γ-NEUTRINO CORRELATION IN NUCLEAR μ-MESON CAPTURE

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The formula for the correlation of the directions of emission of the neutrino and γ-ray quantum with the direction of the μ-meson beam is obtained for the case of allowed K capture. It is shown that the γν correlation for the capture of an unpolarized μ meson is very sensitive to the magnitude and sign of the constant of the induced pseudoscalar interaction. A general formula is given for the correlation in μ-meson K capture of any degree of forbiddenness.

1. INTRODUCTION

At present most of the emphasis in the study of μ-meson capture by protons is directed toward finding out the contribution of the pseudoscalar interaction and of Gell-Mann's "weak magnetism." As is well known,[1] μ capture is described in the framework of the theory of the universal V-A interaction, in which, however, one must take into account contributions from diagrams with one-pion and two-pion intermediate states, which are the respective source of the appearance in the Hamiltonian of the induced pseudoscalar interaction and of Gell-Mann's "weak magnetism."

Inclusion of these diagrams is necessary because the momentum transfer in μ capture is of the order of the mass of the μ meson. The contribution of the first diagram was estimated by Goldberger and Treiman in the simplest approximation of the pole diagram. It turned out that the effective pseudoscalar constant is large in magnitude ($C_P \approx 8C_A$), although the total contribution of the pseudoscalar interaction to the probability is small, since it comes in with a factor m/M, where m is the mass of the μ meson and M is that of the nucleon; the positive value of the ratio follows from a consideration of the weak decay of the π meson through a nucleon-antinucleon pair.

In the Gell-Mann-Feynman theory of the universal weak interaction the vector current is conserved, and the contribution of "weak magnetism" can be expressed in terms of the anomalous magnetic moments of the proton and neutron.[2]

Analysis of the experimental data[3] on the capture probability, the hyperfine splitting of the levels of the mesic atom, and the angular anisotropy of the neutrons from the direct process indicates excellent agreement with the V-A theory, in which the vector current is conserved and the quantity C_P/C_A is positive and large. It must be noted, however, that the contribution of the P interaction and the "weak magnetism" to the partial probability does not exceed ~ 20 percent of the main—Fermi and Gamow-Teller—terms,[1)] and the asymmetry coefficient can come only from the correction terms. There is no such cancellation, however, in the case of a $0^+ \rightarrow 1^+$ transition (for example, $C^{12} \rightarrow B^{12}$), which is convenient because there is no hyperfine splitting of the levels of the mesic atom. Moreover, the asymmetry coefficient is proportaional to the degree of longitudinal polarization of the μ meson in the K orbit, and in light atoms this is not more than 15 to 20 percent of the original complete polarization.

More accurate information about the magnitude and sign of the pseudoscalar interaction constant and the contribution of "weak magnetism" than is given by the experiments described above can evidently be obtained from a study of the angular γν correlation in the capture of unpolarized μ mesons. Such a correlation exists only when there are correction terms, and is ~ 20 percent for $C_P/C_A \sim 8$, whereas when C_P/C_A is small the correlation does not exceed a few percent.

2. CORRELATION OF THE DIRECTIONS OF EMISSION OF THE γ-RAY QUANTUM AND THE NEUTRINO WITH THE DIRECTION OF THE μ-MESON BEAM

In μ-meson capture by a nucleus the energy of the captured μ meson is mainly carried away by

[1)]This can be seen from Eq. (8). For the direct-process neutrons there will be complete cancellation of the main terms, but the absorption of the neutron in the nucleus must be taken into account, and this depends on the structure of the nucleus.

the neutrino. There is, however, a definite probability of capture with the formation of a daughter nucleus in an excited state, which emits a γ-ray quantum.[2)]

The probability for the capture of a polarized μ meson by a nucleus with the subsequent emission of a γ-ray quantum can contain the neutrino momentum **q**, the photon momentum **k**, and the μ-meson polarization axial vector $\boldsymbol{\sigma}$. Furthermore **k** can appear only quadratically, since parity is conserved in γ-ray transitions, and σ cannot appear in degrees higher than the first, since the spin of the μ meson is $1/2$ (there can be only a dipole polarization).

