RADIATION CORRECTIONS TO SCALAR MESON PAIR PRODUCTION IN ELECTRON-POSITRON COLLISIONS

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The perturbation theory series for π-meson pair production in electron-positron collisions has been evaluated in the double logarithmic approximation.

BAĬER and Kheĭfets [1] have shown that taking into account the radiation corrections to the cross section for the production of pairs of fermions in electron-positron collisions leads to a characteristic change in the behavior of the cross section near the threshold for the reaction. They have calculated these radiation corrections in the so-called double logarithmic approximation [2,3]. In the present article an analogous investigation is made for the case of the production of pairs of scalar mesons. It turns out that the change in the behavior of the cross section at the threshold noted in [1] also occurs in the present case.

It is convenient to carry out the concrete investigation in the Duffin-Kemmer formulation which has a great formal similarity with the formalism of the usual spinor electrodynamics. In particular, the class of all possible diagrams of scalar electrodynamics in the formulation indicated above coincides with the class of diagrams for the electrodynamics of fermions.

In order to illustrate the application of the double logarithmic method to the case of scalar electrodynamics we shall evaluate the vertex function $\Gamma_\sigma(p, q; l) = \beta_\sigma + \Lambda_\sigma(p, q; l)$ which we shall need later by assuming that

$$p^2 = q^2 = \mu^2, \qquad l^2 \approx -2(pq) \gg p^2, q^2.$$

In first order perturbation theory we have

$$\Lambda_\sigma^{(2)}(p, q; l) = \frac{e^2}{\pi i}\int \beta_\nu G^\pi(p-k)\,\beta_\sigma G^\pi(q-k)\,\beta_\nu k^{-2}\,d^4k,$$

$$G^\pi(q) = \frac{\hat{q}+\mu+(\hat{q}^2-q^2)/\mu}{q^2-\mu^2} = \frac{\hat{q}(\hat{q}+\mu)}{\mu(q^2-\mu^2)} - \frac{1}{\mu},$$

$$\hat{q} = \beta_\nu q_\nu, \quad \beta_\nu\beta_\sigma\beta_\lambda + \beta_\lambda\beta_\sigma\beta_\nu = \beta_\nu g_{\sigma\lambda} + \beta_\lambda g_{\sigma\nu},$$
$$g_{00} = 1, \qquad g_{11} = g_{22} = g_{33} = -1. \tag{1}$$

We have chosen here the Feynman gauge which is the most convenient one for practical calculations.

Just as in the case of spinor electrodynamics $\Lambda_\sigma^{(2)}(p, q; l)$ has a single logarithmic ultraviolet divergence. Evidently, (1) can be rewritten in the form

$$\Lambda_\sigma^{(2)}(p, q; l)$$
$$= \frac{e^2}{\pi i}\Big\{\int \frac{\beta_\nu\mu^{-1}(\hat{p}-\hat{k})(\hat{p}-\hat{k}+\mu)\beta_\sigma\mu^{-1}(\hat{q}-\hat{k})(\hat{q}-\hat{k}+\mu)\beta_\nu}{[(p-k)^2-\mu^2][(q-k)^2-\mu^2]k^2}d^4k$$
$$+ \frac{\beta_\sigma}{\mu^2}\int\frac{d^4k}{k^2} - \frac{1}{\mu}\Big[\int\frac{\beta_\nu\mu^{-1}(\hat{p}-\hat{k})(\hat{p}-\hat{k}+\mu)\beta_\sigma\beta_\nu}{[(p-k)^2-\mu^2]k^2}d^4k$$
$$+\int\frac{\beta_\nu\beta_\sigma\mu^{-1}(\hat{q}-\hat{k})(\hat{q}-\hat{k}+\mu)\beta_\nu}{[(q-k)^2-\mu^2]k^2}d^4k\Big]\Big\}. \tag{2}$$

Since only the first integral has a double logarithmic part all the other integrals can be omitted in our approximation. In doing this it should be noted that because of the method adopted by us of separating the meson Green's function into two terms this integral has a quadratic divergence in the region of large virtual momenta. However, this divergence is cancelled by the quadratically divergent parts of the third and fourth integrals. For these reasons it need not be taken into consideration and the double logarithmic terms can be considered to be the fundamental ones.

