result of integration by parts.

By utilizing (1.2) and also the symmetry of the deformation tensor we obtain the following symmetry properties of the tensor Y:

$$Y_{ik,abcd} = Y_{ik,cdab} = Y_{ik,bacd} = Y_{ik,abdc}. \quad (1.3)$$

Thus, for small ω and q the odd acoustoelectric current is proportional to $q\omega(u_{ik}^0)^2$, where u_{ik}^0 is the value of the amplitude of the deformation tensor in the sound wave.

Similarly, for an even acoustoelectric current we write

$$j_i^{ac} = \Xi_{i,\, abcd} \langle \dot{u}_{ab} \dot{u}_{cd} \rangle. \quad (1.4)$$

The number of spatial and time derivatives in this expression must be even since a change in the sign of the frequency or of the wave vector corresponds to a change in the direction of propagation of a traveling sound wave. The symmetry properties of the tensor Ξ with respect to the last four indices are the same as those of the tensor Y. An even effect can exist only in crystals without a symmetry center, since only for such crystals can a fifth rank tensor differ from zero. In the present case for small ω and q we have $j^{ac} \sim \omega^2(u_{ik}^0)^2$, i.e., the frequency dependence is the same both for the even and the odd effects. But in fact in the majority of the interesting cases the odd effect is considerably greater than the even one.

It is evident that (1.1) and (1.5) are not the most general quadratic expressions which can be constructed from the quantities $\partial u_{ab}/\partial x_k$ and $\dot{u}_{cd}$, since these quantities are taken at a particular instant of time and at a single point in space. These expressions hold only under the assumption that the correlation of the characteristic electronic quantities defining j^{ac} falls off rapidly at distances much smaller than a wavelength of sound, and at times which are much smaller than the period of the sound wave. It is specifically in this sense that we should interpret the requirement of the smallness of ω and q as a criterion for the applicability of the phenomenological expressions (1.1) and (1.4).

In the odd effect in a number of cases the field E^{ac} is related to the coefficient of sound absorption Γ by a simple order of magnitude estimate[4b]

$$E^{ac} = \Gamma S/eN_0 w \quad (1.5)$$

(where S is the flux density for the sound energy, e is the charge of the current carriers, N_0 is their equilibrium concentration, w is the velocity of sound). This can be obtained by utilizing simple considerations associated with the law of conservation of energy and of momentum. However, one should use this formula cautiously, since we shall see below that sometimes it can lead to grossly incorrect results, although there are cases in which it is either exact or gives the correct order of magnitude.

We now proceed to discuss examples which illustrate the general considerations presented above.

2. PIEZOELECTRIC SEMICONDUCTORS

In a piezoelectric sample the stress tensor s_{ik} is a linear function of the deformation tensor and of the electric field

$$s_{ik} = \lambda_{iklm} u_{lm} + \beta_{l,ik} \mathscr{E}_l, \quad (2.1)$$

where λ_{iklm} is the elastic modulus tensor (at constant $\mathscr{E}$), and $\beta_{l,ik}$ is the piezoelectric tensor. Similarly, the electric displacement vector is given by

$$D_i = \varepsilon_{ik} \mathscr{E}_k - 4\pi \beta_{i,\,kl} u_{kl}, \quad (2.2)$$

where ϵ is the dielectric permittivity tensor.

If the piezoelectric substance is deformed electric fields appear in it which are proportional to the deformation. If a sound wave is propagated in a piezoelectric semiconductor these fields, and also the resulting temperature gradients, give rise to an alternating electric current whose density to the first order in the deformation can be written in the form [2)]

$$j_i^{(1)} = \sigma_{ik} \mathscr{E}_k - eD_{ik} \frac{\partial N}{\partial x_k} - \gamma_{ik} \frac{\partial T}{\partial x_k}. \quad (2.3)$$

Here σ_{ik} is the conductivity tensor, D_{ik} is the tensor of the diffusion coefficients, $N = N_0 + \delta N$ is the concentration of the current carriers, N_0 is its value for $u_{ik} = 0$, T is the temperature.

