ANOMALOUS PLASMA DIFFUSION CAUSED BY OSCILLATIONS

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The effect of forced oscillations on diffusion of plasma across a magnetic field is considered for the particular case of a fully ionized cylindrical plasma column in a strong magnetic field with a random fluctuation source at the plane $z = 0$.

1. It is well known that plasma oscillation can lead to enhanced diffusion across a magnetic field. This effect is found most simply from an analysis of the motion of individual particles in a fluctuating electric field; [1-2] it is found, in particular, that the random motion of an individual particle can be described in terms of diffusion motion with some effective diffusion coefficient.

Unfortunately, in certain cases [2-4] the diffusion coefficient obtained in this way has been applied to the plasma as a whole. In doing this, however the implicit assumption is being made that the motion of an individual particle is not correlated with the motion of the other particles; the error is the same as though one attempted to assign to an entire plasma a diffusion coefficient computed from the motion of an individual ion that diffuses primarily by ion-ion collisions.

In the present work, using the example of a plasma column with an external source of random fluctuations, it is shown that the enhanced diffusion can result only from fluctuations in which both electrons and ions participate. In this case the two diffusion fluxes are equal.

Specifically, we consider the following problem: a cylindrical column of collisionless plasma with a radial density gradient and uniform temperature is located in a strong longitudinal magnetic field **H** directed along the z axis. In the plane $z = 0$ this column is in contact with a ''cathode'' which, on the average, emits exactly the same number of electrons and ions that reach its surface. In addition, at the cathode we specify random fluctuations of the potential jump associated with the double layer; these modulate the electron and ion streams leaving the cathode and thus produce waves in the plasma.

If the correlation length of the potential fluctuations (in radius and azimuth) is appreciably larger than the mean ion Larmor radius and if the frequencies of the fluctuations are appreciably smaller than the ion cyclotron frequency, only drift waves [5] can propagate from this noise source. Under the conditions considered here these waves are weakly damped. We solve the problem of propagation of small amplitude waves by a Fourier transformation in time and a Laplace transformation in z; then, using the quasilinear approximation we compute the electron diffusion flux associated with these waves.

The expression obtained for the diffusion flux shows that near the source (if the density gradient is large enough) the diffusion flux is proportional to $1/H$ and that the distance over which the enhanced diffusion occurs is proportional to H. These results are of interest in themselves because under actual conditions fluctuations of the double layers at the anode and cathode can serve to excite external oscillations.

2. We first consider the propagation of particle density fluctuations and fluctuations in the electric potential from the plane $z = 0$ given the boundary condition

$$\varphi(z=0,\ \mathbf{r}_\perp, t) = \sigma(\mathbf{r}_\perp, t). \tag{1}$$

Here, the perpendicular sign denotes the component transverse to the magnetic field.

If the plasma is assumed to be stable small oscillations can be analyzed in the linear approximation. We consider the linearized kinetic equation for the electrons or ions:

$$\frac{\partial f_1}{\partial t} + \mathbf{v}\frac{\partial f_1}{\partial \mathbf{r}} + \frac{e}{mc}[\mathbf{vH}]\frac{\partial f_1}{\partial \mathbf{v}} = \frac{e}{m}\nabla\varphi\frac{\partial f_0}{\partial \mathbf{v}}. \qquad (2)^*$$

We use a Fourier transformation in time and a Laplace transformation in z:

$$\varphi(\mathbf{r}_\perp, k, \omega) = \int_{-\infty}^{\infty} dt \int_0^{\infty} dz\,\varphi(\mathbf{r}, t)\, e^{-ikz+i\omega t}, \tag{3}$$

where k is complex, $k = k' + ik''$, $k'' < 0$ and ω is real so that all quantities are stationary random fluctuations in time. Thus

*$[\mathbf{vH}] = \mathbf{v}\times\mathbf{H}$.