In order to pick out the latter terms we introduce, as usual [3], the variables u, v and x, whose significance is that they make the principal term in the denominator linear in the variables of integration:

$$k = p\frac{a^2u-av}{a^2-1} + q\frac{a^2v-au}{a^2-1} + k_\perp,$$

$$a = (pq)/\mu^2, \qquad -k_\perp^2 = x > 0;$$

$$(p-k)^2-\mu^2 \sim -2(pq)v, \quad (q-k)^2-\mu^2 \sim -2(pq)u. \tag{3}$$

In carrying out the integration with respect to x one should take the residue at the point $k^2 = 0$, which gives the factor $(-i\pi)$. The regions of integration with respect to u and v are defined by the following inequalities

$$\frac{\mu\Delta\mu}{(pq)} \ll u \ll 1, \quad \frac{\mu\Delta\mu}{(pq)} \ll v \ll 1;$$

$$v/(a + \sqrt{a^2 - 1}) \leqslant u \leqslant v\,(a + \sqrt{a^2 - 1}). \quad (4)$$

The parameter $\Delta\mu$ defined by the equation $p^2 - \mu^2 = 2\mu\Delta\mu$ has been introduced in order to remove the logarithmic infrared divergence in (2).

Omitting everywhere in the numerator small $\hat{k}$ (their presence reduces the logarithmic degree of the integral) we obtain

$$\Lambda_\sigma^{(2)}(p, q; l) = \frac{e^2}{\pi i} \int \frac{\beta_\nu \mu^{-1}\hat{p}\,(\hat{p} + \mu)\,\beta_\sigma \mu^{-1}\hat{q}\,(\hat{q} + \mu)\,\beta_\nu}{[(p-k)^2 - \mu^2]\,[(q-k)^2 - \mu^2]\,k^2}\, d^4k. \quad (5)$$

We assume that the momenta p and q refer to real particles. Then the quantity of interest is not the function $\Lambda_\sigma^{(2)}$ itself but its matrix element $\overline{\varphi}(p)\,\Lambda_\sigma^{(2)}\,\varphi(q)$ which enters into the cross section for the process (scattering by a nonfree photon). But, as can be easily shown by utilizing the properties of the β-matrices, we have

$$\overline{\varphi}(p)\,\beta_\nu \mu^{-1}\hat{p}\,(\hat{p} + \mu) = 2p_\nu \overline{\varphi}(p), \quad \mu^{-1}\hat{q}\,(\hat{q} + \mu)\,\beta_\nu \varphi(q) = 2q_\nu \varphi(q) \quad (6)$$

($p^2 = q^2 = \mu^2$). Therefore, the numerator in (5) can be replaced by $4(pq)\beta_\sigma$. The integral (5) in the region under investigation has been evaluated by Abrikosov[3] [formula (26a)].

We consider now the n-th order approximation of perturbation theory. The numerator in the term of maximum logarithmic order corresponding to one of the diagrams of this order has the form

$$\beta_{\nu_1} \mu^{-1}\hat{p}\,(\hat{p} + \mu)\,\beta_{\nu_2} \ldots \mu^{-1}\hat{p}\,(\hat{p} + \mu)\,\beta_\sigma \mu^{-1}\hat{q}\,(\hat{q} + \mu) \ldots \beta_{\lambda_2} \mu^{-1}\hat{q}\,(\hat{q} + \mu)\,\beta_{\lambda_1}.$$

We now note that

$$\mu^{-1}\hat{p}\,(\hat{p} + \mu)\,\beta_\nu \mu^{-1}\hat{p}\,(\hat{p} + \mu) = 2p_\nu \mu^{-1}\hat{p}\,(\hat{p} + \mu). \quad (7)$$

Assuming again that the momenta p and q refer to real particles, and utilizing (6) and (7), we arrive at the conclusion that the numerator under consideration can be replaced by $[4(pq)]^n \beta_\sigma$.

