ON THE RELATIONS AMONG DIFFERENT METHODS FOR DESCRIBING THE INTERACTION OF HIGH-ENERGY PARTICLES

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The question of the relations among the moving-pole method, the strip approximation, and the one-meson approximation is discussed. It is shown that despite great differences in their mathematical forms and in the concrete problems to which they are applied, all three methods are basically similar: they are based on the neglect of certain quantities in very similar ways.

1. There are three methods now being effectively used in the study of the interaction of elementary particles at high energies:

1. The method of moving poles in the complex plane of the orbital angular momentum l [1-5] (hereafter MPM).

2. The strip approximation[6-9] (hereafter SA).

The one-meson approximation[10-16] (hereafter OMA).

The first two methods can obviously as yet be used only for the description of binary reactions ("four-point" diagrams; elastic scattering, perhaps with changes of the character of the particles). The last method was developed for the description of inelastic interactions, but is naturally carried over to elastic processes as well.

The present paper is devoted to ascertaining what are the relations existing among these methods. We shall try to show that: a) the SA and MPM are very similar as to the significance and scope of the neglects that are made, although the MPM has been much more fruitful and more attractive because its fundamental propositions are neater and more precise; b) in the asymptotic limit of large energies, $s \to \infty$, the expression for the elastic scattering amplitude which can be obtained from the OMA coincides with that which follows from the MPM, and consequently the quantities neglected in these methods are the same.

This correspondence of the fundamental propositions establishes a connection between the MPM and the inelastic processes and allows us to use the analysis of experimental data on inelastic interactions to determine the regions of application of the three methods.

Let us formulate what is to be meant by the methods we have listed (for simplicity we shall consider spinless pseudoscalar particles of mass $\mu = 1$).

1. In the MPM it is assumed that the partial amplitude $f(l, t)$ in the t channel is a meromorphic function of the complex variable l and can have poles only at the points $l = l_i(t)$. It is further assumed that at a finite distance there is a pole at $l = l_0(t)$ which is situated in the l plane to the right of all other poles. For it we have[1)]

$$\begin{aligned} \operatorname{Im} l_0(t) &> 0 \quad \text{for } t > 4, \\ \operatorname{Im} l_0(t) &= 0 \quad \text{for } t < 4. \end{aligned} \tag{1}$$

Thus it lies in the first quadrant and is a moving pole (i.e., $l_i \neq \text{const}$).

On these assumptions

$$f(l, t) = \frac{r(l, t)}{l - l_0(t)} + f_r(l, t), \tag{2}$$

where $f_r(l, t)$ is a function which is regular at $l = l_0(t)$, and in the region asymptotic in s the imaginary part of the amplitude $A_1(s, t)$ in the s channel is of the form

$$A_1(s, t)|_{s\to\infty} \approx r(l_0(t), t)\, s^{l_0(t)} = \varphi(t)\, s^{l_0(t)}. \tag{3}$$

2. The SA is based on the use of double dispersion relations (DDR). Being interested in the asymptotic behavior for large s, we can represent the weight function $\rho(s, t)$ in the DDR in the form[2)]

$$\rho(s, t) = \sum_{m=1}^{\infty} \rho_{2m}(s, t) \tag{4}$$

[1)]We recall that in the MPM the function $l_0(t)$ can have branch points not only at $t = 4$, but also at all higher thresholds: $t = 16$, etc.

[2)]The terms $\rho_{2m+1}(s, t)$ give no contribution to the asymptotic value.[17]

and consequently we can represent $A_1(s,t)$ in the form[3)]

$$A_1(s,t)=\frac{1}{\pi}\sum_{m=1}^{\infty}\int_{t_m(s)}^{\infty}\frac{\rho_{2m}(s,t')}{t'-t}\,dt'. \qquad (5)$$

Here the terms of the series correspond to the diagram shown in Figs. 1 and 2.[4)]

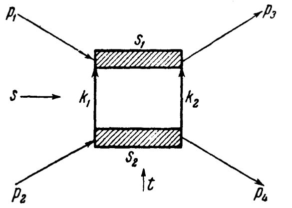

FIG. 1. The diagram which corresponds to $\rho_2(s,t)$.

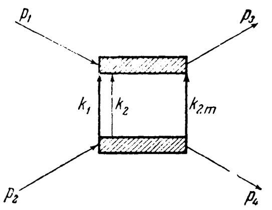

FIG. 2. One of the diagrams corresponding to a term $\rho_{2m}(s,t)$ in Eq. (4) with $m \geq 2$.