The Joule heat produced by this current determines the specific sound absorption in piezoelectric semiconductors. This absorption has been investigated[5], and it was shown that the third term in (2.3) is negligibly small in comparison with the first two, and the following expression

[2)] We assume that the principal interaction of the sound wave with the current carriers is a piezoelectric one, and therefore do not write in (2.3) terms which are proportional, for example, to the gradient of the deformation potential.

was obtained for the density of sound energy absorbed per unit time:

$$\Gamma S = \frac{\beta_{l,ab}\beta_{p,cd}q_l q_p}{\sigma_{mn}q_m q_n}\,\frac{\langle \dot{u}_{ab}\dot{u}_{cd}\rangle}{(1+q^2/\varkappa^2)^2+(\omega\tau_\sigma)^2}\,; \qquad (2.4)$$

$$\varkappa^2 = \frac{4\pi e^2 q^2}{\varepsilon_{rs}q_r q_s}\,\frac{\partial N_0}{\partial\zeta}, \qquad \tau_\sigma = \frac{4\pi\varepsilon_{ik}q_i q_k}{\sigma_{mn}q_m q_n}. \qquad (2.5)$$

We now discuss the odd acoustoelectric effect in piezoelectric substances. It is evident that $\langle j^{(1)}\rangle = 0$. Therefore, in order to obtain after averaging an acoustoelectric current different from zero it is necessary to consider the following terms of the expansion which take into account the dependence of σ, D, and γ on the concentration of the current carriers, the electric field, the deformation tensor, etc. It can be easily verified that a contribution to the odd effect is given by only the following two terms:

$$j_i^{ac} = \langle \delta N \mathscr{E}_k\rangle\,\partial\sigma_{ik}/\partial N - \langle \delta N \partial T/\partial x_k\rangle\,\partial\gamma_{ik}/\partial N\,. \qquad (2.6)$$

It is necessary to indicate the sense in which we should interpret the derivatives with respect to N in (2.6). The kinetic coefficients σ_{ik} and γ_{ik} are functions of the variables which characterize the thermodynamic state of the semiconductor, and also of the concentration of the current carriers N which should be regarded as an additional variable (cf., [6]). To a high degree of accuracy the sound oscillations are adiabatic, and, therefore, the derivatives referred to above should be evaluated at constant entropy S (and likewise deformation u_{ik}). We shall not make a special point of calling attention to this circumstance. The adiabatic derivative can be transformed to the variables N and T and in this way can be expressed in terms of more convenient quantities:

$$\left(\frac{\partial\sigma_{ik}}{\partial N}\right)_S = \left(\frac{\partial\sigma_{ik}}{\partial N}\right)_T + \frac{T}{C_V}\left(\frac{\partial\sigma_{ik}}{\partial T}\right)_N\left(\frac{\partial\zeta}{\partial T}\right)_N, \qquad (2.7)$$

where C_V is the specific heat (at constant volume); ζ is the chemical potential of the current carriers.

With the aid of estimates analogous to those made in [5] it can be verified that the second term in (2.6) is considerably smaller than the first one. We set $\mathscr{E}_i = -\partial\varphi/\partial x_i$, thereby taking into account the fact that the electrical field is longitudinal. Then the remaining term can be rewritten in the following form

$$j_i^{ac} = \langle \delta N \mathscr{E}_\xi\rangle\,\partial\sigma_{i\xi}/\partial N = -\langle \delta N\,\partial\varphi/\partial\xi\rangle\,\partial\sigma_{i\xi}/\partial N,$$

where the ξ axis is directed along q; $\sigma_{i\xi} = \sigma_{ik}q_k/q$.

Utilizing the equation of continuity $e\partial\delta N/\partial t + \partial j_\xi/\partial\xi = 0$ we transform this expression:

$$\begin{aligned} j_i^{ac} &= \frac{\partial\sigma_{i\xi}}{\partial N}\left\langle\frac{\partial\delta N}{\partial\xi}\varphi\right\rangle = -\frac{1}{w}\frac{\partial\sigma_{i\xi}}{\partial N}\left\langle\varphi\frac{\partial\delta N}{\partial t}\right\rangle \\ &= -\frac{1}{w}\frac{\partial\sigma_{i\xi}}{\partial N}\left\langle\left(\varphi+\frac{1}{e}\frac{\partial\zeta}{\partial N}\delta N\right)\frac{\partial\delta N}{\partial t}\right\rangle \\ &= \frac{1}{ew}\frac{\partial\sigma_{i\xi}}{\partial N}\left\langle j_\xi\left(\mathscr{E}_\xi-\frac{1}{e}\frac{\partial\zeta}{\partial\xi}\right)\right\rangle. \end{aligned} \qquad (2.8)$$

In the derivation of (2.8) we have taken into account the fact that in a traveling wave all the quantities depend on the difference $q\xi-\omega t = q(\xi - wt)$, as a result of which we have

$$w\partial\delta N/\partial\xi = -\partial\delta N/\partial t.$$

Strictly speaking, this equation is not quite exact since any traveling sound wave is in actual fact spatially damped like $\exp(-\Gamma\xi/2)$. However, by means of direct estimates it can be verified that the corresponding error is negligibly small as long as

$$\frac{2\pi\beta^2}{\varepsilon\rho w^2}\,\frac{q^2}{\varkappa^2}\,\frac{1}{(1+q^2/\varkappa^2)^2+(\omega\tau_\sigma)^2} \ll 1.$$

This inequality is satisfied in practically all cases. In the following sections we shall also neglect the influence on the acoustoelectric effect of weak damping of sound waves.

