CONSERVATION OF THE VECTOR CURRENT IN WEAK INTERACTIONS AND THE MASS DIFFERENCE IN ISOTOPIC MULTIPLETS. THE $\pi^+ \to \pi^0 + e^+ + \nu$ DECAY

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The validity of the vector current conservation hypothesis in weak interactions is limited similarly to that of the isotopic invariance and hence, generally speaking, one should expect that in the decays of strongly interacting particles the vector coupling constant (which is linear in the particle-mass difference in isotopic multiplets) should be renormalized. In the first part of the paper a proof is given of the theorem (which, in strong interactions, is valid in all orders) that there is no such renormalization for the neutron and Σ hyperon β decay, and for the $\pi^\pm \to \pi^0 + e^\pm + \nu$ decay. The second part of the paper is devoted to a calculation of the probability of the $\pi^+ \to \pi^0 + e^+ + \nu$ decay with radiative corrections taken into account. The use of the above theorem narrows greatly the limits of the theoretically predicted value of the probability.

1. VECTOR CURRENT CONSERVATION AND THE MASS DIFFERENCE

1. The vector current conservation hypothesis [1,2] which was evoked to explain the lack of renormalization of the vector coupling constant in strong interactions has a strictly defined sense only when the mass difference in isotopic multiplets is neglected. The quantity $\delta j_\mu^{(i)}/\delta x_\mu$, where

$$j_\mu^{(i)} = \frac{1}{2}\overline{\Psi}\tau_i\gamma_\mu\Psi - \frac{i}{2}\left(\frac{\partial\varphi^*}{\partial x_\mu}T_i\varphi - \varphi^* T_i\frac{\partial\varphi}{\partial x_\mu}\right) + \text{terms corresponding to other strongly interacting particles and leptons} \quad (1)$$

(here Ψ and φ are the Heisenberg operators of the nucleon and π-meson fields) does not vanish if we take the terms linear in the mass difference into account.

We shall assume that in this case, too, the weak interaction can be written as $Gj_\mu^+ j^\mu$; we would then in general expect a linear (in the mass differences) renormalization of the constant G in the decays of strongly interacting particles. It will, however, be shown in what follows that at least in the β decay of the neutron and of the Σ hyperon, and also in the $\pi^\pm \to \pi^0 + e^\pm + \nu$ decay, the violation of the isotopic invariance and the related renormalization of G are effects quadratic in the mass difference.

2. We assume that the total Hamiltonian H of the field system can be written as the sum

$$H = H_1 + H_w + V + H_{el}, \quad (2)$$

where H_1 is the isotopically invariant part which includes the free Hamiltonian and all strong interactions, H_W is the weak interaction, V is the term containing the operators of the mass shift in isotopic multiplets (which, acting on the functions of physical states, produce the experimentally observed mass differences).

Two points of view are possible concerning the nature of this term. The accepted one is that the mass difference is due to electromagnetic interactions. From this point of view V represents the mass effect of electromagnetic interactions, in the form of a counterterm which permits the mass renormalization. However, by taking V into account, we do not in fact take into account all effects of electromagnetic interactions.

The remaining term H'_{el} describes the interaction of particles with a physical mass, and leads only to vertex effects and to the renormalization of wave functions. We can assume that these effects will be correctly taken into account by calculating the radiative corrections, and in the first part of the paper we do not consider them at all.

Since such an approach is not fully justified, we propose in the following another interpretation. Since the radiative corrections are very small (~ 1–2%), then the degree to which the isotopic invariance is violated in the decay of strongly interacting particles can be determined from the mass differences in isotopic multiplets (which are very large in mesic and hyperonic multiplets). Disregarding the problem of the origin of the mass difference we shall take it into account phenomenologically [(term V in Eq. (1)]. Our thesis is that there is no linear effect in V.

We take the interaction in the form

$$V = \tfrac{1}{2}\delta m_{n,p}\int \overline{\Psi}\,(\tau_\beta + c_N)\,\Psi d^3x + \delta\mu^2_{\pi^0,\pi^\pm}\int \varphi^*\,(T_3^2 + c_\pi)\,\varphi d^3x \text{ terms corresponding to mass differences of other isotopic multiplets} \quad (3)$$

where $\delta m_{n,p} = m_n - m_p$, $\delta\mu^2_{\pi^0,\pi^\pm} = \mu_+^2 - \mu_0^2$, c_N and c_π are arbitrary constants related to the indeterminate values of the unperturbed multiplet masses (these constants determine the center-of-mass shift of the multiplets). The crucial assumption is that the perturbation theory can be applied to the interaction V.

