LIMITATIONS ON THE ASYMPTOTIC VALUES OF CROSS SECTIONS IMPOSED BY THE REGGE POLE HYPOTHESIS

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The asymptotic form of the amplitude determined by an n-th order Regge pole is given. Assuming analyticity of $l(s)$ near $s = 0$ and using the unitarity condition, it is shown that the total cross section does not increase more rapidly than the logarithm of the energy.

THERE seem to be no special reasons in quantum field theory which exclude the possibility of the existence of higher order Regge poles. The multiplicity of the pole can be the consequence of the coincidence of several trajectories or of the intersection of several trajectories at some momentum transfer s. The asymptotic form of the amplitude $A(s, t)$ for $t \to \infty$ determined by an n-th order Regge pole is

$$A(s, t) \approx f(s)\, t^{l(s)} (\ln t)^{n-1}. \tag{1}$$

If this possibility is taken into account, Froissart's theorem [1] implies that $l(0) \leq 1$ and the order of the pole $n \leq 3$ for $l(0) = 1$.

This result can be strengthened if we assume beforehand that the elastic scattering amplitude has the asymptotic form (1). The total cross section for the process is then, by the optical theorem, equal to

$$\sigma_t = 16\pi f_2(0)\, t^{l(0)-1} (\ln t)^{n-1}, \qquad f_2(s) = \operatorname{Im} f(s). \tag{2}$$

The magnitude of the elastic cross section

$$\sigma_e = \int d\Omega \frac{4}{t} |A(s,t)|^2 = \frac{16\pi}{t^2} \int_{-t}^{0} ds\, |A(s,t)|^2 \tag{3}$$

can be estimated for $t \to \infty$ with the help of (1). For small s we have $l(s) \approx l(0) + \gamma s$, where $\gamma > 0$, as shown by Gribov and Pomeranchuk. [2] Then the main contribution to the integral at the upper limit comes from the region

$$-1/2\gamma \ln t \leqslant s \leqslant 0. \tag{4}$$

With increasing t this interval becomes as narrow as we please and the leading singularity will not change within this interval. If the multiplicity of the pole at $s = 0$ is due to the intersection of several trajectories in this point, then it is easily seen that the value of the integral (3) can only decrease if we regard all trajectories as coinciding with the lowest one in the interval (4).

Thus the elastic scattering cross section into small angles and, hence, the total elastic cross section can for $t \to \infty$ be no less than the following expression:

$$\sigma_{1e} = 8\pi\gamma^{-1} |f(0)|^2\, t^{2l(0)-2} (\ln t)^{2n-3}. \tag{5}$$

Since the total cross section is, a fortiori, no less than σ_{1e}, it is clear that $l(0) \leq 1$ and $n \leq 2$ for $l(0) = 1$.

This result can also be obtained in a somewhat different manner. By the unitarity condition the imaginary part of the partial wave amplitude satisfies the condition

$$0 \leqslant \operatorname{Im} f_l(t) \leqslant 1. \tag{6}$$

Assuming that for small l the main contribution to the integral determining $f_l(t)$ comes from the region close to (4), we find that in this case

$$f_l(t) \approx \gamma^{-1} f(0)\, t^{l(0)-1} (\ln t)^{n-2}. \tag{7}$$

With the requirement (6) for $t \to \infty$ we obtain the previous result.

In contrast to the proof of Froissart, these considerations are not based on the absence of anomalous singularities and are therefore also applicable to nuclear processes. [3] Both proofs fail for reactions involving massless particles, since the forward scattering amplitude becomes infinite. [3]

The case of a constant total cross section $l(0) = 1$, $n = 1$ has been investigated by Gribov. [4,5] If $l(0) = 1$ and $n = 2$, the total as well as the elastic cross sections increase logarithmically:

$$\sigma_t = 16\pi f_2(0) \ln t; \qquad \sigma_{1e} = 8\pi\gamma^{-1} |f(0)|^2 \ln t. \tag{8}$$

The partial amplitudes for $l \ll \sqrt{\gamma t \ln t}$ are constant and equal to $f(0)/\gamma$, and for $l \gg \sqrt{\gamma t \ln t}$

they decrease exponentially with increasing l. This form of the partial wave amplitude corresponds to the scattering from a sphere whose radius increases with energy $\sim \sqrt{\gamma \ln t}$ and whose transparency remains constant, in contrast to the case considered by Gribov.

Applying the condition (6) we find that $f_2(0) \leq \gamma$. Setting $\gamma \approx 1/_{50}\,\mu^2$,[6] we obtain an upper limit for the total cross section: $\sigma_t \lesssim \mu^{-2} \ln t$.

We note that the discussion of the case of logarithmically increasing cross sections given by Frye[7] is in error.

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324