The essential point of the SA is the assumption that the predominant contribution to the amplitude comes from the function $\rho_2(s,t)$, so that

$$A_{1\,SA}(s,t)=A_1^{(2)}(s,t)=\frac{1}{\pi}\int_{t_1(s)}^{\infty}\frac{\rho_2(s,t')}{t'-t}\,dt'. \qquad (6)$$

In the region $4 < t < 16$ only $\rho_2(s,t)$ is different from zero. As Mandelstam has shown,[18] the unitarity relation gives

$$\rho_2(s,t)=\frac{2}{\pi t'^{1/2}(t-4)^{3/2}}\int ds_1 ds_2 K A_1(s_1,t)\,A_1^*(s_2,t); \qquad (7)$$

$$K^{-2}=4\,(t-4)^{-2}\,\{(s-s_1-s_2)^2-4s_1s_2-4ss_1s_2/(t-4)\}, \qquad (7a)$$

and the integration is over the region $K^2 > 0$. In the SA the expressions (6) and (7) are extended to the entire region $t > 4$; after this they are usually regarded as an integral equation for the determination of the amplitude $A_{1SA}(s,t)$:

$$A_{1\,SA}(s,t)=\frac{2}{\pi^2}\int_{t_1(s)}^{\infty}dt'\int ds_1 ds_2\,\frac{K A_{1\,SA}(s_1,t')\,A_{1\,SA}^*(s_2,t')}{(t'-t)\,t'^{1/2}(t'-4)^{3/2}}. \qquad (8)$$

This equation, however, does not determine $A_{1SA}(s,t)$ uniquely, but only up to an arbitrary function.

The solution of this equation is[19,20]

$$A_{1\,SA}=\varphi_{SA}(t)\,s^{l_{SA}(t)}, \qquad (9)$$

where $l_{SA}(t)$ is an arbitrary function which has the property (1) but does not branch at the higher threshold values.

3. In the OMA one can also describe inelastic processes, corresponding to diagrams which consist of two (or more) blocks connected by only one meson line. Such diagrams are of the form shown in Fig. 3. (The elastic process which they cause is represented by the diagram of Fig. 1.[5)])

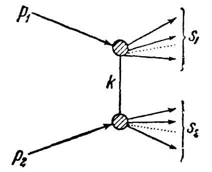

FIG. 3. Diagram for an inelastic process in the OMA.

Accordingly, in the OMA the total cross section can be written in the form (for $s \to \infty$)

$$\sigma_{tot}(s)=\frac{(16\pi)^2 4}{s\,(2\pi)^4}\int d^4k\,\frac{\bar{A}_1(s_1,t=0,k^2)\,\bar{A}_1(s_2,t=0,k^2)}{(k^2+1)^2}. \qquad (10)$$

By using the unitarity relation in the s channel in the framework of the OMA we can find the quantity $A_{1OMA}(s,t)$ in the region $t < 0$ (cf. [12], where it is written in somewhat different variables). It is[6)]

$$A_{1\,OMA}(s,t)=\frac{2}{\pi^3}\int d^4k_1 d^4k_2\,\frac{\delta(p_3-p_1-k_1-k_2)}{(k_1^2+1)(k_2^2+1)}$$
$$\times\{\bar{A}_1(s_1,t,k_1^2,k_2^2)\,\bar{A}_1^*(s_2,t,k_1^2,k_2^2)$$
$$+\bar{A}_1^*(s_1,t,k_1^2,k_2^2)\,\bar{A}_1(s_2,t,k_1^2,k_2^2)\}. \qquad (11)$$

We note that with this formulation of the OMA there is an arbitrariness in it associated with the definition of $A_1(s,t)$. This arbitrariness is due to the fact that the OMA describes a rather wide set of processes, which can be of different natures. To explain this we consider two versions.

A. We may assume that all of the quantities $\bar{A}_1$ are also one-meson quantities, i.e., $\bar{A}_1(s,t,k_1^2$

3)We drop the second term in the dispersion relation for $A_1(s,t)$, since it is unimportant for large s. For the same reason we have dropped the second term in Eq. (7). Also we shall not specify the number of subtractions with respect to t, since this does not affect the results to be obtained.

4)To avoid misunderstanding we must emphasize, however, that the higher terms of the series also contain contributions from the exchange of smaller numbers of mesons; for example, in particular the term ρ_4 also contains a contribution from two-meson exchange, simply because of the unitarity relation.