In the case of allowed μ capture the vector **q** cannot appear in degrees higher than the second. Under these conditions the following scalar and pseudoscalar quantities can be constructed from **q**, **k**, and σ: from σ and **q**, the pseudoscalar $\boldsymbol{\sigma}\cdot\mathbf{q}$, which determines the angular distribution of the recoil nuclei in the capture of polarized μ mesons; from **q** and **k**, the scalar $(\mathbf{q}\cdot\mathbf{k})^2$, which determines the angular correlation of the γ-ray quantum and the recoil nucleus in the capture of unpolarized μ mesons; and from the two vectors **k** and **q** and the axial vector σ there can be constructed two pseudoscalars[3)] $(\mathbf{k}\cdot\boldsymbol{\sigma})(\mathbf{k}\cdot\mathbf{q})$ and $([\mathbf{k}\times\mathbf{q}]\cdot\boldsymbol{\sigma})(\mathbf{k}\cdot\mathbf{q})$, which determine the correlation of the directions of emission of the γ-ray quantum and the recoil nucleus with the direction of the μ-meson beam.

As can be seen, an angular anisotropy of the γ-ray quanta relative to the polarization vector of the μ mesons cannot be observed without a correlation with the direction of emergence of the recoil nuclei. This is a consequence of the fact that a polarized μ meson can give only a dipole polarization to the nucleus.

Thus in an allowed μ capture the total probability of angular $\gamma\nu$ correlation with the direction of the μ meson beam is of the form

$$W = 1 - (\alpha + \eta k^2)\langle\sigma\rangle\,\boldsymbol{\sigma}\mathbf{q} - \beta P_2(\mathbf{qk}) - \eta\langle\sigma\rangle(\mathbf{k}\boldsymbol{\sigma})(\mathbf{kq}) - \xi\langle\sigma\rangle([\mathbf{kq}]\,\boldsymbol{\sigma})(\mathbf{kq}) \qquad (1)^*$$

Here $\langle\sigma\rangle$ is the degree of longitudinal polarization of the μ meson at the instant it gets into the K orbit; σ, **q**, and **k** are unit vectors in the respective directions; $P_2(\mathbf{q}\cdot\mathbf{k})$ is the Legendre polynomial; and α, β, η, and ξ are functions of the energy of the neutrino, which depend on the nuclear matrix elements and the weak-interaction constants.

The general theory of angular correlations in nuclear transitions is expounded in a paper by Dolginov.[5] In the case of nuclear μ-meson capture with subsequent γ-ray emission the transition probability is

$$W = \langle |H_\mu H_\gamma|^2\rangle, \qquad (2)$$

where H_γ is the matrix element of the radiative transition, as given, for example, by Dolginov,[5] and H_μ is the matrix element of the μ capture, as given, for example, by Morita and Fujii[6]:

$$H_\mu = \int \psi^*_{j_1\mu_1}\overline{H}\tau_-\psi_{j_0\mu_0}d\mathbf{r}, \qquad (3)$$

($\psi_{j_0\mu_0}$ and $\psi_{j_1\mu_1}$ are the wave functions of the initial and final states of the nucleus, and τ_- is the operator for conversion of proton into neutron).

In the nonrelativistic approximation for the nucleons

$$\overline{H} = C_V L(1) + C_A\boldsymbol{\sigma}L(\boldsymbol{\sigma}) + (C_V/2M)\,[2L(\boldsymbol{\alpha})\,\mathbf{p} + \mathbf{p}L(\boldsymbol{\alpha})] + (C_A/M)\,L(\gamma_5)\,\boldsymbol{\sigma}\mathbf{p} + (C_A - C_P)\,\boldsymbol{\sigma}\mathbf{p}L(\beta\gamma_5)/2M + (1+\mu_p-\mu_n)\,C_V i\,(\boldsymbol{\sigma}\,[\mathbf{p}L(\boldsymbol{\alpha})]/2M. \qquad (4)$$

Here $L(\sigma) = \psi^*_\nu(1+\gamma_5)\sigma\psi_\mu$, and so on, μ_p and μ_n are the anomalous magnetic moments of the proton and neutron, and σ, α, β, and γ_5 are Dirac matrices. The differential operator **p** on the left of the lepton invariant L acts only on it, and not on the nucleon wave function.