$$\left[i\,(kv_z - \omega) + \mathbf{v}_\perp \frac{\partial}{\partial \mathbf{r}_\perp} + \frac{e}{mc}[\mathbf{v}_\perp \mathbf{H}] \frac{\partial}{\partial \mathbf{v}_\perp} \right] f_1(\mathbf{r}_\perp, \mathbf{v}, k, \omega) - v_z f_1(z = 0, \mathbf{r}_\perp, \mathbf{v}, \omega) = \frac{e}{m}\left[ik \frac{\partial f_0}{\partial v_z} + \frac{\partial f_0}{\partial \mathbf{v}_\perp}\frac{\partial}{\partial \mathbf{r}_\perp}\right] \varphi(\mathbf{r}_\perp, k, \omega) - \frac{e}{m}\sigma(\mathbf{r}_\perp, \omega)\frac{\partial f_0}{\partial v_z}. \quad (4)$$

The boundary conditions on f_1 and φ appear explicitly in (4). When $v_z > 0$ we assume that $f_1(z = 0, \mathbf{r}_\perp, \mathbf{v}, \omega) = 0$; when $v_z < 0$ obviously boundary conditions do not affect f_1 in (2) and (4) and are omitted.

Assume that in the cylindrical coordinate system (r, α, z) the density gradient is in the direction of r. We introduce in (4) the new variables:[6]

$$v_r(\tau) = v_\perp \cos(\omega_H \tau + \alpha), \quad v_\alpha(\tau) = -v_\perp \sin(\omega_H \tau + \alpha),$$

$$\mathbf{r}_\perp(\tau) = \mathbf{r}_\perp + \int_0^\tau \mathbf{v}_\perp(\tau)\,d\tau \equiv \mathbf{r}_\perp + \mathbf{R}(\tau), \qquad \omega_H = eH/mc, \quad (5)$$

where $\mathbf{R}(\tau)$ is of the order of the Larmor radius. Then, multiplying (4) by $e^{i(kv_z-\omega)\tau}$ we see that the left part becomes

$$\frac{\partial}{\partial \tau}[f_1(\mathbf{r}_\perp(\tau), \mathbf{v}(\tau), k, \omega)\, e^{i(kv_z-\omega)\tau}].$$

Integrating (4) with respect to $d\tau$ from $\operatorname{sign} v_z \cdot \infty$ to 0 (taking account of the sign of k'', the imaginary part of k) we have

$$f_1(\mathbf{r}_\perp, \mathbf{v}, k, \omega) = \frac{e}{m}\int_{\operatorname{sgn} v_z \cdot \infty}^{0} \left\{ \left[\frac{\partial f_0}{\partial \mathbf{v}_\perp(\tau)}\frac{\partial}{\partial \mathbf{r}_\perp} + ik\frac{\partial f_0}{\partial v_z}\right] \varphi(\mathbf{r}_\perp(\tau), k, \omega) - \sigma(\mathbf{r}_\perp(\tau), \omega)\frac{\partial f_0}{\partial v_z}\right\} e^{i(kv_z-\omega)\tau}\,d\tau. \quad (6)$$

When $v_z < 0$ the last term of the integral in (6) is omitted because of the boundary conditions. We shall limit ourselves here to low-frequency oscillations $\omega \ll \omega_H$ and assume that the Larmor radius of the ions is small compared with the wavelengths of the transverse waves.

As is well known, in this case the stationary function must be of the form

$$f_0 = f_0(r + v_\alpha/\omega_H, \mathbf{v}_\perp^2, v_z); \quad (7)$$

integrating (6) with respect to $d\mathbf{v}$, in the linear approximation in $1/\omega_H$ we have[5]

$$n(\mathbf{r}_\perp, k, \omega) = \int f_1\,d\mathbf{v} = \frac{e}{m}k\varphi(\mathbf{r}_\perp, k, \omega)\int_{-\infty}^{\infty}\frac{\partial f_0(r, v_z)/\partial v_z}{kv_z - \omega}dv_z - \frac{ic}{Hr}\frac{\partial \varphi(\mathbf{r}_\perp, k, \omega)}{\partial \alpha}\frac{\partial}{\partial r}\int_{-\infty}^{\infty}\frac{f_0(r, v_z)}{kv_z - \omega}dv_z - \frac{ie}{m}\sigma(\mathbf{r}_\perp, \omega)\int_0^\infty \frac{\partial f_0(r, v_z)/\partial v_z}{kv_z - \omega}dv_z. \quad (8)$$

In the last term in (8) the limits of integration are established from the condition that when $v_z < 0$ the boundary conditions in (4) must be omitted.