5)We note that in the SA one uses diagrams in which all of the lines are on the mass shell, i.e., $k^2 = -1$ everywhere. In the OMA one uses Feynman diagrams which differ from the foregoing only in that the lines k (in Fig. 3) and k_1 and k_2 (in Fig. 1) are not on the mass shell ($k^2, k_1^2, k_2^2 > 0$). The forms of the diagrams remain the same.

6)In what follows we shall find it convenient to use this symmetrized formulation. It is clear that it does not contain any additional restrictions, since the terms in the curly brackets in Eq. (11) differ only by a relabelling of the variables of integration.

$= k_2^2 = -1) = A_{1OMA}(s, t)$. Processes of this type have been studied by Amati and others [21] (multiperipheral processes). We shall show later that this approximation is completely equivalent to the SA.

B. In the more general case one must take into account the fact that many-meson processes can also contribute to the amplitudes A_{1OMA}. Then in Eq. (11) we must set $\bar{A}_1 = A_{1\,tot}$, where $A_{1\,tot}$ is the total amplitude for the process,[7)] which for $k_1^2 = k_2^2 = -1$ and $t = 0$ is connected with the total cross section by the optical theorem $\sigma_{tot} = (16\pi/s)A_1\,(s, t = 0)$. In this case with $t = 0$ we can rewrite Eq. (12) in the form

$$\sigma_{OMA}(s)\,|_{s\to\infty} = \frac{4}{(2\pi)^4 s}\int d^4k\,\frac{s_1 s_2 \sigma(s_1, k^2)\,\sigma(s_2, k^2)}{(k^2+1)^2}. \qquad (12)$$

This expression has already been studied in earlier papers.[10-16]

2. Let us now turn to the question of the relation between the OMA and the SA. To settle it, it is necessary to find the expression for $A_1(s, t)$ given by Eq. (11) (OMA) in the region $t > 0$. First let us change in Eq. (11) to integration over the variables s_1, s_2, $-k_1^2$, $-(p_3 - p_1 - k_1)^2$. With the use of the relations

$$s = -(p_1+p_2)^2,\quad t = -(p_1-p_3)^2,\quad s_1 = -(p_1+k_1)^2,$$
$$s_2 = -(p_2-k_1)^2,$$

a simple but lengthy calculation brings Eq. (11) into the form

$$A_{1OMA}(s, t) = \frac{1}{2\pi^3 s}\int ds_1\,ds_2\,dr\,dv\,\frac{\{\bar{A}_1(s_1, t, r, v)\,\bar{A}_1^*(s_2, t, r, v) + \text{c. c.}\}}{[(r-2)^2 - v^2]\,[F(s, t, s_1, s_2, r) - v^2]^{1/2}}, \qquad (13)$$

where we have introduced the following notations:

$$r = -k_1^2 - (p_3 - p_1 - k_1)^2,\quad v = -k_1^2 + (p_3 - p_1 - k_1)^2,$$
$$F = t^2\,(s - s_1 - s_2 + r - 2)^2/(s-4)\,(4 - s - t)$$
$$+\ t\,(r^2 - 2rs_1 - 2rs_2 + 2rs - 4r)/(s-4)$$
$$+\ 4t\,(s_1 s_2 s + s_1^2 + s_2^2 - s_1 s - s_2 s - 2s_1 s_2 + s)/s\,(s-4). \qquad (14)$$

For $t < 0$ the integral over v, r is taken over the region in which $F(s, t, s_1, s_2, r) > v^2$. Furthermore there is the restriction

$$(r-2)^2 > F(s, t, s_1, s_2, r). \qquad (15)$$

[7)]It must be pointed out that this form also does not embrace all one-meson processes, but only the main contribution from them, corresponding to the pole in $f(l, t)$ (see below). This is due to the fact that the higher terms of the series (5) contain, besides entirely many-meson terms, also the contribution from the exchange of two mesons (see footnote 3)

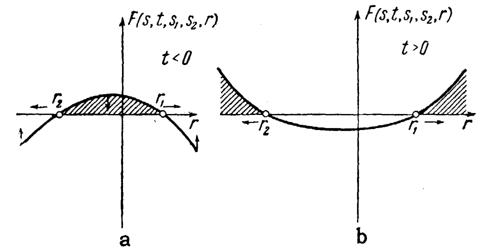

FIG. 4. A comparison of regions a (the shaded area is the region of integration over v and r for $t < 0$; the arrows indicate the directions of motion of the points r_1 and r_2 and of deformation of the curve $F(r)$ as t increases) and b (the region $F > v^2$ for $t > 0$ is shaded) shows that it is impossible to continue the representation (13) from the region $t < 0$ to the region $t > 0$, since the region $F > v^2$ changes discontinuously at $t = 0$.

