ON THE SHAPE OF THE MAGNETIZATION CURVE OF SUPERCONDUCTING ALLOYS NEAR THE SECOND CRITICAL FIELD

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The conclusions of Abrikosov's theory [1] on the shape of the magnetization curves of superconducting alloys near the second critical field are compared with experiment. The results of experiments published at different times are first treated in an appropriate manner. The comparison shows satisfactory agreement between the theory and experiment.

In recent years considerable attention has been paid to superconducting alloys with high critical magnetic fields. In connection with this the work of Abrikosov [1] is of interest. Starting from the Ginzburg-Landau theory, [2] Abrikosov considered the behavior in a magnetic field of a superconductor with $\kappa > 1/\sqrt{2}$, where κ is the characteristic parameter of the Ginzburg-Landau theory. It was found that such superconductors (superconductors of the second group) have two critical magnetic fields: H_{c1} and H_{c2}. In an external field $H < H_{c1}$ we have the superconducting state and the magnetic field in the superconductor is zero. When $H_{c1} < H < H_{c2}$ there is a mixed state, and the magnetic induction (the average intensity of the magnetic field in the superconductor) is $B \equiv \overline{H}_i < H$. Finally, when $H > H_{c2}$ (H_{c2} is the field in which the electrical resistance reappears) the superconductor is in the normal state.

This characteristic behavior of superconductors of the second group in a magnetic field is related to the fact that when $\kappa > 1/\sqrt{2}$ the surface energy of the boundary between the normal and superconducting phases becomes negative. According to Abrikosov the dependence B(H) is linear when H approaches H_{c2} from below and the magnetic moment per unit volume $M = (B - H)/4\pi$ is given by the formula

$$-4\pi M/(H_{c2} - H) = 1/1.18\,(2\kappa^2 - 1). \quad (1)$$

The purpose of the present work was to compare Abrikosov's theory with experiment by a suitable treatment of the published experimental data on the magnetization curves of superconducting alloys.

In all the papers examined the magnetic induction measurements were carried out on long samples placed in longitudinal magnetic fields so that the demagnetization factor could be neglected. According to [1,2] $H_{c2} = \kappa\sqrt{2}\,H_c$, where H_c is the thermodynamic critical field:

$$^1/_2 H_c^2 = \int_0^{H_{c2}} (H - B)\, dH.$$

We obtain H_c by graphical integration of the experimental B(H) curve; having found H_{c2} experimentally we determine κ. Substituting this value of κ into the right-hand part of Eq. (1) we obtain a quantity which, as in [1], will be denoted by $\tan\alpha_{theor} = 1/1.18(2\kappa^2 - 1)$. The left-hand part of Eq. (1), which can be written as $(H - B)/(H_{c2} - H)$, gives the experimental value of $\tan\alpha$, which is found directly from the magnetization curve.

A comparison of $\tan\alpha_{theor}$ and $\tan\alpha_{exp}$ is the purpose of the present work (in [1] this comparison was carried out only for the data in [3], good agreement being obtained between the theory and experiment).

Stout and Guttman [4] investigated the magnetization curves of superconducting Tl–In alloys. They used both single-crystal and polycrystalline samples. The magnetization curves were recorded as follows. A sample cooled below the critical temperature was placed in a magnetic field $H > H_{c2}$. The magnetic field was then removed and some trapped magnetic flux was retained in the sample. This flux produced a residual induction B_0 the value of which was found by Stout and Guttman for each sample and each temperature. The ballistic galvanometer method was then used to record the dependence $B_{meas}(H)$ on gradual increase of the external magnetic field.

The dependence $B_{meas}(H)$ obtained in [4] cannot be used directly for comparison with Abrikosov's theory since the trapped magnetic flux is