Expanding (8) in a Fourier series in α, we have

$$n_s(r, k, \omega) = e\left(k\mu_z + \frac{cs}{eH}\frac{1}{r}\frac{\partial}{\partial r}\mu_r\right)\varphi_s(r, k, \omega) + i\,e\mu_0\sigma_s(r, \omega), \quad (9)$$

where s is the number of the Fourier component. Here we have introduced the notation:

$$\mu_z = \frac{1}{m}\int_{-\infty}^{\infty}\frac{\partial f_0(r, v_z)/\partial v_z}{kv_z - \omega}dv_z, \quad (10)$$

$$\mu_r = \int_{-\infty}^{\infty}\frac{f_0(r, v_z)}{kv_z - \omega}dv_z, \quad (11)$$

$$\mu_0 = \frac{1}{m}\int_0^{\infty}\frac{\partial f_0(r, v_z)/\partial v_z}{kv_z - \omega}dv_z. \quad (12)$$

The integrals in (10)—(12) are defined in the lower half plane of k ($k'' < 0$, ω real) and are analytically continued into the upper half plane of k.

We write

$$f_0(r, v_z) = N(r)\sqrt{m/\pi T}\exp\left(-\frac{mv_z^2}{2T}\right). \quad (13)$$

It then follows from (10), (11), and (13) that

$$\frac{cs}{eH}\frac{1}{r}\frac{\partial}{\partial r}\mu_r = \frac{s\gamma}{\omega}\left(k\mu_z + \frac{N}{T}\right), \quad (14)$$

$$\gamma = \frac{cT}{eH}\frac{1}{r}\frac{\partial \ln N}{\partial r}. \quad (15)$$

Below we denote $|e|$ by e.

Since we are only interested in low-frequency oscillations the plasma can be regarded as quasi-neutral $n_i = n_e$. Then from (9) and (14)

$$\varphi_s(k, \omega) = -\frac{\mu_0^e + \mu_0^i}{(1 + s\gamma_i/\omega)k\mu_z^i + (1 - s\gamma_e/\omega)k\mu_z^e}\, i\sigma_s(\omega), \quad (16)$$

$$n_{si}(k, \omega) = e(1 + s\gamma_i/\omega)k\mu_z^i\varphi_s + (Nes\gamma_i/T_i\omega)\varphi_s + ie\mu_0^i\sigma_s(\omega), \quad (17)$$

$$n_{se}(k, \omega) = -e(1 - s\gamma_e/\omega)k\mu_z^e\varphi_s + (Nes\gamma_e/T_e\omega)\varphi_s - ie\mu_0^e\sigma_s(\omega). \quad (18)$$

3. To compute the mean radial velocity V_r we multiply by mv_α and integrate with respect to $d\mathbf{v}$ the equation for the mean distribution function:

$$\mathbf{v}\frac{\partial f_0}{\partial \mathbf{r}} + \frac{e}{mc}[\mathbf{vH}]\frac{\partial f_0}{\partial \mathbf{v}} = \frac{e}{m}\left\langle \nabla\varphi\frac{\partial f_1}{\partial \mathbf{v}}\right\rangle, \quad (19)$$

whence

$$V_r = \frac{1}{N}\int v_r f_0\,d\mathbf{v} = -\frac{c}{NHr}\left\langle \frac{\partial \varphi}{\partial \alpha} n\right\rangle. \quad (20)$$

It is evident from (14) that the condition of neutrality of the plasma flow (equality of the electron and ion fluxes NV_{re} and NV_{ri}) is satisfied in the general case so long as $\langle \nabla^2\varphi\, \partial\varphi/\partial\alpha \rangle = 0$; this condition holds for all random functions of α that are homogeneous in α, as is the case for φ in the present analysis. From this condition and (20), in view of the relation $n_e = \nabla^2\varphi/4\pi e + n_i$ we find $V_{ri} = V_{re}$.