We take the neutrino wave function in the form of the expansion of a plane wave in terms of spherical vectors (cf. [5]):

$$\psi_\nu = \sum i^{-l}(-)^{J+M}\sqrt{2l+1}\,v_\rho C^{J\rho}_{1/2\rho l0}D^J_{-M\rho}(\varphi\vartheta 0)\,\psi_{JM\lambda}(\mathbf{r}),$$

$$\psi_{JM\lambda}(\mathbf{r}) = \begin{pmatrix} -if_k\hat{Y}^{-\lambda}_{JM}(\mathbf{r}) \\ g_k\hat{Y}^{\lambda}_{JM}(\mathbf{r}) \end{pmatrix}, \qquad (5)$$

where $\hat{Y}^{\tau}_{L\Lambda}(\mathbf{r})$, $D^J_{\mu\mu'}(\alpha,\beta,\gamma)$ and $C^{c\gamma}_{a\alpha b\beta}$ are defined in [5]; v_ρ are spinor components in the rest system of the neutrino; l, J, M are respectively the orbital and total angular momentum and the projection of the total angular momentum of the neutrino on the axis of quantization;

$$k = 2\lambda\,(J + 1/2) = \begin{cases} l, & k > 0 \\ -l-1, & k < 0 \end{cases} \qquad (6)$$

with $\lambda = \pm 1/2$; and f_k and g_k are the radial parts of the neutrino wave function.

The summation in Eq. (5) is over the possible values of all the quantum numbers.

We take the wave function of the μ meson in the form $\psi_\mu = \sum a_\sigma\psi^\sigma_\mu$, where a^ξ_σ determines the degree of polarization of the meson in the K orbit of the

[2)]For example, in the case of μ-meson capture by C^{12} nuclei 10 percent of the transitions are to an excited level of the B^{12} nucleus.

[3)]The pseudoscalar $([\mathbf{k}\times\mathbf{q}]\cdot\sigma)(\mathbf{k}\cdot\mathbf{q})$ is not invariant under time reversal.

*$(\mathbf{qk}) = \mathbf{q}\cdot\mathbf{k}$, $[\mathbf{qk}] = \mathbf{q}\times\mathbf{k}$.

mesic atom. The quantities a^{ξ}_{σ} fix the density matrix $\rho(\sigma,\sigma') = \langle a^{\xi *}_{\sigma} a^{\xi}_{\sigma'} \rangle_{\xi}$, where $\langle \ldots \rangle_{\xi}$ means averaging over the statistical ensemble. Choosing the axis of quantization along the direction of the initial momentum of the μ meson (before it is stopped), we get the density matrix in diagonal form.

Confining ourselves to the nonrelativistic approximation[4)] for the wave function of the μ meson in the K orbit, we have

$$\psi^{\sigma}_{\mu} = \begin{pmatrix} 0 \\ g_{\mu}\chi^{\sigma} \end{pmatrix},$$

where χ^{σ} is a normalized spinor and

$$g_{\mu} = 2\,(\alpha Z m)^{3/2} e^{-\alpha Z m r}.$$

When in the matrix element H_{γ} we expand the potential of the electromagnetic wave in a series of multipoles and substitute ψ_{ν} and ψ_{μ} in H_{μ}, we get the general expression for the angular correlation[5)] of **q**, **k**, and σ:

$$W = \sum i^{l-l'} (2J+1)(2J'+1)\sqrt{2l'+1}\, C^{l0}_{l'0f0}\, W\left(\tfrac{1}{2} J l' f;\, l J'\right)$$
$$\times W(j_0 j_1 I S;\, I' j_1)\, B_S \Big\{(-)^{J'-J}\, W\left(\tfrac{1}{2} I' J S;\, J' I\right)(2S+1)$$
$$\times \delta_{fS} P_S(\mathbf{qk}) + \sqrt{6}\,\langle\sigma\rangle (-)^{I'-I} X\left(J J' f I I' S \tfrac{1}{2} \tfrac{1}{2} 1\right)$$
$$\times F_{S1f}(\mathbf{qk}\boldsymbol{\sigma})\Big\} B^{l\rho}_{l} B^{l'\rho'*}_{l'}. \qquad (7)$$

The sum is taken over all values of the indices for which $B^{l\rho}_{l}$ and B_S are different from zero.

Here there has already been a summation over the polarizations of the γ ray (S is even) and the directions of the neutrino polarization. The factor $1+\gamma_5$ allows us to sum over the helicities of the neutrino. In the derivation of Eq. (7) there has been a summation over the polarizations of the μ meson.

The quantities $B^{l\rho}_{l}$, $F_{S_1 f}(\mathbf{qk}\sigma)$, and B_S are defined in Appendix I; W(abcd;ef) and X(abcdefghi) are the Racah and Fano functions, respectively (cf. e.g., [5]); and $P_S(\mathbf{qk})$ is a Legendre polynomial.

For light nuclei, $A \lesssim 16$, we have $R/\lambda\!\!\!^{-} \lesssim 1-2$ (R is the radius of the nucleus and $\lambda\!\!\!^{-}$ is the wavelength of the neutrino), and the expansion of the Bessel functions involved in f_k and g_k in powers of $R/\lambda\!\!\!^{-}$ still gives a series which converges well, so that the μ capture can be classified as to degrees of forbiddenness.

In the case of allowed μ capture, $\Delta j = 0, \pm 1$ (no), the possible values of the indices are:

$$J, J' = 1/2, 3/2; \quad l, l' = 0, 1, 2; \quad f = 0, 1, 2;$$
$$I, I' = 0, 1; \quad S = 0, 2.$$

Retaining in Eq. (7) the sum over these values of the indices, we see that the expression (7) reduces to the form (1), where α, β, η, and ξ are now given by

$$\alpha W_0 = \left(\frac{C_V}{C_A}\right)^2 \left|\int 1\varphi_{\mu}\right|^2 \left(1 + \frac{q}{M} - \frac{1}{3} q^2 \langle r^2\rangle_e\right)$$
$$- \frac{1}{3}\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right|^2 \Big[1 - \frac{1}{3} q^2 \langle r^2\rangle_A$$
$$- \frac{q}{M}\left(1 - \frac{C_P}{C_A} + 2(1+\mu_p-\mu_n)\frac{C_V}{C_A}\right)\Big]$$
$$- \frac{2}{3}\frac{C_V}{C_A}\frac{q}{M}\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \left|\int [\mathbf{rp}]\varphi_{\mu}\right| - \frac{2}{3}\frac{q}{M}\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \left|\int i\mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\varphi_{\mu}\right|$$
$$- \frac{2}{15} q^2 \left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \left(\left|\int \mathbf{r}(\boldsymbol{\sigma}\mathbf{r})\varphi_{\mu}\right| - \frac{1}{3}\left|\int r^2\boldsymbol{\sigma}\varphi_{\mu}\right|\right)$$
$$- \frac{2}{3}\left(\frac{C_V}{C_A}\right)^2 \frac{q}{M}\left|\int 1\varphi_{\mu}\right| \left|\int i\mathbf{rp}\varphi_{\mu}\right|,$$

$$\beta W_0 = \frac{q}{M}\Lambda^{L}_{j_0 j_1}\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \Big\{\Big[1 - \frac{C_P}{C_A} + (1+\mu_p-\mu_n)\frac{C_V}{C_A}\Big]\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right|$$
$$- \frac{C_V}{C_A}\left|\int [\mathbf{rp}]\varphi_{\mu}\right| - 2\left|\int i\mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\varphi_{\mu}\right| - \frac{3}{10} qM\Big(\left|\int \mathbf{r}(\boldsymbol{\sigma}\mathbf{r})\varphi_{\mu}\right|$$
$$- \frac{1}{3}\left|\int r^2\boldsymbol{\sigma}\varphi_{\mu}\right|\Big)\Big\},$$