On the other hand, the quantity $\langle j_i(\mathscr{E}_i - e^{-1}\partial\zeta/\partial x_i)\rangle$ is nothing other than the density of sound energy absorbed by the electrons per unit time ΓS. Therefore,

$$j_i^{ac} = \frac{\partial\sigma_{i\xi}}{\partial N}\,\frac{\Gamma S}{ew}\,. \qquad (2.9)$$

If $\sigma_{ik} \sim N$, while the second term in (2.7) is small in comparison with the first one, then expression (1.5) is obtained for E^{ac}.

On substituting (2.4) into (2.9) we finally obtain

$$j_i^{ac} = -\frac{1}{e}\,\frac{\partial\sigma_{ik}}{\partial N}\,\frac{\beta_{l,ab}\beta_{p,cd}}{\sigma_{mn}q_m q_n}\,\frac{q_l q_p\langle \dot{u}_{cd}\partial u_{ab}/\partial x_k\rangle}{(1+q^2/\varkappa^2)^2+(\omega\tau_\sigma)^2}\,. \qquad (2.10)$$

Expressions (2.4) and (2.10) are applicable if the tensor σ does not depend on ω and q. Conditions under which this holds have been discussed in [5]. We shall assume that these conditions are satisfied also in all the following sections.

On comparing (2.10) and (1.1) in the limiting case of small ω and q we obtain

$$Y_{ik,\,abcd} = -\frac{1}{e}\,\frac{\partial\sigma_{ik}}{\partial N}\,\frac{\beta_{l,ab}\beta_{p,cd}q_l q_p}{\sigma_{mn}q_m q_n}\,. \qquad (2.11)$$

Thus, in the present case the tensor Y depends in a complicated manner on the direction of the vector q. The conditions for the smallness of ω and q which were mentioned in the preceding section have the form $q \ll \kappa$, $\omega \ll 1/\tau_\sigma$. In concluding the present section we note that everything that we have said on the limits of applicability of the theory refers to the case when there is only one kind of current carriers. However, if there are several kinds, i.e., if there exist several groups of carriers such that the time for the establishment of equilibrium between these groups is relatively large, then the problem becomes more complicated, and it must be solved with the aid of Eqs. (3.2) and (4.1) discussed in the two following sections.

3. IMPURITY SEMICONDUCTOR WITH SEVERAL ENERGY MINIMA

We discuss here conductors in which the current carriers occupy a number of regions in p-space (energy minima) which go over into one another under symmetry transformations. The volume of such regions is assumed to be small compared to the volume of the first Brillouin zone, so that the components of the tensor of the deformation potential Λ_{ik}^{α} do not depend on p to a high degree of accuracy, but, of course, depend on the number of the region α.

Examples of such conductors are n-germanium and n-silicon. The absorption of sound and the acoustoelectric effect in n-Ge have been discussed in the paper by Weinreich et al [3] in the special case of transverse sound propagating along a fourfold axis. Here we shall discuss the general theory of this effect.

On assuming that there exist s equivalent minima (in n-Ge $s = 4$; in n-Si $s = 6$) we shall first determine the increment δn_{α}^{0} in the concentration of the electrons at the α-th minimum of n_0 as a result of a static deformation. In the case of such a deformation the energy of the electron at the α-th minimum will change by an amount $\delta\epsilon_{\alpha} + e\varphi_0 = \Lambda_{ik}^{\alpha}u_{ik} + e\varphi_0$.

The electrostatic potential φ_0 ought to be determined as a solution of the Poisson equation. However, in the case of long waves, when $q^2/\kappa^2 \ll 1$, this equation can be replaced by the condition of neutrality

$$\sum_{\alpha=1}^{s} \delta n_{\alpha}^{0} = 0.$$

From this it follows that

$$\delta n_{\alpha}^{0} = -Q\,(\delta\varepsilon_{\alpha} - \overline{\delta\varepsilon}), \qquad Q = \frac{\partial n_0}{\partial \zeta}, \quad \overline{\delta\varepsilon} = \frac{1}{s}\sum_{\alpha=1}^{s}\delta\varepsilon_{\alpha}. \tag{3.1}$$