Thus, just as in the case of spinor electrodynamics we have

$$\overline{\varphi}(p)\,\Gamma_\sigma(p, q; l)\,\varphi(q) = \overline{\varphi}(p)\,\beta_\sigma \varphi(q) \exp\{-e^2 f/2\pi\}, \quad (8)$$

where

$$f = \int \frac{du}{u}\frac{dv}{v} \quad (9)$$

within the limits shown in (4).

It should be noted here, apparently, that in contrast to the usual electrodynamics where the electron mass diverges logarithmically, the meson mass in scalar electrodynamics diverges quadratically. This circumstance should be taken into account in calculations in the double logarithmic approximation. Indeed, one should first carry out a renormalization of the meson mass and then use such a meson Green's function which has been renormalized with respect to mass. The remaining divergent terms in the Green's function will, as a result of Ward's identity, cancel with the corresponding terms of the vertex part, so that we can utilize for the internal lines the "nonovergrown" functions with renormalized mass.

Let us now discuss scattering accompanied by the emission of real quanta. The crossed meson line in the generalized Abrikosov diagram corresponds, as can be easily seen, to the factor

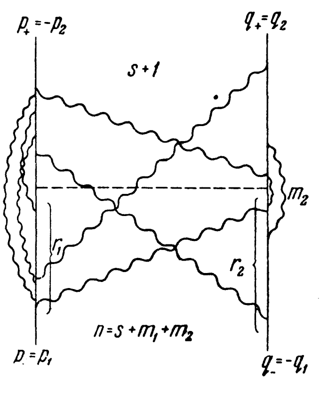
$$\frac{\hat{p}\,(\hat{p} + \mu)}{2\pi} \int_{E>0} \delta(p^2 - \mu^2)\, d^4p \quad (10)$$

with the normalization $\overline{\varphi}\beta_0\varphi = 2\mu E$. It is clear from this that the discussion of scattering accompanied by the creation of real quanta in virtue of Eq. (7) does not differ at all from that carried out by Abrikosov for the case of ordinary electrodynamics.

The foregoing discussion is sufficient for our main aim: the calculation of the cross section for the creation of a pair of scalar mesons accompanying the annihilation of an electron-positron pair. The general form of the diagram of order $2(n+1)$ which gives a double logarithmic contribution is shown in the figure (the notation coincides with that utilized in [4], the dotted line denotes a strong quantum). This diagram corresponds to the integral

$$\begin{aligned} I = \int \Big\langle \bar{u}\gamma G^e\Big[p_2 - \binom{\varkappa}{k}\Big]\gamma \ldots \gamma G^e\Big[p_2 - \sum_{r_1+1}^{s} k_i - \sum_{1}^{m_1} \varkappa_i\Big] \\ \times \gamma_\sigma G^e\Big[p_1 + \sum_{1}^{r_1} k_i - \sum_{1}^{m_1} \varkappa_i\Big]\gamma \\ \ldots \gamma G^e\Big[p_1 \pm \binom{k}{\varkappa}\Big]\gamma u\Big\rangle \Big\langle \overline{\varphi}\beta G^\pi\Big[q_2 \mp \binom{\varkappa}{k}\Big]\beta \\ \ldots \beta G^\pi\Big[q_2 + \sum_{r_2+1}^{s} k_i - \sum_{1}^{m_2} \varkappa_i\Big]\beta_\sigma G^\pi\Big[q_1 - \sum_{1}^{r_2} k_i - \sum_{1}^{m_2} \varkappa_i\Big]\beta \\ \ldots \beta G^\pi\Big[q_1 - \binom{\varkappa}{k}\Big]\beta\varphi\Big\rangle \prod_{i=1}^{s} \frac{d^4k_i}{k_i^2} \prod_{i=1}^{m_1} \frac{d^4\varkappa_i}{\varkappa_i^2} \prod_{j=1}^{m_2} \frac{d^4\varkappa_j}{\varkappa_j^2} \Big(l - \sum_{1}^{s} k_i\Big)^{-2} \end{aligned} \quad (11)$$

(each γ-matrix is contracted with one of the β-matrices). Here k is the momentum of the "ladder" quantum, κ is the momentum of the "vertex" quantum, $G^e(p)$ is the "nonovergrown" Green's function for the electron and $G^\pi(q)$ is the "non-



overgrown'' Green's function for the meson (in the sense indicated above).

