Letters to the Editor

PERTURBATION THEORY AND FERMION REGGE POLES IN ELECTRODYNAMICS

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IN a recent paper of Gell-Mann and Goldberger[1] it is suggested that the asymptotic behavior of the amplitude for the Compton effect on an electron for large s and finite u as derived from perturbation theory may be explained on the assumption that the electron is a Regge pole. In this note it is shown that the asymptotic behavior due to a fermion Regge pole with definite signature is in contradiction with perturbation theory.

This result can be understood without calculations. Suppose that the main contribution to the amplitude comes from the single pole with the biggest value of the angular momentum j and a definite signature. Then the amplitude is proportional to the expression

$$A_{\pm} = r(u)\, s^{l(u)}\left[1 \pm e^{-i\pi l(u)}\right] / \sin \pi l(u), \qquad l = j - \tfrac{1}{2}. \quad (1)$$

The ratio Im A/Re A is equal to $\tan(\pi l/2)$ for a positive signature and to $\cot(\pi l/2)$ for a negative signature. Let us expand the quantity $l(u)$ in a power series in the coupling constant α: $l(u) = 0 + \alpha l_1(u) + \ldots$ For a positive signature, if the expansion starts with terms of order α, Im A_+ differs from Re A_+ only by the factor $\alpha\pi l_1/2$ and does not vanish in this order. For a negative signature Im $A_- >$ Re A_-.

Let us consider now perturbation theory. The second order amplitude contains terms of order $\alpha \ln s$ relative to the main pole diagram so that the expansion, indeed, starts with α. In fact only one diagram contributes to the amplitude, the box diagram with the external photon lines permuted. The imaginary part of this diagram in the u channel vanishes. In this manner we arrive at a contradiction.

The reason for this contradiction is clear; if the signature is definite then one has symmetry (or antisymmetry) with respect to the substitution $s \to -s$, which under our conditions corresponds to the substitution $s \to t$; but the perturbation theory diagrams possess no symmetry with respect to s and t. On the contrary, in order to obtain the absence of symmetry that follows from perturbation theory it is necessary to postulate the existence of a fermion Regge pole with negative signature related in a definite manner to a pole with positive signature.

In actual calculations it is necessary to take into account the existence, for negative u, of two complex conjugate poles with opposite parities. As a result the amplitude is defined not by Eq. (1) but by the following combinations:

$$H = m(A + \bar{A}), \qquad B = \sqrt{-u}(A - \bar{A});$$

$\bar{A}$ differs from A by the replacement of the pole trajectory j(u) and the residue r(u) by their complex conjugate values. Comparing the expressions following from the Regge pole hypothesis[2] with perturbation theory[1] one can obtain a power series expansion in α for j(u) and r(u). In particular

$$\operatorname{Re} j = \tfrac{1}{2} - \alpha\left[I_0(m^2 - u) + I_1 u\right]/2\pi,$$
$$\operatorname{Im} j = -\alpha m\sqrt{-u}\, I_1/2\pi. \quad (2)$$

Here I_0 and I_1 are integrals introduced in [1]. In perturbation theory the amplitude is real and therefore in fact one is comparing Re H and Re B. On the other hand the values of j(u) and r(u) obtained from Re H and Re B uniquely determine Im H and Im B, which turn out to be equal not to zero but to the following expressions:

$$\operatorname{Im} H = -\alpha(m^2 - u)(I_0 - I_1)/4,$$
$$\operatorname{Im} B = -\alpha(m^2 - u) I_0/4. \quad (3)$$

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[1] M. Gell-Mann and M. L. Goldberger, Phys. Rev. Lett. 9, 275 (1962).

[2] Gorshkov, Pekalo, and Frolov, JETP 45, No. 2 (1963) (in press).

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