We note that the representation (13) [and consequently, Eq. (11)] cannot be directly continued into the region $t > 0$. In fact, the regions in which $F > v^2$ are entirely different for $t < 0$ and $t > 0$ (see Fig. 4, a, b) and do not go over into each other continuously. Therefore it is necessary first to take the integral over v for $t < 0$ and then to continue the resulting expression into the region $t > 0$. It is shown in the Appendix that the dependence of $A_1(s_i, t, r, v)$ on v is unimportant (when very weak restrictions are placed on the character of this dependence), since all that matters for what follows is the zeroth term of the expansion of the numerator of the expression (13) in powers of v^2. Carrying out the integration over v, we get

$$A_{1OMA}(s, t) = \frac{1}{2\pi^2 s}\int ds_1\,ds_2\int_{r_2(t)}^{r_1(t)} dr\,\frac{\{\bar{A}_1(s_1, t, r)\,\bar{A}_1^*(s_2, t, r) + \text{c. c.}\}}{(r-2)\,[(r-2)^2 - F(s, t, s_1, s_2, r)]^{1/2}}. \qquad (16)$$

Continuing this expression into the region[8)] $t > 0$, we come to the point t_0 at which $r_1(t_0) = 2$. For $t > t_0(s_1, s_2, s)$ the point $r = 2$ falls in the region of integration and we must take $1/(r-2)$ to mean

$$\frac{1}{r-2} = i\pi\,\delta\,(r-2) + \mathrm{P}\frac{1}{r-2}.$$

Therefore an imaginary part of A_{1OMA} appears,

$$\operatorname{Im} A_{1OMA}(s, t) = \rho_{OMA}(s, t) \qquad (17)$$
$$= \frac{1}{2\pi s}\int ds_1 ds_2\,\frac{\{\bar{A}_1(s_1, t)\,\bar{A}_1^*(s_2, t) + \text{c. c.}\}}{[-F(s, t, s_1, s_2, 2]^{1/2}}.$$

A direct comparison of Eqs. (13) and (8) shows that

[8)]We note that there is no singularity of the integral (13) at $t = 0$, since there $r_1 < 2$.

$$F(s, t, s_1, s_2, 2) \equiv t(t-4)^3 K^{-2}/4s(4-s-t). \qquad (18)$$

If we assume that $\bar{A}_1 \equiv A_{1OMA}$ (see above), then when we use Eq. (18) the expression (17) leads to the equation (8), from which it follows that this version of the OMA is completely equivalent to the SA (as we already mentioned above).

In version B, $\bar{A}_1 = \bar{A}_{1\,tot}$, the OMA is not equivalent to the SA, since the OMA indirectly includes the effect of processes which are inelastic in the t channel (details see below). In this case we cannot regard the expressions (17) and (18) as an integral equation for the determination of A_1, since the left and right members involve different functions. These expressions can be used to find A_{1OMA} if A_{tot} is known.

3. Let us now investigate the connections among the SA, MPM, and OMA. Using the DDR [in particular, Eqs. (5) and (6)], we can easily get the following expression for $f(l, t)$ (we note that there is so far no use of the SA; l is larger than the number of subtractions in s).

$$f(l, t) = \frac{1}{\pi}\int_4^\infty A_1(s, t)\, Q_l\left(1 + \frac{2s}{t-4}\right)\frac{2ds}{t-4}$$
$$= \frac{2}{\pi(t-4)}\int_4^\infty ds\, Q_l\left(1+\frac{2s}{t-4}\right)\left\{\int_{t_1(s)}^\infty \frac{\rho_2(s, t')}{t'-t}dt' + \sum_{m=2}^\infty \int_{t_m(s)}^\infty dt' \frac{\rho_{2m}(s, t')}{t'-t}\right\}, \qquad (19)$$

where $Q_l(z)$ is the Legendre function of the second kind.

Let us consider the interval $4 < t < 16$ and find out what terms of Eq. (19) can provide poles of the function $f(l, t)$. Acting in the framework of the SA, we can substitute in Eq. (19) the function $\rho_2(s, t)$ in the form (7), and then $A_1(s, t)$ in the form (9). Integrating over s in the region $l > n$ (n is the number of subtractions in s) and continuing the result analytically in l,[9)] we can easily get the function $f(l, t)$ in the region $l \sim l_{SA}(t)$ in the form $f(l, t) = \varphi_{SA}(t)[l - l_{SA}(t)]^{-1}$ (this has already been given in [19,20]).