$$\eta W_0 = \Lambda^{L}_{j_0 j_1}\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \Big\{\Big[1 + \frac{q}{2M}\Big(1 - \frac{C_P}{C_A} - (1+\mu_p$$
$$- \mu_n)\frac{C_V}{C_A}\Big)\Big]\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right|$$
$$+ \frac{C_V}{C_A}\frac{q}{2M}\left|\int [\mathbf{rp}]\varphi_{\mu}\right| - \frac{q}{M}\left|\int i\mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\varphi_{\mu}\right|$$
$$- \frac{1}{20} q^2 \Big(\left|\int \mathbf{r}(\boldsymbol{\sigma}\mathbf{r})\varphi_{\mu}\right| - \frac{1}{3}\left|\int r^2\boldsymbol{\sigma}\varphi_{\mu}\right|\Big)\Big\},$$

$$\xi W_0 = \frac{3}{5}\sqrt{\frac{3}{2}}\frac{q}{M}\Lambda^{L}_{j_0 j_1}\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \Big\{\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \Big[\operatorname{Im}\frac{C_P^*}{C_A}$$
$$- (1+\mu_p-\mu_n)\operatorname{Im}\frac{C_V^*}{C_A}\Big] + \left|\int [\mathbf{rp}]\varphi_{\mu}\right| \operatorname{Im}\frac{C_V^*}{C_A}\Big\}. \qquad (8)$$

Here W_0 is equal, apart from a common factor, to the probability of μ capture (cf. [6]):

$$W_0 = \left(\frac{C_V}{C_A}\right)^2 \left|\int 1\varphi_{\mu}\right|^2 \left(1 + \frac{q}{M} - \frac{1}{3} q^2 \langle r^2\rangle_e\right)$$
$$+ \left|\int \boldsymbol{\sigma}\varphi_{\mu}\right|^2 \Big[1 - \frac{1}{3} q^2 \langle r^2\rangle_A$$
$$+ \frac{q}{3M}\left(1 - \frac{C_P}{C_A} - 2(1+\mu_p-\mu_n)\frac{C_V}{C_A}\right)\Big]$$
$$+ \frac{2}{3}\frac{C_V}{C_A}\frac{q}{M}\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \left|\int [\mathbf{rp}]\varphi_{\mu}\right| - \frac{2}{3}\frac{q}{M}\left|\int \boldsymbol{\sigma}\varphi_{\mu}\right| \left|\int i\mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\varphi_{\mu}\right|$$
$$- \frac{2}{3}\left(\frac{C_V}{C_A}\right)^2 \frac{q}{M}\left|\int 1\varphi_{\mu}\right| \left|\int i\mathbf{rp}\,\varphi_{\mu}\right|,$$

[4)]The binding energy of the μ meson in the K orbit is much smaller than its rest energy.

[5)]We omit common factors which do not affect the correlation.

where q is the energy of the neutrino.

For electric multipole radiation of character 2^L in a transition $j_0 \xrightarrow{\mu} j_1 \xrightarrow{\gamma} j_2$ we have

$$\Lambda^L_{j_0j_1} = -\sqrt{2/3}\,[1 - 3/L(L+1)]\sqrt{5(2L+1)}(2j_1+1) \times W(j_2Lj_12;\, j_1L)\, W(j_0j_112;\, 1j_1)\, C^{L0}_{L020}.$$

In the expressions for W_0, α, β, and η it is assumed that the constants C_i are real. If they are complex (failure of invariance under $t \to -t$) we must replace C_i'/C_i by $\mathrm{Re}(C_i'^*/C_i)$ in these expressions. The expressions for $\int 1\varphi_\mu$ and so on are given in Appendix II.