In the numerator of (11) we can again delete all the $\hat{k}$ and $\hat{\kappa}$ which reduce the logarithmic degree. After this the reduction of the numerator presents no difficulties. By utilizing (6) and (7) together with the analogous equations

$$(\hat{p}+m)\gamma_\nu u = 2p_\nu u, \quad \bar{u}\gamma_\nu(\hat{p}+m) = 2p_\nu\bar{u},$$
$$(\hat{p}+m)\gamma_\nu(\hat{p}+m) = 2p_\nu(\hat{p}+m), \tag{12}$$

we obtain for it the value $[4(pq)]^n \langle\bar{u}\gamma_\sigma u\rangle\langle\bar{\varphi}\beta_\sigma\varphi\rangle$, with all the scalars of the problem being taken equal, as in [4].

The reduction of the denominator does not differ from that carried out in [5]. For each virtual quantum we introduce the variables u_i, v_i, x_i (for ladder quanta) or φ_i, ψ_i, y_i (for the vertex quanta). It is important to establish the sign of the matrix element with fixed s, r_1, r_2, m_1, m_2. In order to do this we note that in $s - r_1 - m_1$ propagator functions the scalar products p_2k_i and $p_2\kappa_i$ occur with a minus sign. Similarly, there is a minus sign in $r_2 + m_2$ propagator functions containing the products q_1k_i and $q_1\kappa_i$. Moreover, one should perform the replacement $\varphi_i, \psi_i \to -\varphi_i, -\psi_i$ in $m_1 + m_2$ cases (as many times as there are Green's functions in which p_1k_i occurs with a plus sign and $p_1\kappa_i$ occurs with a minus sign, or in which q_2k_i occurs with a plus sign and $q_2\kappa_i$ occurs with a minus sign). As a result of this the matrix element acquires the factor $(-1)^{s+r_1+r_2}$. For subsequent transformations it is convenient to use the following rule: we shall introduce the variables u and v (ladder) and φ and ψ (vertex) in such a way that the variables u or φ correspond to the end of the line while the variables v or ψ correspond to the beginning of the line. Naturally, the beginning and the end of the line can be chosen in an arbitrary manner. As a result of these transformations and of subsequent integration we obtain

$$I = j_0\left(-\frac{2\pi i}{(pq)}\right)^n(-1)^{s+r_1+r_2}A^sA_e^{m_1}A_\pi^{m_2}$$
$$\times\frac{f^sf_e^{m_1}f_\pi^{m_2}}{(r_1+m_1)!(s-r_1+m_1)!(r_2+m_2)!(s-r_2+m_2)!}, \tag{13}$$

where A_i are the Jacobians of the corresponding transformations:

$$A = (m\mu/4\pi)\,ab\,(ab-1)^{-1/2}, \quad A_e = (m^2/4\pi)\,a^2\,(a^2-1)^{-1/2},$$
$$A_\pi = (\mu^2/4\pi)b^2\,(b^2-1)^{-1/2},$$
$$j_0 = l^{-2}\langle\bar{u}\gamma_\sigma u\rangle\langle\bar{\varphi}\beta_\sigma\varphi\rangle, \tag{14}$$

while f_i are integrals of the type (9) taken between appropriate limits.

It is important to note that if we move the end of one of the ladder quanta past a strong quantum (a change in one of the r by unity) only the sign of the matrix element is altered. Since, on the other hand, in any order of perturbation theory for any given diagram there also exists another diagram which is analogous to the given one in all respects except that one of the ends of a weak ladder quantum has been taken past a strong quantum, it is clear that in summing over all r the contributions of such diagrams cancel one another, and the only significant contribution is the one coming from the vertex quanta in the diagram shown in the figure with $r_1 = r_2 = s = 0$.