We assume that $\sigma(\mathbf{r}_\perp, t)$ is a stationary random function of t and α. Then

$$\langle \sigma_l (r, \omega') \sigma_s (r, \omega)\rangle = \sigma_s^2 (r, \omega)\, \delta (\omega + \omega')\, \delta_{-l, s}. \quad (21)$$

Substituting the Fourier expansion for n and φ we have from (14)

$$V_r (r, z) = \frac{c}{NH} \frac{i}{r} \sum_{s=-\infty}^{\infty} s \int \langle \varphi_{-s} (r, k_1, -\omega)\, n_s (r, k_2, \omega)\rangle \times e^{i (k_1+k_2) z}\, dk_1 dk_2 d\omega. \quad (22)$$

In computing (22) it will be found convenient to use the density expression (18). Substituting (13) in (22) we have

$$V_r (r, z) = -\frac{ce}{NH} \frac{i}{r} \sum_s s \int \left(1 - \frac{s\gamma_e}{\omega}\right) k_2 \mu_z^e (k_2, \omega) \times \langle \varphi_{-s} (k_1, -\omega)\, \varphi_s(k_2, \omega)\rangle\, e^{i (k_1+k_2) z}\, dk_1 dk_2 d\omega$$
$$+ \frac{ce}{NH} \frac{1}{r} \sum_s s \int \mu_0^e (k_2, \omega) \langle \varphi_{-s} (k_1, -\omega)\, \sigma_s (\omega)\rangle \times e^{i (k_1+k_2) z}\, dk_1\, dk_2\, d\omega. \quad (23)$$

It is evident that in the first term we need only take account of the imaginary part of μ_z^e. The remaining terms vanish because they are odd with respect to s and ω.

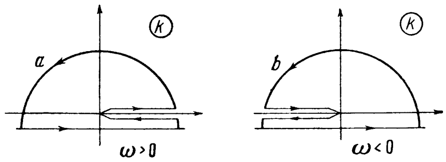
In accordance with the discussion leading to (20), we note that the difference between (23) and the analogous relation expressed in terms of (17) is only an apparent one. They are essentially the same, since $n_i = n_e$.

We now consider the individual terms in (23). The function μ_0 appearing in (23) [through (16)] is expressed as a singular integral (12) with one finite limit. Hence, its imaginary part suffers a discontinuity at the real axis of k when $k' > 0$, $\omega > 0$ or $k' < 0$, $\omega < 0$ (i.e., for diverging waves); this jump is given by:[7]

$$i \sqrt{\frac{2\pi m}{T}}\, \frac{|\omega|}{Tk^2} \exp\left\{-\frac{m}{2T}\left(\frac{\omega}{k}\right)^2\right\}. \quad (24)$$

Consequently, the integration in (23) in the complex planes of k_1 and k_2 is conveniently taken over the contours a or b shown in the figure.

Furthermore, (16) has poles in the upper half plane of k; these can make a contribution in (23) in the integration along the contours (cf. figure).



We consider only those poles for which the imaginary part of k is small since it is these that make the primary contribution to diffusion at large distances from the plane z = 0. In this case we can limit ourselves to the region

$$T_i/m_i \ll \omega^2/k^2 \ll T_e/m_e, \quad |k'| \gg k'', \quad (25)$$

where the following expansion holds:

$$\frac{1}{N} k\mu_z^e \approx -\frac{1}{T_e} - i \sqrt{\frac{\pi m_e}{2T_e}}\, \frac{|\omega|}{kT_e} \exp\left\{-\frac{m_e}{2T_e}\left(\frac{\omega}{k}\right)^2\right\},$$
$$\mathrm{Re}\frac{\mu_0^e}{N} \approx -\frac{1}{2T_e k},$$
$$\frac{1}{N} k\mu_z^i \approx \frac{k^2}{m_i\omega^2} - i \sqrt{\frac{\pi m_i}{2T_i}}\, \frac{|\omega|}{kT_i} \exp\left\{-\frac{m_i}{2T_i}\left(\frac{\omega}{k}\right)^2\right\},$$
$$\mathrm{Re}\frac{\mu_0^i}{N} \approx \frac{1}{\sqrt{m_i T_i}\,\omega}. \quad (26)$$

Substituting this expansion in the denominator of (16) we find that it vanishes at the points

$$k = k' + ik'' = \pm\, \omega \sqrt{m_i \vartheta_s / T_e} + i \sqrt{\pi m_e/8T_e}\, |\omega| \exp(-m_e\vartheta_s/2m_i) + i\sqrt{\pi m_i/8T_i}\, |\omega| \sqrt{\vartheta_s} \exp(-T_e\vartheta_s/2T_i);$$
$$\vartheta_s = (\omega - s\gamma_e)/(\omega + s\gamma_i). \quad (27)$$

