Letters to the Editor

QUASI-OPTICAL MODEL AND THE ASYMPTOTIC BEHAVIOR OF THE SCATTERING AMPLITUDE

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It has been shown[1] that the system of two particles in quantum field theory can be described by an equation of the Schrödinger type with a generalized complex potential. Such a quasi-optical description permits, on the one hand, the finding of the scattering matrix and, on the other, the study of the structure of bound and resonant states (virtual processes). If we pose the problem of describing only the relativistic scattering matrix, then the generalized potential will depend only on the energy and the momentum transfer of the system and the equation for the wave function will have the form

$$(E^2 - q^2 - m^2)\,\psi_\pm(q) = \frac{1}{\sqrt{q^2+m^2}}\int V^\pm(q, q'; E)\,\psi_\pm(q')\,d^3q', \tag{1}$$

where q and q′ are the three-dimensional momenta, V^+ is the potential for states that are even in $\cos\theta$ (θ is the scattering angle), and V^- is the corresponding potential for odd states.

For simplicity we consider here the case of scalar particles. For the potential $V^\pm(q, q', E)$ we can take the following spectral representation:

$$V^\pm(q, q', E) = \frac{1}{\pi}\int_{\mu^2}^{\infty}\frac{U^\pm(E,\nu)}{\nu+(q-q')^2}\,d\nu, \tag{2}$$

where $U(E, \nu)$ is a spectral function depending on the energy of the system and is complex in the region $E^2 > m_1^2$.

If the amplitude of the process $M(E, t)$ satisfies the dispersion relation in the time variable t for any fixed values of the energy E, then the projection of the amplitude $M^\pm$ on even and odd states will be represented in the form

$$M^\pm(E, t) = \int_{\mu^2}^{\infty}\frac{\sigma^\pm(E,\nu)}{\nu+(q-q')^2}\,d\nu. \tag{3}$$

With the aid of the method suggested in [1], we can thus construct a potential in the form (2). In [2] it was shown that for a description of bound states of the system we also can construct from field theory the potential of the form (2) without the hypothetical dispersion relations. In expression (3) we did not take into account the subtraction polynomial, but, depending on its degree n, our potential will correctly describe only waves with $l > n$, and, consequently, our conclusions on the asymptotic behavior hold if $n < 1$.

The imaginary part V characterizes the inelastic scattering processes. Regge showed that when the potential is a superposition of Yukawa potentials, the asymptotic behavior of the scattering amplitude, as $t \to \infty$, has the form

$$M(E, t) = g(E)\,t^{\alpha(E)}, \qquad t = -(q-q')^2, \tag{4}$$

where q and q′ are the initial and final momenta of the particle.

For investigation of the asymptotic behavior in the s channel, Eq. (1) must be written in the t channel: here t is connected with the energy through the relation $t = 4E^2$. Then the variable s will be the momentum transfer ($s \le 0$).

In this note we would like to draw attention to the fact that the potential of type (2) leads to the Regge asymptotic behavior (4). This can be shown if we use the method developed by Fubini and Straffolini for ordinary potential scattering.[3] We write the equation for the amplitude $T^\pm$:

$$T^\pm(q, q') = V^\pm((q-q')^2, E) + \int\frac{V^\pm((q-p)^2, E)\,T^\pm(p, q')}{[(E+i\varepsilon)^2 - m^2 - p^2]\sqrt{p^2+m^2}}\,d^3p. \tag{5}$$

(On the mass surface the transition amplitude is the same as in the ordinary scattering matrix.) We seek a solution of Eq. (5) in the form

$$T^\pm(q, q') = \frac{1}{\pi}\int_0^\infty\frac{\tau^\pm(q'^2, q^2, \nu)}{\nu - s}\,d\nu. \tag{6}$$

Substituting (2) and (6) into (5), we obtain an equation for the spectral function τ:

$$\tau^\pm(q'^2, q^2, \nu, E) = U^\pm(E, \nu) + \iint\frac{Q^\pm(q'^2, \nu, u', t', q^2, E)}{(E^2 - m^2 - u')\sqrt{u'+m^2}}\,\tau^\pm(q'^2, u', t', E)\,du'dt';$$

$$Q^\pm(q'^2, \nu, u', t', q^2, E) = \frac{1}{2}\int K(q'^2, \nu, u', t', \nu_1, q^2)\,U^\pm(E, \nu_1)\,d\nu_1. \tag{7}$$

Here K is a well-known kernel (see, for example, [4]).

