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ON THE CHARGED VECTOR THEORY OF LEE AND YANG

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The charged vector meson theory of Lee and Yang (ξ limiting process) is derived within the framework of the Lagrangian formalism and can thus be treated (for finite values of ξ) as a more consistent variant of the Pauli-Villars regularization method. An interpretation of the two postulates of Lee and Yang employed in going to the limit $\xi \to 0$ is presented. It is found that, while the second postulate can be formulated in accordance with the usual concepts of quantum field theory, the validity of the first postulate is highly questionable, since it allows without any justification, the exclusion of the most important terms in nonrenormalizable theories.

In recent times there has been increasing interest in theories of vector mesons. However, it is universally known[1] that the theory is unrenormalizable in the case of charged vector mesons, since finite matrix elements can be obtained only by the introduction of counterterms with an infinite number of derivatives in the Lagrangian. Recently, Lee and Yang[2] developed a variant of the charged vector meson theory, which is called the ξ limiting process. The principal aim in introducing this process is the conversion of vector electrodynamics into an approximately renormalizable theory. In order to appraise the theory of Lee and Yang one must understand how well-founded their approximations are from the point of view of the basic problem of the nonrenormalizability of such a theory. Unfortunately, Lee and Yang have formulated the ξ limiting process itself within the Hamiltonian formalism, which complicates the procedure unnecessarily. We have therefore reformulated this theory within the Lagrangian formalism,[1] which makes the whole method thoroughly transparent.1)

The Lagrangian formalism for vector electrodynamics is built up most conveniently from a free field Lagrangian which automatically takes into account the subsidiary condition:2)

$$L_{\text{free}} = -\frac{1}{2}\left(\frac{\partial\varphi_\alpha^*}{\partial x^\beta} - \frac{\partial\varphi_\beta^*}{\partial x^\alpha}\right)\left(\frac{\partial\varphi_\alpha}{\partial x^\beta} - \frac{\partial\varphi_\beta}{\partial x^\alpha}\right) + m^2\varphi_\alpha^*\varphi_\alpha. \quad (1)$$

This leads to a Green's function of the form[1]

$$D^c_{\alpha\beta}(k) = \left(g^{\alpha\beta} - \frac{k_\alpha k_\beta}{m^2}\right)\frac{1}{m^2 - k^2 - i\varepsilon}. \quad (2)$$

Following Bogolyubov and Shirkov,[1] we obtain the usual expression for the S matrix corresponding to

$$L_{int} = -ie\left(\varphi_\alpha^*\frac{\partial\varphi_\alpha}{\partial x^\beta} - \frac{\partial\varphi_\alpha^*}{\partial x^\beta}\varphi_\alpha\right)A_\beta + ie\left(\varphi_\alpha^*\frac{\partial\varphi_\beta}{\partial x^\alpha} - \frac{\partial\varphi_\beta^*}{\partial x^\alpha}\varphi_\alpha\right)A_\beta$$
$$- e^2\varphi_\alpha^*\varphi_\alpha A_\beta A_\beta + e^2\varphi_\alpha^*\varphi_\beta A_\alpha A_\beta. \quad (3)$$

As is known, the theory defined by (2) and (3) is unrenormalizable. The simplest way of making it into a renormalizable theory consists in applying the Pauli-Villars regularization method to (2), i.e., in replacing (2) by

$$\widetilde{D}^c_{\alpha\beta}(k) = \frac{g^{\alpha\beta}}{m^2 - k^2 - i\varepsilon} - \frac{k_\alpha k_\beta}{m^2}\left(\frac{1}{m^2 - k^2 - i\varepsilon} - \frac{1}{m^2/\xi - k^2 - i\varepsilon}\right), \quad (4)$$

where we must set $\xi \to 0$ in the final formulas, since an indefinite metric is required if $\xi > 0$.3) In principle, one might be content with this, since the theory defined by (3) and (4) differs from renormalizable scalar electrodynamics (with $\xi \neq 0$) only

1)We note that the theorem I of [2] is not a theorem but a definition,[3] for $\langle T[\varphi_\alpha(x)\,\varphi^*_\beta(y)]\rangle_0$ is not defined for $x = y$ and is usually determined by the requirement that the S matrix be independent of the shape of the intermediate surfaces. As to theorems II and III of [2], analogous theorems have actually been formulated already in [3] on the example of a theory with derivative couplings.

2)Although the quantization scheme for a vector field in which the subsidiary condition is imposed separately on the Lagrangian formalism[1] gives analogous results for the free field, its direct translation to the case of vector electrodynamics leads to the vanishing of the magnetic moment of the vector particle.