Alloy	T_c, °K	T, °K	H_c, Oe	H_{c2}, Oe	$\varkappa$	$\tan\alpha_{exp}$	$\tan\alpha_{theor}$	Reference
95% Pb, 5% Tl	7	2.52	798	1520	1.39	0.32	0.32	[3]
		2.96	756	1424	1.35	0.31	0.33	[3]
		3.46	709	1300	1.30	0.31	0.36	[3]
15% Tl, 85% In, single crystal	3.25	1.293	257	293	0.807	2.46	2.83	[4]
		2.946	51	55.1	0.753	4.3	6.4	[4]
20% Tl, 80% In, single crystal	3.23	1.286	272	421	1.09	0.56	0.60	[4]
		2.591	106	140	0.93	1.13	1.16	[4]
30% Tl, 70% In, polycrystal	3.30	1.400	283	480	1.20	0.46	0.45	[4]
		2.084	211	348	1.17	0.44	0.49	[4]
38% Tl, 62% In, polycrystal	2.94	1.399	264	470	1.26	0.42	0.39	[4]
		2.083	170	290	1.21	0.46	0.44	[4]
36% Ta, 64% Nb, single crystal	6.9	4.2	714	1900	1.88	0.16	0.14	[5]
		6.0	416	1680	2.75	0.062	0.055	[5]
Nb_3Sn	18	4.2	14800	72000	3.59	0.030	0.034	[6]

still involved in the results. Let the total cross section of the sample be S and the trapped magnetic flux pass a cross section S_0. The magnetic induction in the region with trapped magnetic flux is not known. Inspection of the experimental magnetization curves of nonuniform superconducting alloys indicates that hysteresis begins at fields greater than H_{c1} and smaller than H_{c2}. At moderate values of κ (in the case considered $\kappa \le 1.26$) when H_{c2} is not much greater than H_{c1}, we may assume without committing a large error that the magnetic induction is equal to H_{c2} in the region with trapped magnetic flux. Then we have the equality

$$H_{c2}S_0 = B_0 S.$$

The measured magnetic induction B_{meas} given in [4] is then equal to $B_{meas} = B(S - S_0)/S + B_0$, where B is the magnetic induction in the region of the superconductor which is free from the trapped magnetic flux. It is this region to which all the conclusions of Abrikosov apply. Therefore all the curves in [4] have been recalculated using the formula

$$B = (B_{meas} - B_0)/(1 - B_0/H_{c2}),$$

and the dependence B(H) has been compared with the theory. The results are given in the table.

Calverley and Rose-Innes [5] reported the magnetization curves of the alloy $Ta_{36}Nb_{64}$. Using special heat treatment these authors were able to make the alloy highly uniform and consequently there was a complete absence of trapped magnetic flux. The magnetization curves of this alloy are those of a typical superconductor of the second group. The experimental results in [5] are given in the form of $\alpha(H)$ curves, where α is the deflection of a ballistic galvanometer. For $H < H_{c2}$ this deflection is $\alpha = k(BS + HS_0)$, where k is the coefficient of proportionality, S is the cross section of the sample, and S_0 is the area represented by the gap between the ballistic coil and the sample. For $H > H_{c2}$ the deflection is $\alpha = kH(S + S_0)$ and when $H < H_{c1}$ the induction is $B = 0$ and $\alpha = kS_0H$.

From the curves given in [5] we can find kS_0 and kS. The magnetic induction in the sample is $B = (\alpha - kS_0H)/kS$. The dependence of this quantity on H is compared with the theory of Abrikosov [1]. The results are given in the table.

The superconducting alloy Nb_3Sn is currently of great interest: it has the high critical field $H_c = 14{,}800$ Oe. The dependence of the magnetic moment M on the intensity of the external magnetic field, given in [6], allows us to check the applicability of Abrikosov's theory to this alloy. The results of this check are also given in the table.

It is appropriate to mention here that the Ginzburg-Landau theory, and consequently the results of Abrikosov, are obviously valid in a field $H \approx H_{c2}$ for temperatures considerably smaller than T_c. This is related to the fact that at the second critical field the transition from the normal to the mixed state is a phase transition of the second kind. This is also shown clearly in the table, where there is satisfactory agreement between the theory and experiment at temperatures considerably smaller than T_c; the table also shows the temperature dependence of κ.

Analysis of the published data was very difficult because small-scale graphs had to be used. This applies particularly to [6]. Therefore the agreement between the theory and experiment within the limits of 10–20% should be regarded as satisfactory. Obviously it would be desirable to carry out a special study, the results of which would allow us to judge accurately the validity of Abrikosov's conclusions.

From the results of the present work it is evident that there is good agreement with the theory for all the magnetization curves of superconducting alloys obtained on different occasions and in different places and by different authors. In connection with this it should be noted that the treatment of Goodman[7], which is close to the ideas of Abrikosov[1], does not give such agreement with experiment. This is admitted by Goodman himself.[8]

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