However, if the deformation varies with time, then the increment in the concentration δn_{α} will be determined by the system of equations

$$e\frac{\partial \delta n_{\alpha}}{\partial t} + \frac{\partial}{\partial \xi}\left[-\frac{1}{e}\sigma_{\xi\xi}^{(\alpha)}\frac{\partial}{\partial\xi}(\delta\varepsilon_{\alpha} - \overline{\delta\varepsilon}) - \sigma_{\xi\xi}^{(\alpha)}\frac{\partial\varphi'}{\partial\xi} - eD_{\xi\xi}^{(\alpha)}\frac{\partial\delta n_{\alpha}}{\partial\xi}\right]$$
$$= \sum_{\beta=1}^{s}\left[w_{\beta\alpha}(\delta n_{\beta} - \delta n_{\beta}^{0}) - w_{\alpha\beta}(\delta n_{\alpha} - \delta n_{\alpha}^{0})\right], \tag{3.2}$$

where $w_{\alpha\beta}$ is the probability of transition for a current carrier from the α-th to the β-th minimum. This equation is applicable if the time for the establishment of equilibrium within a given minimum is much smaller than $1/w_{\alpha\beta}$. The microscopic derivation of these equations and of the expression for $w_{\alpha\beta}$ is given in the Appendix.

The electric field $-\partial\varphi'/\partial\xi$ vanishes in the case of a static deformation. In the case of a dynamic deformation, if $q^2/\kappa^2 \ll 1$ and $\omega\tau_{\sigma} \ll 1$, then to determine it we can again use the condition of neutrality which has the form

$$\sum_{\alpha}\delta n_{\alpha} = 0. \tag{3.3}$$

As before, the principal contribution to the acoustoelectric current is related to the dependence of $\sigma_{ik}^{(\alpha)}$ on n_{α}

$$j_i^{ac} = -\sum_{\alpha}\frac{\partial\sigma_{i\xi}^{(\alpha)}}{\partial n_{\alpha}}\left\langle \delta n_{\alpha}\frac{\partial}{\partial\xi}(\delta\varepsilon - \overline{\delta\varepsilon} + e\varphi')\right\rangle. \tag{3.4}$$

In order to be specific we shall evaluate this expression for the case of germanium when $s = 4$, $w_{\alpha\beta} = 1/\tau$. Then (3.2) assumes the form

$$(-i\omega + D_{\xi\xi}^{(\alpha)}q^2 + 4/\tau)\,\delta n_{\alpha} + e^{-1}\sigma_{\xi\xi}^{(\alpha)}q^2\varphi'$$
$$= -\frac{1}{e^2}\sigma_{\xi\xi}^{(\alpha)}q^2\delta\varepsilon_{\alpha} + \frac{Q}{\tau}\sum_{\beta}(\delta\varepsilon_{\beta} - \delta\varepsilon_{\alpha}). \tag{3.5}$$

We shall further restrict ourselves to the case of long waves, when $q^2D\tau = q^2\sigma\tau/e^2Q \ll 1$. Then (3.5) becomes considerably simplified, and on taking (3.3) into account its solution can be written in the form

$$\delta n_{\alpha} = \delta n_{\alpha}^{0}/(1 - i\omega\tau_M), \quad \tau_M = \tau/4,$$
$$e\varphi' = -\left\{\sum_{\alpha}\sigma_{\xi\xi}^{(\alpha)}\delta\varepsilon_{\alpha} - \overline{\delta\varepsilon}\sum_{\alpha}\sigma_{\xi\xi}^{(\alpha)} + e^2\sum_{\alpha}D_{\xi\xi}^{(\alpha)}\delta n_{\alpha}\right\}\Big/\sum_{\alpha}\sigma_{\xi\xi}^{(\alpha)}. \tag{3.6}$$

On substituting (3.6) into (3.4) we obtain

$$j_i^{ac} = \frac{1}{16e}\frac{\partial N_0}{\partial\zeta}\frac{\tau_M}{1 + (\omega\tau_M)^2}$$
$$\times\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}(\Lambda_{ab}^{\beta} - \Lambda_{ab}^{\alpha})(\Lambda_{cb}^{\alpha} - \widetilde{\Lambda}_{cd})\frac{\partial\sigma_{ik}^{(\alpha)}}{\partial n_{\alpha}}\left\langle\frac{\partial u_{ab}}{\partial x_k}\dot{u}_{cd}\right\rangle,$$
$$\widetilde{\Lambda}_{cd} = \sum_{\alpha}\Lambda_{cd}^{\alpha}\sigma_{\xi\xi}^{(\alpha)}\Big/\sum_{\alpha}\sigma_{\xi\xi}^{(\alpha)}. \tag{3.7}$$