In the derivation of Eq. (8) the second terms in the expansions of the Bessel functions have been included only in the main terms that do not contain the small parameter q/M. Furthermore, in terms of types **pL** and [**pL**] there are two Bessel functions satisfying the rules for an allowed transition, and we have kept only the contribution from the Bessel function of lower order. In applying the operator **p** to L we have not kept the result of the action of **p** on the μ-meson wave function, since this brings in the small parameter αZ.

The asymmetry coefficient in the angular distribution of the recoil nuclei relative to the μ-meson beam agrees with the corresponding expression in the paper of Morita and Greenberg.[7]

As can be seen from Eq. (8), the coefficient β, which determines the angular correlation of the neutrino and the recoil nucleus in the capture of an unpolarized μ meson, depends only on the interference of the correction terms with the Gamow-Teller main term for the transition[6)] ($\int \sigma\varphi_\mu$). This is a consequence of the fact that the correlation $P_2(\mathbf{qk})$ must contain the "d waves of the neutrino" (the main contribution to the correlation is the interference of the "s waves and d waves of the neutrino"), which in an allowed μ capture can arise only from the terms in the Hamiltonian that contain the operator **p** or from terms in the nuclear matrix element that contain $j_2(qr)$ [$j_2(qr)$ is a Bessel function].

In the case of a pure Gamow-Teller transition, which predominates in μ capture with excitation of the nucleus (cf. e.g., [8]), the formula for the angular correlation in the capture of an unpolarized μ meson is particularly simple:

$$W = 1 + \varkappa\,(q/M)\,P_2(\mathbf{qk}),$$

$$\varkappa = \Lambda^L_{j_0j_1}\Big\{\frac{C_P}{C_A} + \mu_p - \mu_n + \Big[2\Big|\int i\mathbf{r}\,(\boldsymbol{\sigma}\mathbf{p})\,\varphi_\mu\Big| - \Big|\int [\mathbf{rp}]\,\varphi_\mu\Big| + \frac{3}{10}qM\Big(\Big|\int \mathbf{r}\,(\boldsymbol{\sigma}\mathbf{r})\,\varphi_\mu\Big| - \frac{1}{3}\Big|\int r^2\boldsymbol{\sigma}\varphi_\mu\Big|\Big)\Big]\Big|\int \boldsymbol{\sigma}\varphi_\mu\Big|^{-1}\Big\}$$

$$\times\Big\{1 - \frac{1}{3}q^2\langle r^2\rangle_A + \frac{q}{3M}\Big[3 - \frac{C_P}{C_A} + 2(\mu_p - \mu_n) - 2\Big(\Big|\int i\mathbf{r}\,(\boldsymbol{\sigma}\mathbf{p})\,\varphi_\mu\Big| + \Big|\int [\mathbf{rp}]\,\varphi_\mu\Big|\Big)\Big|\int \boldsymbol{\sigma}\varphi_\mu\Big|^{-1}\Big]\Big\}^{-1}. \qquad (9)$$

Here $C_V = -C_A$.

As can be seen, calculation of nuclear matrix elements is necessary only for the estimation of corrections that do not contain the P interaction and the "weak magnetism," but it is necessary to take these corrections into account in determining the lower limit on the asymmetry coefficient. We can see from Eq. (9) that in the case of a $0 \to 1 \to 1$ transition the anisotropy coefficient $\kappa q/M$ for E_2 radiation reaches values ~ 20 percent for $C_P/C_A \sim 8$, while the sign of the anisotropy coefficient is determined by the sign of the pseudoscalar constant.

Anisotropic emission of circularly polarized γ rays relative to the μ-meson beam will be observed in the radiative capture of polarized μ mesons (cf. [9]). The maximum of the internal bremsstrahlung is at ~ 50 MeV, however, whereas the largest energy of nuclear γ-radiation in μ capture does not exceed ~ 8 MeV. Furthermore the probability of bremsstrahlung is not more than $\sim 10^{-4}$ of the probability of nuclear γ-radiation.