Two cases can occur:

1. $a \gg 1$ and $b \gg 1$. In this case $A = A_e = A_\pi = (pq)/4\pi$ and (13) takes on the form

$$I = j_0(-i/2)^n f_e^{m_1}f_\pi^{m_2}/(m_1!)^2\,(m_2!)^2. \tag{15}$$

Here, in accordance with the foregoing, we have set $r_1 = r_2 = s = 0$.

2. $a \gg 1$ and $b \sim 1$. In this case $A = A_e = (pq)/4\pi$. As regards A_π, for $b \to 1$ we have $A_\pi \to \infty$, which follows from the fact that at the point $b = 1$ the transformation (3) is not applicable. However, in virtue of the condition

$$\psi^\pi/(b+\sqrt{b^2-1}) \leqslant \varphi^\pi \leqslant \psi^\pi\,(b+\sqrt{b^2-1}),$$

the quantity $A_\pi f_\pi$ tends for $b \to 1$ to a finite and, moreover, a single logarithmic limit: $(\mu^2/2\pi)\ln(\mu/\Delta\mu)$. Therefore, in our approximation we should omit the terms with $m_2 \neq 0$:

$$I = j_0(-i/2)^n\,(m!)^{-2}f_e^m. \tag{16}$$

In this formula m is the number of quanta corresponding to the electron vertex.

On multiplying (15) and (16) by the number of different diagrams with given m_1 and m_2, which is evidently equal to $m_1!m_2!$, and also on taking into account the fact that to each photon line in the

Feynman diagram there corresponds the factor $1/\pi i$, and to each vertex there corresponds the factor e, we obtain after summation over m_1, m_2 and n, subject to the condition $m_1 + m_2 = n$,

$$d\sigma = d\sigma_0 \exp\left\{-\frac{e^2}{\pi}(f_e + f_\pi)\right\} \tag{17}$$

in the first case, and an analogous formula in which f_π has been omitted in the second case. Here $d\sigma_0$ is the cross section for the process in the lowest order of perturbation theory.

Thus, in order to obtain the desired cross section it is sufficient to utilize the simplest diagram of perturbation theory treating it as a skeleton diagram. In doing this the photon Green's function should be regarded as "nonovergrown," while for the vertices we should take the one found by Abrikosov in the case of fermions, and the one obtained in the early part of this article in the case of mesons.

The cross section (17) depends on the arbitrary parameters Δm and $\Delta\mu$ and tends to zero as $\Delta m \to 0$, $\Delta\mu \to 0$. In order to eliminate this dependence it is necessary to take into account the possibility of emission of real quanta, since only the total cross section for the elastic and the inelastic processes is of physical interest.

The use of generalized Abrikosov diagrams for the calculation of such a cross section has been demonstrated in the case of spinor electrodynamics by Baĭer[5] and by Kheĭfets (thesis). In our case, as has been demonstrated above using the vertex as an example, the situation is completely analogous[5]. Therefore, we can immediately write down the final expressions for the desired cross section:

$$d\sigma = d\sigma_0 \exp\left\{-\frac{4\alpha^2}{\pi}\left(\ln\frac{E}{\Delta E_e}\ln\frac{E}{m} + \ln\frac{E}{\Delta E_\pi}\ln\frac{E}{\mu}\right)\right\}. \tag{18}$$

In this formula ΔE_e is the energy emitted by the fermions, while ΔE_π is the energy emitted by the mesons.

At the threshold we obtain instead of (18)

$$d\sigma = d\sigma_0 \exp\left\{-\frac{4\alpha^2}{\pi}\ln\frac{E}{\Delta E_e}\ln\frac{E}{m}\right\}. \tag{19}$$

Formula (19) shows that the threshold effect noted in [1] also occurs in the case under investigation. The latter formula is also valid if all the emitted mesons are recorded independently of their energy ($\Delta E_\pi \sim E$).

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