From here we obtain for $\omega \ll 1/\tau_M$

$$Y_{ik,\,ab\,cd} = \frac{1}{16e}\frac{\partial N_0}{\partial\zeta}\tau_M\sum_{\alpha}\sum_{\beta}\frac{\partial\sigma_{ik}^{(\alpha)}}{\partial n_{\alpha}}(\Lambda_{ab}^{\beta} - \Lambda_{ab}^{\alpha})(\Lambda_{cd}^{\alpha} - \widetilde{\Lambda}_{cd}). \tag{3.8}$$

In order to determine when the relation (1.5) holds we shall evaluate the absorption of sound. In order to do this we make use of the Mandel'shtam-Leontovich method[7] which can be used in all those cases when a process characterized by a long time for the establishment of equilibrium can occur in a body. Following this method we shall determine the increment in the moduli of elasticity due to the interaction between sound and the current carriers:

$$\lambda_{iklm} = \frac{\partial s_{ik}}{\partial u_{lm}} = \left(\frac{\partial s_{ik}}{\partial u_{lm}}\right)_{n_\alpha} + \sum_\alpha \left(\frac{\partial s_{ik}}{\partial n_\alpha}\right)_{u_{lm}} \frac{\partial \delta n_\alpha}{\partial u_{lm}}, \quad (3.9)$$

where s_{ik} is the stress tensor.

Utilizing (3.6) we obtain

$$\lambda_{iklm} = (\lambda^0_{iklm} - i\omega\tau_M \lambda^\infty_{iklm})/(1 - i\omega\tau_M), \quad (3.10)$$

$$\lambda^0_{iklm} = \left(\frac{\partial s_{ik}}{\partial u_{lm}}\right)_{n_\alpha} + \sum_\alpha \left(\frac{\partial s_{ik}}{\partial n_\alpha}\right)_{u_{lm}} \frac{\partial \delta n^0_\alpha}{\partial u_{lm}}; \quad (3.11)$$

$$\lambda^\infty_{iklm} = (\partial s_{ik}/\partial u_{lm})_{n_\alpha}. \quad (3.12)$$

Here λ^0 is the modulus of elasticity for the crystal at zero frequency,[3)] while λ^∞ is the modulus of elasticity for the crystal at frequencies $\omega \gg 1/\tau_M$, or in the absence of current carriers.

The increment in the energy of the system of electrons as a result of the deformation is given by

$$\delta E = \sum_\alpha \varepsilon_\alpha \delta n_\alpha, \quad (3.13)$$

while the corresponding increment in the stress tensor is given by

$$\delta s_{ik} = \left(\frac{\partial \delta E}{\partial u_{ik}}\right)_{n_\alpha} = \sum_\alpha \frac{\partial \varepsilon_\alpha}{\partial u_{ik}} \delta n_\alpha = \sum_\alpha \Lambda^\alpha_{ik} \delta n_\alpha. \quad (3.14)$$

From this it follows that

$$(\partial s_{ik}/\partial n_\alpha)_{u_{lm}} = \Lambda^\alpha_{ik}. \quad (3.15)$$

On substituting (3.15) and (3.6) into (3.11) and (3.10) we obtain

$$\delta\lambda_{iklm} = \lambda_{iklm} - \lambda^\infty_{iklm} = \frac{1}{4} \sum_{\alpha\beta} \Lambda^\alpha_{ik} (\Lambda^\beta_{lm} - \Lambda^\alpha_{lm}) \frac{Q}{1 - i\omega\tau_M}. \quad (3.16)$$

On substituting the renormalized moduli of elasticity into the equation of motion we can obtain both the correction to the phase velocity of sound

$$\delta w = \mathrm{Re}\, \delta\lambda_{ilkm} u_i u_k q_l q_m / 2\rho q^2 u^2 w, \quad (3.17)$$

and also the absorption coefficient

$$\Gamma = 2\delta w \omega^2 \tau_M / w w_g, \quad (3.18)$$

where w_g is the group velocity of sound which appears in the expression for the flux density of sound energy.

By utilizing (3.16), (3.17), and (3.18), we obtain

$$\Gamma S = \frac{1}{32} \frac{\omega^2\tau_M}{1 + (\omega\tau_M)^2} \sum_{\alpha\beta} \delta\varepsilon_\alpha (\delta\varepsilon_\beta - \delta\varepsilon_\alpha) \frac{\partial N_0}{\partial \zeta}$$
$$= \frac{1}{16} \frac{\partial N_0}{\partial \zeta} \frac{\tau_M}{1 + (\omega\tau_M)^2} \sum_{\alpha\beta} \Lambda^\alpha_{ik} (\Lambda^\beta_{lm} - \Lambda^\alpha_{lm}) \langle \dot{u}_{ik} \dot{u}_{lm} \rangle. \quad (3.19)$$

Sometimes if the sound is propagated in the crystal along some direction possessing a high degree of symmetry it may turn out that $\mathbf{j}^{ac}$ and $\mathbf{E}^{ac}$ are also both parallel to this direction. If, moreover, all the $\sigma^{(\alpha)}_{\xi\xi}$ are equal to one another and are proportional to n_α, then relation (1.5) holds. Just such a situation occurs in the case discussed in [3]. But in all the other cases relation (1.5), generally speaking, does not hold.