3. Let us consider the $\pi^+ \to \pi^0 + e^+ + \nu$ decay. The matrix element of this process is

$$M = (2\pi i)\,\delta\,(E^+ - E^0 - q_0)\,G l^\mu \int \langle\pi^0|j_\mu^{(-)}(\mathbf{x}, 0)|\pi^+\rangle e^{-i\mathbf{q}\mathbf{x}} d^3x, \quad (4)$$

$$\int\langle\pi^0|j_\mu^{(-)}(\mathbf{x}, 0)|\pi^+\rangle e^{-i\mathbf{q}\mathbf{x}}\, d^3x = \frac{\delta_{\mathbf{p}^+,\,\mathbf{p}^0+\mathbf{q}}}{\sqrt{4E^+E^0}}\,[f\,(q^2)\,(p^+ + p^0)_\mu + h\,(q^2)\,q_\mu]. \quad (5)$$

where $p^+ = (E^+, \mathbf{p}^+)$ and $p^0 = (E^0, \mathbf{p}^0)$ are the momenta of the π^+ and π^0 mesons, $q = (q_0, \mathbf{q})$ is the momentum transferred to leptons, $l_\mu = \bar{u}_\nu\gamma_\mu(1 + \gamma_5)v_e$ is the leptonic part of the current, and $j_\mu^{(-)} = 2^{-1/2}(j_\mu^{(1)} - ij_\mu^{(2)})$.

If the current $j_\mu^{(i)}$ is conserved, then $f(0) = 1$ and $h(q^2) \equiv 0$. In general $h(q^2) \sim \delta m$, where δm is the generalized symbol for the mass differences in Eq. (3).[1]

The time-like component of Eq. (5) for q = 0, at the point where $\delta_{\mathbf{p}^+,\mathbf{p}^0+\mathbf{q}} = 1$, in the rest-mass system of the π^+ meson is:

$$\int\langle\pi^0|j_0^{(-)}(\mathbf{x}, 0)|\pi^+\rangle\, d^3x = \frac{\mu_+ + \mu_0}{\sqrt{4\mu_+\mu_0}}\left[f\,(0) + h\,(0)\frac{\mu_+ - \mu_0}{\mu_+ + \mu_0}\right] = f\,(0). \quad (6)$$

Eq. (6) is correct to terms quadratic in the mass differences.

[1]It should be noted that the term with $h(q^2)$ in Eq. (5) is not important in the expression for the decay probability since in addition to being small ($\approx \delta m$) as compared to the first term in Eq. (5), it contains the additional parameter m_e/μ_+ (where m_e is the electron mass) which is due to the action of q_μ on the leptonic bracket.

Let us introduce the notation

$$|\pi^+\rangle = \Psi^{(+)}(0) \equiv \Psi^{(+)},$$

$$|\pi^0\rangle = \Psi^{(0)}(0) \equiv \Psi^{(0)}, \qquad j_\mu^{(i)}(\mathbf{x}, 0) = J_\mu^{(i)}(\mathbf{x}),$$

where $J_\mu^{(1)}(\mathbf{x})$ is the current operator in the Schroedinger representation, and $\Psi^{(+)}(t)$ and $\Psi^{(0)}(t)$ are the stationary single-meson states of the Hamiltonian H_1. It should be noted that

$$j_\mu^{(i)}(\mathbf{x}, t) = e^{i(H_1+V)t}\,J_\mu^{(i)}(\mathbf{x})\,e^{-i(H_1+V)t}.$$

The quantity $e^{iH_1t}J_\mu^{(1)}(\mathbf{x})\,e^{-iH_1t}$ which represents the current (in the V-interaction representation) is at the same time a Heisenberg operator with respect to the isotopically invariant interactions included in H_1. We have therefore

$$\frac{\partial}{\partial x_\mu}\,[e^{iH_1t}J_\mu^{(i)}(\mathbf{x})\,e^{-iH_1t}] = 0.$$

In terms of the current $J_\mu^{(1)}(\mathbf{x})$, this relation becomes

$$\operatorname{div}\mathbf{J}^{(i)}(\mathbf{x}) = i\,[H_1, J_0^{(i)}(\mathbf{x})]. \quad (7)$$

Integrating over the whole space we obtain the basic relation

$$[H_1, t^{(-)}] = 0, \quad (8)$$

where $t^{(-)} = \int J_0^{(-)}(\mathbf{x})\,d^3x$. Finally we obtain:

$$f\,(0) = (\Psi^{(0)}, t^{(-)}\Psi^{(+)}), \quad (9)$$

where the whole dependence on the mass differences (or the interaction V) is contained in $\Psi^{(0)}$ and $\Psi^{(+)}$. It is evident that

$$(H_1 + V)\,\Psi^{(+);\,(0)} = \mu_{+;\,0}\Psi^{(+);(0)}. \quad (10)$$

Let us consider the complete set of states Φ_n of the unperturbed Hamiltonian H_1. We have then the general formula of the perturbation theory for stationary states:

$$\Psi^{(+)} = \Phi^{(1)} + \sum_n{}'\frac{(\Phi_n, V\Phi^{(1)})}{E_n - \mu}\,\Phi_n, \quad (11)$$

$$\Psi^{(0)} = \Phi^{(0)} + \sum_n{}'\frac{(\Phi_n, V\Phi^{(0)})}{E_n - \mu}\,\Phi_n, \quad (11')$$

where $\Phi^{(1)}$ and $\Phi^{(0)}$ are the π^+ and π^0 meson states (for $\mathbf{p}^{+;0} = 0$) of the Hamiltonian H_1. (The indices $^{(1)}$ and $^{(0)}$ characterize the z component of the isotopic spin.) The summation $\sum_n{}'$ in Eq. (11) is carried out over the complete set Φ_n excluding the π^+-meson state $\Phi^{(1)}$. Correspondingly, the summation in Eq. (11′) does not include the π^0-meson state $\Phi^{(0)}$.