In (27) we can set up rather sharp boundaries in ω between which $k'' \ll |k'|$; specifically,

$$(1 + (1 + T_i/T_e)\, 2m_e/m_i) s\gamma_e \equiv a_1 s\gamma_e < \omega < a_2 s\gamma_e \equiv s\gamma_e\, 5T_i/(4T_i - T_e). \quad (28)$$

In this frequency range (27) can be replaced approximately by

$$k = \pm\, \omega \sqrt{m_i\vartheta_s/T_e} + i \sqrt{\pi m_e/8T_e}\, |\omega|. \quad (29)$$

We see that contributions in (23) come only from the residues at the poles and the jump at the branch cut (24). For integration along the contour shown in the figure it can be shown that the basic contribution comes from the product of the residues at the poles of the first term. The other terms are either odd in (ω, s) and give zeros or, like the product of the branch cut and the residue at the poles in the second term in (23), are proportional to γ^2 and need not be considered.

We compute the product of the residues in the first term in (23) by substituting the Green's function (16). The required parity in (k, ω) is achieved here by the odd parity of the product $\mathrm{Re}[\mu_0^e, \mu_0^i]$

with respect to (k, ω) [cf. (26)], which is one of the consequences of the difference of the interaction between converging and diverging waves with particles in the phase velocity region in (28). In the numerator a contribution comes only from the expression

$$k_2 \operatorname{Im}[\mu_z^e(k_2, \omega)] \operatorname{Re}\{\mu_0^e(k_1, -\omega)\mu_0^i(k_2, \omega) + \mu_0^i(k_1, -\omega)\mu_0^e(k_2, \omega)\}. \tag{30}$$

From a circuit of the poles in (29) we have

$$V_r = -\sqrt{\frac{\pi^5 m_e}{2m_i}}\frac{ce}{HT_e}\frac{1}{r}\sum_{s=-\infty}^{\infty} s\int_{l_s}\frac{(\omega+s\gamma_i)^2}{|\omega|(\omega-s\gamma_e)} \times \exp\left(-\sqrt{\frac{\pi m_e}{2T_e}}|\omega|z\right)\sigma_s^2(r, \omega)\,d\omega, \tag{31}$$

where l_s denotes the range of integration in (28); $\sigma_s(\omega)$ is the boundary value of the Fourier component of the electric potential φ_s. We assume that the oscillation spectrum of the electric field at the boundary is independent of s and ω over some range of variation of ω and s; specifically,

$$\sigma_s^2(r, \omega) \approx r^2E^2/s^2\Omega, \quad 1 < s < S, \quad 0 < \omega < \Omega, \tag{32}$$

and that the spectrum vanishes for large values of s and ω. Here, Ω is the maximum frequency of oscillation at the boundary while S is determined by the minimum scale length of the correlation δ or, if γ is large enough, by the quantity Ω/γ_e [cf. (28)] i.e., $S = \min(\Omega/\gamma_e, r/\delta)$.

Substituting (32) and (28) in (31) and making the substitution of variables $\omega = s\gamma_e x$ we have

$$V_r(z, r) = -\sqrt{\frac{2\pi^5 m_e}{m_i}}\frac{ceE^2}{HT_e\Omega}r\gamma_e\sum_{s=1}^{S}\int_{a_1}^{a_2}\frac{(x+T_i/T_e)}{x(x-1)} \times \exp\left\{-\sqrt{\frac{\pi m_e}{2T_e}}s\gamma_e zx\right\}dx. \tag{33}$$

The expression in (33) increases with increasing T_e/T_i because the upper limit a_2 increases. This is because ion acoustic waves can be excited when T_e/T_i is high enough.