In the framework of the OMA we must use $\rho_2(s, t)$ in the form (17), but in the right member we must substitute $A_{1\,tot}(s, t)$, i.e., the amplitude $A_1(s, t)$ in the form (3). Then in a similar way we get $f(l, t)$ in the region $l \sim l_0(t)$ in the form $f(l, t) = \varphi(t)[l - l_0(t)]^{-1}$. Thus a pole of the function $f(l, t)$ is provided by the first term in (19), i.e., by the quantity $\rho_2(s, t)$.

For $4 < t < 16$ the second term in Eq. (19), which contains ρ_{2m}, $m \geq 2$, is real for real l and t, and consequently can have only symmetric singularities in the l plane. They cannot contribute to the pole at $l \to l_0(t)$, since then there would be a pole at $l = l_0^*(t)$, and in the MPM this is forbidden by the condition (1).

Thus if we confine ourselves to the consideration of the pole contributions to the amplitude $f(l, t)$ in the MPM and in the OMA, the amplitudes $A_1(s, t)$ in the two methods will be the same. The functions ρ_{2m} with $m \geq 2$ do not contribute to these amplitudes. Owing to this we can assert that the neglects made in the MPM and in the OMA are equivalent in their meaning and scope.

We note that the neglect of the nonpole contributions in the MPM is the main hypothesis of the method. In the OMA, besides the contribution from the pole of the function $f(l, t)$, which leads to an asymptotically constant and positive definite term in $A_{1OMA}(s, 0)$, the expression (12) can contain a contribution from the cut in the l plane for the function $f(l, t)$ (our attention was called to this by V. N. Gribov). This last contribution can lead only to an oscillating term in $A_{1OMA}(s, 0)$, which, as can be shown at least for binary reactions, is compensated by the one-meson terms contained in ρ_{2m}, $m \geq 2$. The neglect of this contribution to A_{1OMA} is equivalent to the choice of the function $A_{tot}(s_i, t = 0, k^2)$ so as to lead to a positive definite expression for the integrand in Eq. (12) and for the integral itself. In the papers that use (12) [10-16] this condition has in fact always been imposed in the OMA.

Let us see what effect the functions ρ_{2m} with $m \geq 2$ have on the quantities that remain arbitrary within the framework of the three methods [these functions are: in the MPM and the SA, the value of the residue $\varphi(t)$ and the position $l_0(t)$ of the pole; in the OMA, the functions $\sigma(s_i, k^2)$]. In the MPM for $4 < t < 16$,

$$f(l, t) = f^*(l^*, t) \Big/ \left[1 - 2i\left(\frac{t-4}{t}\right)^{1/2} f^*(l^*, t)\right]. \qquad (20)$$

Since the function $f^*(l^*, t)$ is not singular near the pole of the function $f(l, t)$, terms ρ_{2m} with $m \geq 2$ can contribute to it. Namely, they can affect the positions of poles of the function $f(l, t)$ and the value of the residue $\varphi(t)$. In principle there will necessarily be such an influence, because the functions ρ_{2m} are not completely independent, but are connected by the unitarity relation. This indirect contribution is not included in the SA, and owing to this the SA and the MPM are not completely equivalent. This manifests itself particularly in the fact that in general the functions $l_0(t)$ and

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9)This procedure is correct if the pole of the function $f(l, t)$ at $l \to l_0(t)$ is an isolated singular point, which is always assumed in the framework of the MPM.

$l_{SA}(t)$ are different (the latter cannot have branch points at $t = 16$ and the higher thresholds). This difference, however, cannot affect the characteristics of elastic scattering that can be measured in practice [i.e., $\varphi(0)$ and $\gamma = l'(0)$].

In the OMA this indirect effect is included and is a reflection of the fact that the junctions of the diagram (Fig. 1) have a complex structure which depends on the character of many-meson (in the t channel) processes. In fact, if we try to concretize the structure of the junctions of this diagram it can be seen that they receive contributions from all diagrams of the type shown in Fig. 2. If, however, these processes directly make a nonvanishing additive contribution to $A_1(s, t)$, described by the terms ρ_{2m}, $m \geq 2$, then this contribution will be omitted in all three methods.