It must be noted that at nuclear excitation energies > 1 MeV there can be an appreciable probability of conversion into electron pairs for the range of light nuclei we are considering. The most probable transition $0 \to 0$ is uninteresting, however, because in this case there is no angular correlation between the components of the pair and the recoil nucleus.

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APPENDIX I

The quantities $B_l^{I\rho}$ which appear in Eq. (7) determine the order of forbiddenness in μ capture:

$$C^{j_0\mu_0}_{I\Lambda j_1\mu_1} B_l^{I\rho} = \int \psi^*_{j_1\mu_1} S_l^{I\rho}\tau_-\psi_{j_0\mu_0}\,d\mathbf{r},$$

$$r^{-I}S_l^{I\rho} = \sum_{\omega=\pm 1/2}\Big\{\Big[C_V Y_{I\Lambda}a_l^I - \sqrt{3}C_A r^\rho\boldsymbol{\sigma}\mathbf{Y}^\rho_{I\Lambda}a_l^{I\rho} - \sqrt{3}\,\frac{C_V}{2M}Y_{I\Lambda}b_l^I - 3\sqrt{2}\,(1+\mu_p-\mu_n)\,\frac{C_V}{2M}\,(r^{\rho+1}\boldsymbol{\sigma}\mathbf{Y}^1_{I\Lambda}C_l^{I,\rho+1} - r^{\rho-1}\boldsymbol{\sigma}\mathbf{Y}^{-1}_{I\Lambda}C_l^{I,\rho-1})$$

$$- \frac{C_A - C_P}{2M}\,(r\boldsymbol{\sigma}\mathbf{Y}^1_{I\Lambda}d_l^{I,1} - r^{-1}\boldsymbol{\sigma}\mathbf{Y}^{-1}_{I\Lambda}d_l^{I,-1})\Big]g_\mu - \sqrt{3}\,\frac{C_V}{M}\,ir^\rho\mathbf{Y}^\rho_{I\Lambda}m_l^{I\rho}g_\mu\mathbf{p} + \frac{C_A}{M}\,iY_{I\Lambda}n_l^I g_\mu\boldsymbol{\sigma}\mathbf{p}\Big\},$$

[6)] Terms proportional to $(q/M)^2$ are omitted.

where

$$r^l a_l^l = g_k \delta_{\lambda\omega}, \quad r^{l+\rho} a_l^{l\rho} = g_k U_{l\rho} \delta_{\lambda\omega},$$

$$r^l b_l^l = (2l+1)^{-1/2} \Big[l^{1/2} \Big(\frac{d}{dr} - \frac{l-1}{r} \Big)$$

$$+ (l+1)^{1/2} \Big(\frac{d}{dr} + \frac{l+2}{r} \Big) \Big] f_k U_{l\rho} \delta_{\rho 1} \delta_{\lambda-\omega},$$

$$r^{l+\rho+1} C_l^{l,\,\rho+1} = (l+\rho+1)^{1/2} W(11ll+\rho;\,1l$$

$$+\rho+1) \Big(\frac{d}{dr} - \frac{l+\rho}{r} \Big) f_k U_{l\rho} \delta_{\lambda-\omega},$$

$$r^{l+\rho-1} C_l^{l,\,\rho-1} = (l+\rho)^{1/2} W(11ll+\rho;\,1l$$

$$+\rho-1) \Big(\frac{d}{dr} + \frac{l+\rho+1}{r} \Big) f_k U_{l\rho} \delta_{\lambda-\omega},$$

$$r^{l+1} d_l^{l,1} = [(l+1)/(2l+1)]^{1/2} (d/dr - l/r) f_k \delta_{\rho 0} \delta_{\lambda-\omega},$$

$$r^{l-1} d_l^{l,-1} = [l/(2l+1)]^{1/2} (d/dr + (l+1)/r) f_k \delta_{\rho 0} \delta_{\lambda-\omega},$$

$$r^{l+\rho} m_l^{l\rho} = f_k U_{l\rho} \delta_{\lambda-\omega}, \quad r^l n_l^l = f_k \delta_{\rho 0} \delta_{\lambda-\omega}.$$