In a cubic crystal we have

$$\Lambda^\alpha_{ik} = A\delta_{ik} + k^{(\alpha)}_i k^{(\alpha)}_k k^{-2} B, \quad (3.20)$$

where the vector $\mathbf{k}^{(\alpha)}$ determines the position of the α-th minimum in p-space.

In n-germanium the minima occur on three-fold axes. On substituting (3.20) into (3.16) it can be easily verified that in this case only the modulus λ_{xyxy} is renormalized as a result of transitions between minima. Therefore, absorption of this type will be experienced only by those oscillations the expressions for the velocity of sound for which contain this modulus.[4)] This fact was discovered experimentally in the work of Pomerantz et al[9].

Similar calculations show that in the case of n-silicon, in which the minima are situated on four-fold axes, the modulus λ_{xyxy} is not renormalized, but the moduli λ_{xxxx} and λ_{xxyy} are renormalized.

4. CONDUCTOR WITH CARRIERS OF OPPOSITE SIGN

We shall assume that the recombination time for carriers of opposite sign is much larger than the time for the establishment of equilibrium within a system of carriers of a given sign. Therefore, we can speak of the absorption of sound and of the acoustoelectric current associated with the lagging of the concentration of carriers with respect to its

[3)]The problem of the renormalization of the static moduli of elasticity has been experimentally investigated in the paper by Bruner and Keyes.[8] The same paper also contains a theoretical analysis applicable to the present case.

[4)]By an analysis of (3.5) simultaneously with (3.3) it can be shown that taking into account terms of order q^2D does not alter this conclusion.

equilibrium value corresponding to a given value of the deformation.

Equations analogous to (3.2) have the following form

$$e_1 \frac{\partial \delta n_1}{\partial t} + \operatorname{div} \mathbf{j}_1 = -e_1 \left(\frac{\delta n_1 - \delta n_1^0}{\tau_r} + \frac{\delta n_2 - \delta n_2^0}{\tau_r} \right),$$

$$e_2 \frac{\partial \delta n_2}{\partial t} + \operatorname{div} \mathbf{j}_2 = -e_2 \left(\frac{\delta n_1 - \delta n_1^0}{\tau_r} + \frac{\delta n_2 - \delta n_2^0}{\tau_r} \right),$$

$$e_1 = -e_2, \quad \delta n_1 = \delta n_2,$$

$$\delta n_1^0 = \delta n_2^0 = -Q_1 Q_2 (\delta \varepsilon_1 + \delta \varepsilon_2)/(Q_1 + Q_2),$$

$$\delta \varepsilon_1 = \Lambda_{ik}^{(1)} u_{ik}, \quad \delta \varepsilon_2 = \Lambda_{ik}^{(2)} u_{ik}. \tag{4.1}$$

The solution of these equations can be written in the form

$$\delta n_1 = \delta n_1^0/(1 - i\omega\tau_M), \tag{4.2}$$

$$\frac{1}{\tau_M} = \frac{2}{\tau} + \frac{(\sigma_{ik}^{(1)} D_{lm}^{(2)} + \sigma_{ik}^{(2)} D_{lm}^{(1)}) q_i q_k q_l q_m}{(\sigma_{rs}^{(1)} + \sigma_{rs}^{(2)}) q_r q_s}. \tag{4.3}$$

By analogy with the preceding section we obtain for the sound energy absorbed per unit time

$$\Gamma S = \frac{\tau_M}{1 + (\omega\tau_M)^2} \frac{Q_1 Q_2}{Q_2 + Q_2} (\Lambda_{ab}^{(1)} + \Lambda_{ab}^{(2)}) (\Lambda_{cd}^{(1)} + \Lambda_{cd}^{(2)}) \langle \dot{u}_{ab} \dot{u}_{cd} \rangle, \tag{4.4}$$