4. It is an interesting question, what properties an inelastic process must have if the elastic scattering it causes has the properties prescribed by the moving-pole hypothesis. On the basis of the foregoing, we can assert that these properties will coincide with those found in the OMA. The principal properties in question are [12]:

a) in the center-of-mass system the secondary particles divide into at least two narrowly collimated jets;

b) in the case of two such jets the momentum transfer k^2 (from jet to jet) is small; namely, for $s \to \infty$ we have $k^2 \sim (\ln s)^{-1}$.

c) the distribution of the jets as to "masses," i.e., as to the quantities s_1 and s_2, is such that on the average $s_1 s_2/s \sim (\ln s)^{-1}$.

It must be pointed out that by no means all of the models for inelastic processes that have been considered have these properties. For example, the Fermi-Landau multiple-production process [22] does not have them.[10] Therefore a critical question is, do all experimentally observed inelastic processes actually have the properties listed here? At present the answer is not completely clear, and there is need for further analysis of the existing experiments, and also for more accurate values of some of the data. If the answer is affirmative, this will mean that the MPM and the OMA describe the interaction process completely for $s \to \infty$. If, however, it turns out that there exist processes which do not vanish for $s \to \infty$ and which do not have the properties a)—c) (for example, if it turns out that the Fermi-Landau process occurs), then this will mean that neither the SA, nor the OMA, nor the MPM can give a complete description of the processes. It will then be necessary to ascertain whether the MPM can be extended (and if so, in what way) to take such processes into account.

In conclusion the authors express their deep gratitude to E. L. Feĭnberg for extremely valuable advice and fruitful discussions.

APPENDIX

We shall prove that if the expression

$$B(s_1, s_2, t, r, v) = \tfrac{1}{2}\,[A_1(s_1, t, r, v)\,A_1^*(s_2, t, r, v) + \text{c. c.}] \tag{A.1}$$

with $v^2 = (r-2)^2$ can be expanded in a power series in v^2 in the region $v^2 < F(s, t, s_1, s_2, r)$,[11] in which it is permissible to regroup terms and integrate term by term, and if the coefficients $C_n(t, s_1, s_2, r)$ of the suitably regrouped series have no poles in the region $r_2 < r < r_1$, then the expression (17) for the function $\rho_{OMA}(s, t)$ is unchanged.

In fact, under the conditions indicated we have

$$B(s_1, s_2, t, r, v) = B(s_1, s_2, t, r, (r-2)) + \sum_{n=1}^{\infty} b_n(s_1, s_2, t, r)\,[v^2-(r-2)^2]^n. \tag{A.2}$$

Substituting Eq. (A.2) in Eq. (13) and regrouping the terms of the resulting series in the numerator, we have

$$A_{1\,OMA}(s, t) = \frac{1}{2s\pi^3}\int ds_1\, ds_2 \left\{\pi \int_{r_2}^{r_1} \frac{B(s_1, s_2, t, r, (r-2))}{(r-2)\,[(r-2)^2 - F]^{1/2}} + \int_{r_2}^{r_1} dr \int_{-\sqrt{F}}^{+\sqrt{F}} dv \frac{\sum_{n=0}^{\infty} c_n(s_1, s_2, t, r)\, v^{2n}}{(F - v^2)^{1/2}}\right\}. \tag{A.3}$$

But

$$\int_{-\sqrt{F}}^{+\sqrt{F}} \frac{v^{2n}\, dv}{(F - v^2)^{1/2}} = \frac{(2n-1)!!}{(2n)^n}\,\pi F^n. \tag{A.4}$$

It can now be seen that the second term in the curly brackets in Eq. (A.3) is equal to

$$\pi \int_{r_2}^{r_1} dr \sum_{n=0}^{\infty} c_n(s_1, s_2, t, r)\, F^n(s, t; s_1, s_2, r)\,\frac{(2n-1)!!}{(2n)^n} \tag{A.5}$$

and consequently is always real in the region in which we are interested, since according to Eq. (A.1) the coefficient c_n is real for real arguments. But the first term simply gives an expression for $\mathrm{Im}\, A_{1OMA}$ which is identical with Eq. (17).

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[10] It is not hard to verify that in this process one cannot divide the secondary particles into two jets so that the momentum transfer k^2 falls off logarithmically with increasing energy.

[11] Since $B(s_1, s_2, t, r, v)$ is a symmetric function of k_1^2 and $(p_3 - p_1 - k_1)^2$ it can depend only on the even powers of v.

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