Here $U_{I\rho} = [2(2I+1)]^{1/2} W(J\tfrac{1}{2}I+\rho 1;\, I\tfrac{1}{2})$, $\rho = 0, \pm 1$. The quantum numbers l, k, λ, and J are connected by the relation (6). For terms that do not contain the operator **p** acting on nucleon wave functions the smallest possible values of these numbers for a given order of forbiddenness N are determined from the conditions

$$J + \omega = \begin{cases} l = N \\ l + \rho = N,\ N+2 \\ l + \rho - 1 = N \\ l + \rho + 1 = N, N+2 \end{cases}$$

For the terms with the coefficients $m_l^{I\rho}$ and $n_l^{I\rho}$ the possible values of l and J are determined from the conditions

$$J + \omega = l + \rho = N-1, N+1, \quad J + \omega = l = N+1.$$

The quantities $B_l^{I\rho}$ which do not conserve parity can be obtained from those given above if we make the replacements

$$g_k \rightleftarrows if_k, \quad \delta_{\lambda \pm \omega} \to \delta_{\lambda \mp \omega}.$$

The quantities $F_{S1f}(\mathbf{qk}\sigma)$ which appear in Eq. (7) are defined in the following way:

$$F_{S1f}(\mathbf{qk}\sigma) = 4\pi \sum_{\tau_1} C_{10S\tau_1}^{f\tau_1} Y_{S\tau_1}(\theta\Phi) Y_{f\tau_1}^*(\vartheta\varphi)$$

in the coordinate system where **q** has angles $(\vartheta\varphi)$, **k** angles $(\theta\Phi)$, and σ angles (00). Particular values of these quantities are given in [5].

For the case of electric multipole radiation of character 2^L the quantity B_S which appears in Eq. (7) is of the form

$$B_S = [1 - S(S+1)/2L(L+1)]$$

$$\times \sqrt{(2S+1)(2L+1)(2j_1+1)}$$

$$\times W(j_2 L j_1 S;\, j_1 L) C_{L0S0}^{L0}.$$

For the case of mixed magnetic multipole radiation of character 2^L and electric radiation of character 2^{L+1} we must use instead of B_S the expression (13) from the paper of Dolginov and Toptygin.[10]

APPENDIX II

Our notation for the nuclear matrix elements is:

$$\int O_i \varphi_\mu \equiv \int \psi_{j_1\mu_1}^* e^{-\alpha Zmr} O_i \tau_- \psi_{j_0\mu_0}\, d\mathbf{r}.$$

When the exponential is replaced by unity $(\varphi_\mu \to 1)$ these are the same as the nuclear matrix elements for β decay in the notation of Konopinski and Uhlenbeck.

We give the values of O_i for the various nuclear matrix elements:

$\int 1 \varphi_\mu$:	$1 C_0$
$\int \sigma \varphi_\mu$:	$C_1 \sigma \mathbf{Y}_{1\Lambda}^{-1}$
$\int \mathbf{rp}\, \varphi_\mu$:	$C_0 r \mathbf{p} \mathbf{Y}_{00}^1$
$\int [\mathbf{rp}]\, \varphi_\mu$:	$\sqrt{2/3}\, i C_1 r \mathbf{p} \mathbf{Y}_{1\Lambda}^0$
$\int \mathbf{r}(\sigma \mathbf{p})\, \varphi_\mu$:	$\sqrt{1/3}\, C_1 r Y_{1\Lambda} \sigma \mathbf{p}$
$\int \mathbf{r}(\sigma \mathbf{r})\, \varphi_\mu - \frac{1}{3}\int r^2 \sigma \varphi_\mu$:	$\frac{1}{3}\sqrt{2} C_1 r^2 \sigma \mathbf{Y}_{1\Lambda}^1$

Here $C_I = [C_{I\Lambda_0 j_0 \mu_0}^{j_1 \mu_1}]^{-1}$.

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268