In the asymptotic region $s \to \infty$, Eq. (7) takes the form

$$\tau_{as}^{\pm}(q'^2, q^2, \nu, E) = \frac{1}{s}\int_0^s d\nu \int_0^\infty Q_{as}^{\pm}\left(\frac{\nu}{s}, q'^2, u', E\right) \frac{\tau_{as}^{\pm}(q'^2, u', \nu, E)}{(E^2 - m^2 - u')\sqrt{u' + m^2}} du',$$

$$Q_{as}^{\pm}(x, u, u', E) = \int \frac{\theta(u' - ux - \nu_1 x/(1-x))\, U^{\pm}(E, \nu_1)}{(1-x)^{1/2}(u' - ux - \nu_1 x/(1-x))^{1/2}} d\nu_1. \quad (8)$$

This equation has a solution of the form

$$\tau^{\pm}(q'^2, q^2, \nu, E) = \tau_\alpha^{\pm}(q'^2, q^2, E)\, \nu^{\alpha(E)}. \quad (9)$$

Here the function τ_α satisfies the equation

$$\tau_\alpha^{\pm}(u, s, E) = \int R_\alpha^{\pm}(u, u', s, E) \frac{\tau_\alpha^{\pm}(u', s, E)}{(E^2 - m^2 - u')\sqrt{u' + m^2}} du'.$$

$$R_\alpha^{\pm}(u, u', s, E) = \int U^{\pm}(E, \nu)\, d\nu \int_0^1 \frac{dx \cdot x^\alpha}{(1-x)^{1/2}} \frac{\theta(u' - ux - \nu x/(1-x))}{[u' - ux - \nu x/(1-x)]^{1/2}}. \quad (10)$$

From Eq. (10) we can determine the eigenfunction τ_α and the eigenvalue α, which is a function of E. For $E^2 < m_1^2$ the function $U(E, \nu)$ is real and, consequently, α is real.

Inserting (9) into (6) we obtain for large values of s

$$T(q'^2, q^2, s, E) = s^{\alpha(E)} \tau_\alpha(q'^2, q^2, E) \frac{[1 + e^{-i\pi\alpha(E)}]}{\sin \pi\alpha(E)}. \quad (11)$$

We can also obtain similar results directly from Eq. (1) by going over to partial waves.[3]

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[1] A. A. Logunov and A. N. Tavkhelidze, Preprint E-1145, Joint Institute of Nuclear Research, 1962.

[2] Logunov, Tavchelidze, Todorov, and Chrustalev, Nuovo cimento (1963), in press.

[3] S. Fubini and R. Straffolini, Lectures given by S. Fubini, 1962, Trieste.

[4] R. Omnes, Nuovo cimento 25, 806 (1962).

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NEUTRON-NEUTRON TOTAL CROSS SECTION AT 8.3 GeV

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THE neutron-neutron total cross section was measured at the proton synchrotron of the Joint Institute of Nuclear Research from the attenuation of a neutral beam under conditions of good geometry ($\theta/2 = 0.228°$).

As a neutron detector we used a telescope consisting of scintillation counters and a Cerenkov counter with complete absorption in lead glass. The detector recorded only those neutrons which, in interactions in an aluminum converter 10 cm thick, produced secondary particles (mainly neutral and charged pions) whose energy release in the Cerenkov-counter radiator was somewhat greater than the threshold energy. The energy thresholds of the neutron counter were calibrated from measurements of the energy of accelerated protons in the accelerator and with an electron beam. A system of fast discriminators with a resolving time of 1.0 μsec was used for the pulse-height analysis of the Cerenkov-counter output. A second identical system of discriminators permitted simultaneous counts of random coincidences. As a monitor we used a telescope consisting of three scintillation counters.[1] The neutron-neutron cross section was measured by the difference method with 50.01- and 55.60-g/cm^2 targets of H_2O and D_2O.

To decrease the effect of fluctuations in the measuring equipment and in the accelerator during the measurements, the ordinary and heavy water targets were exposed alternately for approximately 10—12 cycles of accelerator operation.

The experimentally obtained value of the n-n total cross section at an effective energy of about 8.3 GeV is

$$\sigma_{nn} = 31.5 \pm 1.7 \text{ mb}$$

The error is statistical.

Glauber[2] showed that the cross section for the interaction of high-energy particles with deuterons should be less than the sum of cross sections for free neutrons and protons. In order to obtain the true value for the total n-n cross section, it is necessary to take into account the effect of the