3)The necessity of using an indefinite metric in a theory with an intermediate meson for a realistic variant of the V-A interaction has already been noted in [4]. There also the mass of such a meson was given as $m \approx 5m_\mu$, which leads to best agreement with experiment.

by the more complicated system of indices. However, if one wants to be consistent, one must take into account that the modification of the Green's function corresponds to a modification of L_{free}. For $\tilde{D}^c_{\alpha\beta}(k)$ corresponds to an L_{free} which differs from (1) by the additional term $-\xi(\partial\varphi^*_\alpha/\partial x^\alpha)\times(\partial\varphi_\beta/\partial x^\beta)$. Admitting this term in L_{free} implies a violation of the subsidiary condition, so that the indefinite metric introduced into the theory by the finiteness of ξ is naturally connected with the scalar components of the vector field.

The modification of L_{free} leads to the appearance of the additional terms

$$-ie\xi\left(\varphi^*_\beta\frac{\partial\varphi_\alpha}{\partial x^\alpha}-\frac{\partial\varphi^*_\alpha}{\partial x^\alpha}\varphi_\beta\right)A_\beta-e^2\xi\varphi^*_\alpha\varphi_\beta A_\alpha A_\beta \tag{5}$$

in L_{int}. In this way we have obtained very simply a renormalizable variant (with $\xi\neq 0$) of vector electrodynamics in which L_{int} is given by the sum of the terms (3) and (5) and the Green's function is defined by (4), which leads to the same Feynman rules as in [2].[4)] Thus the ξ limiting process of Lee and Yang can be regarded as a variant of the Pauli-Villars regularization method, which is more refined and consistent, for here not only the Green's function (4), but also the vertices (5) depend on ξ.

It is now easy to understand that, although a finite S matrix can be obtained in the usual way, it will be nonunitary as long as $\xi\neq 0$. Therefore we must let ξ tend to zero in such a way that the unitarity of the S matrix is reestablished without returning to the original unrenormalizable theory. In the theory of Lee and Yang, the approximate method of the limiting process $\xi\to 0$ was specified by two postulates whose meaning is not sufficiently clear. In order to clarify the meaning of these postulates, let us consider the expression for the self-energy of the vector meson $\Sigma_2^{\alpha\beta}(p)$ in second order (with $\xi\neq 0$). After the usual regularization and the limiting transition $M_i^2\to\infty$ we obtain a finite expression for $\Sigma_2^{\alpha\beta}(p)$ which depends on ξ. In the limit $\xi\to 0$ we obtain, omitting terms independent of ξ,

$$\Sigma_2^{\alpha\beta}(p,\xi)=\frac{e^2\pi^2}{i}\Big\{\frac{1}{2}(g^{\alpha\beta}p^2-p_\alpha p_\beta)\frac{1}{\xi}\ln\xi - [g^{\alpha\beta}(p^2-m^2)-p_\alpha p_\beta]\ln\xi+\frac{1}{3}(g^{\alpha\beta}p^2-p_\alpha p_\beta)\frac{p^2}{m^2}\ln\xi\Big\}, \tag{6}$$

i.e., all terms of $\Sigma_2^{\alpha\beta}(p,\xi)$ are quasilocal for $\xi\to 0$.

By the first postulate of Lee and Yang [2] we must retain in (6) only the first term as the most singular one for $\xi\to 0$. This procedure can be based on the fact that these terms are the worst from the point of view of the nonrenormalizability of the theory, and if they can be removed with the help of the second postulate of Lee and Yang, the greatest difficulty has been surmounted.

Indeed, if one accepts the first postulate, then the second postulate, which allows one to separate out the finite part of the most singular term (for $\xi\to 0$) remaining in (6), can be interpreted in the following way. Since this term is a quasilocal operator of second order, it can be removed from the S matrix by introducing a corresponding counterterm in the Lagrangian, but with coefficients depending on ξ,[5)] where this counterterm is of the "renormalizable" type. At the same time one can also introduce a counterterm of the same structure which, however, is independent of ξ and has an arbitrary finite coefficient. Lee and Yang have, in fact, proposed a procedure which allows one to fix this arbitrariness, but it is, of course, not unique.

If one believes in the validity of the first postulate, one must at once neglect in (6) the term with the highest power of the momentum, which is the most dangerous from the point of view of nonrenormalizability, for the removal of such terms from the S matrix requires the introduction into the Lagrangian of terms with an ever increasing number of derivatives. It is easy to show that, although the peculiarities of the ξ limiting process slow down this increase, the power of the momentum in these terms will increase without limit in the highest orders. Thus the validity of the first postulate of Lee and Yang [2] seems highly questionable, since it allows, without any justification, the exclusion of the most important terms in nonrenormalizable theories. Thus the concrete results obtained by this method can have only limited value.

In conclusion I express my gratitude to the coworkers of the departments of theoretical physics of the Mathematical and Physical Institutes of the Academy of Sciences, U.S.S.R. for fruitful discussions.

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4)We shall not consider the term with the anomalous magnetic moment in L_{int}, since it can be included in our formalism without difficulty.

5)We note that the terms with M_i^2 (for $\xi\neq 0$) also contain factors depending on ξ, so that their removal requires the introduction of counter terms with ξ dependent coefficients anyway.

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