and for the acoustoelectric current

$$j_i^{ac} = \frac{1}{e_1 (\sigma_{lm}^{(1)} + \sigma_{lm}^{(2)}) q_l q_m} \left(\frac{\partial \sigma_{ik}^{(1)}}{\partial n_1} \sigma_{pq}^{(2)} - \frac{\partial \sigma_{ik}^{(2)}}{\partial n_2} \sigma_{pq}^{(1)} \right)$$
$$\times \frac{Q_1 Q_2}{Q_1 + Q_2} (\Lambda_{ab}^{(1)} + \Lambda_{ab}^{(2)}) (\Lambda_{cd}^{(1)} + \Lambda_{cd}^{(2)})$$
$$\times \frac{\tau_M}{1 + (\omega\tau_M)^2} \left\langle \frac{\partial u_{ab}}{\partial x_k} \dot{u}_{cd} \right\rangle. \tag{4.5}$$

The absorption of sound in semiconductors with two kinds of current carriers has been discussed by Hopfield[10] for the case when the diffusion of the current carriers could be neglected, the tensors σ_{ik} are nonsymmetric because of the presence of a strong magnetic field **H** and, moreover, there exists in a direction perpendicular to **H** a constant electric field **E** the presence of which can change the sign of the coefficient Γ. For E = 0 the corresponding expression of Hopfield is a special case of our formula (4.4) which is obtained if in the right hand side of (4.3) we neglect the second term.

In the paper by Dumke and Haering[11] the same problem was solved taking diffusion into account. For $E \neq 0$ a calculation analogous to the one carried out in the text yields

$$\Gamma S = \frac{\tau_M (\omega - \mathbf{qV})}{\omega [1 + \tau_M^2 (\omega - \mathbf{qV})^2]} \frac{Q_1 Q_2}{Q_1 + Q_2} (\Lambda_{ab}^{(1)} + \Lambda_{ab}^{(2)}) (\Lambda_{cd}^{(1)} + \Lambda_{cd}^{(2)})$$
$$\times \langle \dot{u}_{ab} \dot{u}_{cd} \rangle,$$

where $\mathbf{V} = c\mathbf{E} \times \mathbf{H}/H^2$ is the drift velocity in the crossed electric and magnetic fields. This expression differs from the corresponding formula of [11].

CONCLUSION

In conclusion we will dwell briefly on several other mechanisms which lead to the absorption of sound and to the acoustoelectric effect.

In the case of a simple zone a homogeneous but variable deformation does not lead to absorption of sound if we neglect the small dependence of the deformation potential on the quasimomentum. The point is that the constant increment to the energy due to the deformation is equivalent to a change in the reference origin and should not lead to any physical effects. As a result of this the coefficient of sound absorption turns out to be proportional for small ω and q not to $\langle \dot{u}_{ab} \dot{u}_{cd} \rangle$, but to $\langle (\partial \dot{u}_{ab}/\partial x_i)(\partial \dot{u}_{cd}/\partial x_k) \rangle$ as a result of which it should contain an extra power of q^2.

Indeed, calculation shows that in the case of longitudinal sound

$$\Gamma = \frac{q^3 \Lambda^2 \varepsilon}{4\pi e^2 \rho w^2} \frac{\omega\tau_\sigma}{(1 + q^2/\varkappa^2)^2 + (\omega\tau_\sigma)^2}.$$

If $\sigma \sim N$ and the second term in (2.7) is small, then Γ is related to E^{ac} by Eq. (1.5).

Another possible mechanism for the absorption of sound is due to the change in the position of the impurity level as the result of deformation. If the impurities are not completely ionized, then the equilibrium concentration corresponding to a given deformation is established only after a certain time τ_i which can be large. The existence of such a time can also be associated with a mechanism of sound absorption of Mandel'shtam-Leontovich type and with a corresponding acoustoelectric current.

In the case of an impurity semiconductor with a simple zone the change in the position of the impurity level $\delta\epsilon = H_{ik} u_{ik}$, where H_{ik} is an energy of the order of the depth of the impurity level. The corresponding coefficient for the absorption of longitudinal sound is

$$\Gamma = \frac{\omega^2 \tau_i}{1 + (\omega\tau_i)^2} \frac{N_0 H_{\xi\xi}^2}{\rho w^3 kT}.$$

This coefficient is relatively small. For a semiconductor with many minima cases are possible in which the shift in the impurity levels as a result of deformation is fairly large, and consequently the absorption of sound should also be large. However, such a shift is of a fairly complicated nature, and this effect requires special discussion.