We consider the case $T_i = T_e = T$. In this case the basic contribution in (23) comes from the lower limit of ω. Hence summing over s we write

$$NV_r = -N\sqrt{\frac{32\pi^5 m_e}{m_i}}\frac{ceE^2}{HT\Omega}r\gamma\int_{1+4m_e/m_i}^{2}\frac{dx}{x-1} \times \frac{\exp\{-\sqrt{\pi m_e/2T}\,\gamma zx\} - \exp\{-S\sqrt{\pi m_e/2T}\,\gamma zx\}}{1-\exp\{-\sqrt{\pi m_e/2T}\,\gamma zx\}}. \tag{34}$$

When $\sqrt{\pi m_e/2T}\,\gamma z \ll 1/S$ (34) becomes

$$NV_r \approx -N\sqrt{\frac{32\pi^5 m_e}{m_i}}\ln\left(\frac{m_i}{m_e}\right)\frac{ceE^2}{HT\Omega}r\gamma S, \tag{35}$$

where, in place of s, we must substitute the smaller of r/δ or Ω/γ. When $1/S \ll \sqrt{\pi m_e/2T}\,\gamma z \ll 1$ the diffusion flux is

$$NV_r \approx -N\sqrt{\frac{64\pi^5}{m_i T}}\ln\left(\frac{m_i}{m_e}\right)\frac{ceE^2}{H\Omega}\frac{r}{z}. \tag{36}$$

As $\sqrt{\pi m_e/2T}\,\gamma z$ increases further the diffusion flux falls off exponentially with exponent $-\sqrt{\pi m_e/2T}\,\gamma z$.

Using (15) we can estimate $\gamma \sim \rho_i v_i/a^2$ where ρ_i is the Larmor radius, a is the characteristic scale size of the inhomogeneity and v_i is the ion thermal velocity. Thus, the exponential decay in V_r starts when $z/a > \sqrt{m_i/m_e}\,a/\rho_i S$, that is to say, at a rather large distance from the cathode.

All of our calculations hold in the wave region $|k'| \gg k''$. Hence, the results apply only when $z > 1/|k'|$. Since the largest contribution to diffusion comes from oscillations characterized by $\omega \approx s\gamma$ [cf. (28)], k' can be estimated as follows:

$$k' = \omega\sqrt{\frac{m_i}{T}\frac{\omega - s\gamma_e}{\omega + s\gamma_i}} \sim \omega\sqrt{\frac{m_e}{T}} \sim \frac{\Omega}{v_e},$$

where Ω is the highest frequency so that $z \gtrsim v_e/\Omega$. If $\Omega \sim \omega_{Hi}$, then $z \gtrsim \sqrt{m_i/m_e}\,\rho_i$.

4. Thus we have shown that far from the plane $z = 0$ [at which the random noise source is located], when $T_i = T_e$ the enhanced diffusion is caused only by drift waves,[5] which can propagate in an inhomogeneous plasma. These waves are absorbed by the electrons as a consequence of Landau absorption (29). It is precisely by virtue of this absorption that the electrons can diffuse across the magnetic field.

The mechanism responsible for ion diffusion can be established from the following simple considerations. Since the phase velocity of the waves treated here is much greater than the ion thermal velocity all the ions will execute oscillations along the magnetic field with the same amplitude $\xi_z = -eE_z/m_i\omega^2$. Consequently the electric field acting on the ions in the transverse direction will be $E_\perp = E_\perp(z + \xi_z) \approx E_\perp(z) + \xi_z dE_\perp(z)/dz$. Whence we find that each ion on the average will drift across the magnetic field with a velocity

$$V_r = \frac{c\langle E_\perp\rangle}{H} = -\frac{c}{H}\frac{e}{m_i\omega^2}\left\langle E_z\frac{\partial E_\perp}{\partial z}\right\rangle \approx -\frac{c}{H}\frac{e}{m_i\omega^2}\frac{\partial}{\partial z}(E_z^0 E_\perp^0),$$

where E^0 is the amplitude of the electric field oscillation, that is to say $E = E^0\exp(is\alpha - i\omega t + ikz)$; the angle brackets denote time averages.

Thus, the mean (diffusion) ion velocity is also proportional to the wave absorption although the ions themselves do not participate in the absorption. The calculations given in this work show that the diffusion fluxes of the ions and electrons are exactly the same, being given by a common expression (33). For certain values of z this formula yields the simpler expressions (35) and (36). It is evident from the basic expression (36) that there is a region of z, the boundaries of which depend on the density gradient, for which the diffusion flux is independent of $\partial N/\partial r$ and proportional to $1/H$. We note that the diffusion flux falls off as T^{-1}. This is explained by the fact that in the region (25) the electron absorption is proportional to $T^{-3/2}$ [cf. (26)].

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