APPENDIX

We shall give a microscopic derivation of Eqs. (3.2). In order to avoid awkward expressions we shall consider the case when we can neglect the Fermi degeneracy. Let $F_{\alpha p}$ be the distribution function for electrons belonging to the α-th minimum with respect to the momentum p. It is determined by an equation of the following form:

$$\frac{\partial F_{\alpha p}}{\partial t}+\frac{\partial F_{\alpha p}}{\partial \mathbf{r}}\frac{\partial \varepsilon_\alpha}{\partial \mathbf{p}}-\frac{\partial F_{\alpha p}}{\partial \mathbf{p}}\frac{\partial \varepsilon_\alpha}{\partial \mathbf{r}}=\left[\frac{\partial F_{\alpha p}}{\partial t}\right]_{\mathrm{st}}. \qquad \text{(A.1)}$$

We integrate (A.1) over the momenta, and in doing so transform the third term on the left hand side by integrating it by parts. As a result we obtain

$$\frac{\partial n_\alpha}{\partial t}+\frac{1}{e}\operatorname{div}\mathbf{j}_\alpha=2\int\left[\frac{\partial F_{\alpha p}}{\partial t}\right]_{\mathrm{st}}d\tau_p, \qquad \text{(A.2)}$$

$$d\tau_p=\frac{d^3p}{(2\pi\hbar)^3},\quad n_\alpha=2\int F_{\alpha p}\,d\tau_p,\qquad \mathbf{j}_\alpha=2e\int\frac{\partial\varepsilon_\alpha}{\partial\mathbf{p}}F_{\alpha p}\,d\tau_p. \qquad \text{(A.3)}$$

Further we have

$$\int d\tau_p\left[\frac{\partial F_{\alpha p}}{\partial t}\right]_{\mathrm{st}}=-V_0\int d\tau_p\int d\tau_{p'}$$
$$\times\sum_{\alpha'}(W_{\alpha p,\alpha' p'}F_{\alpha p}-W_{\alpha' p',\alpha p}F_{\alpha' p'}), \qquad \text{(A.4)}$$

where $W_{\alpha p,\alpha' p'}$ is the transition probability, V_0 is the volume of the crystal.

Summation in (A.4) is in fact carried out over all $\alpha' \neq \alpha$, since the transitions within a given minimum conserve the number of particles in the minimum.

Further, we shall assume that the characteristic transition time between minima is much greater than the time for the establishment of equilibrium within a given minimum. As a result of this, as can be verified by means of direct estimates, up to small terms proportional to the ratio of these times the distribution function for the carriers is the quasiequilibrium function of the form

$$F_{\alpha p}=F_0(\varepsilon^0_{\alpha p}+\delta\varepsilon_\alpha-\overline{\delta\varepsilon}-\zeta^0-\delta\zeta_\alpha), \qquad \text{(A.5)}$$

where F_0 is the Boltzmann function. A deviation of the system from equilibrium manifests itself in the fact that the distribution function within each minimum has its own "chemical potential." On the other hand, it is clear that a distribution function of the form $F_0(\epsilon^0_{\alpha p}+\delta\epsilon_\alpha-\delta\epsilon-\zeta^0)$ makes the collision integral vanish, and consequently also (A.4).

On substituting (A.5) into (A.4) and on keeping the first nonvanishing term of the expansion in terms of $\delta\zeta_\alpha$ we obtain

$$\int d\tau_p\left[\frac{\partial F_{\alpha p}}{\partial t}\right]_{\mathrm{st}}=V_0\sum_{\alpha'}\delta\zeta_{\alpha'}\int d\tau_p\int d\tau_{p'}W_{\alpha' p'\alpha p}\frac{\partial F^0_{\alpha' p'}}{\partial\zeta}$$
$$-V_0\sum_{\alpha'}\delta\zeta_\alpha\int d\tau_p\int d\tau_{p'}W_{\alpha p,\alpha' p'}\frac{\partial F^0_{\alpha p}}{\partial\zeta},$$
$$F^0_{\alpha p}=F_0(\varepsilon^0_{\alpha p}-\zeta^0). \qquad \text{(A.6)}$$

(A.6) can be rewritten in the form

$$\int\left[\frac{\partial F_{\alpha p}}{\partial t}\right]_{\mathrm{st}}d\tau_p=\frac{1}{2}\sum_{\alpha'}[w_{\alpha'\alpha}(\delta n_{\alpha'}-\delta n^0_{\alpha'})$$
$$-w_{\alpha\alpha'}(\delta n_\alpha-\delta n^0_\alpha)], \qquad \text{(A.7)}$$

where

$$w_{\alpha'\alpha}=\frac{2}{Q}V_0\int d\tau_p\int d\tau_{p'}W_{\alpha' p',\alpha p}\frac{\partial F^0_{\alpha' p'}}{\partial\zeta}. \qquad \text{(A.8)}$$

On substituting (A.7) into (A.1) we obtain equation (3.2).

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