Since the interaction V is diagonal in the charge states, then the π^0 and π^- mesons are also absent in Eq. (11); accordingly, there is no π^- or π^+ among the n states in Eq. (11′). Consequently, single-meson states are absent in Eqs. (11) and (11′) among the n states, and the denominators in these formulas never vanish. (We assume that in the spectrum of the operator H_1 there are no other states with $p_n^2 = \mu^2$ apart of the single-meson states.)

Furthermore, using Eq. (9), we obtain

$$f(0) = (\Phi^{(0)}, t^{(-)}\Phi^{(1)}) + \sum_n \frac{1}{E_n - \mu}\{(\Phi_n, V\Phi^{(1)})(\Phi^{(0)}, t^{(-)}\Phi_n) + (\Phi_n, V\Phi^{(0)})^*(\Phi_n, t^{(-)}\Phi^{(1)})\}. \quad (12)$$

Since $t^{(-)}$ is an integral of motion [cf. Eq. (8)], and since it obviously commutes with the total momentum of the field, it is necessary that $p_n^2 = \mu^2$. We found, however, that there are no such states among the n states in Eq. (12) and, therefore, there is no linear effect in V, i.e.,

$$f(0) = (\Phi^{(0)}, t^{(-)}\Phi^{(1)}). \quad (13)$$

It can be easily seen that $t^{(i)}$ are infinitesimal rotation operators in the isotopic spin space. Since $t^{(-)} = 2^{-1/2}(t^{(1)} - it^{(2)})$, then the matrix element $(\Phi^{(0)}, t^{(-)}\Phi^{(1)})$ in Eq. (13) is equal to unity [2)], thus proving the theorem that the constant G is not renormalizable in the approximation linear in the mass differences.

4. The proof of the theorem is carried out analogously for the β decay ($n \to p + e + \nu$) and for the β decay of Σ hyperons. Thus, e.g., the β decay vertex operator Γ_μ should be written in the form

$$\Gamma_\mu = a(q^2)\gamma_\mu + b(q^2)q_\mu + c(q^2)\sigma_{\mu\nu}q^\nu. \quad (14)$$

The contribution of the form factors b and c is negligibly small, and for the quantity a(0) we find easily a relation of the type (6):

$$a(0) = \sqrt{2}\int \langle p | j_0^{(+)}(\mathbf{x}, 0) | n \rangle d^3x,$$

$$j^{(+)} = 2^{-1/2}(j^{(1)} + ij^{(2)}). \quad (15)$$

The consequent calculation is carried out in Sec. 3. As a result, we obtain an equation analogous to Eq. (13)

$$a(0) = \sqrt{2}(\Phi^{(1/2)}, t^{(+)}\Phi^{(-1/2)}), \quad (16)$$

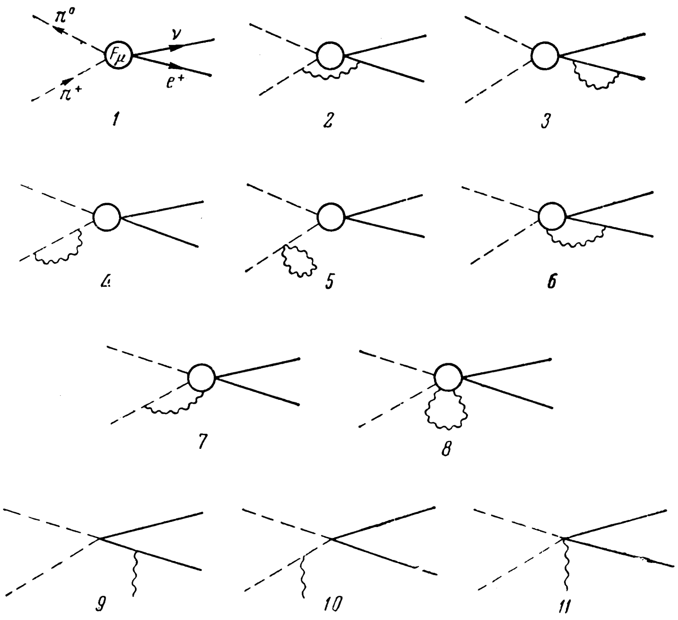
where $\Phi^{(-1/2)}$ and $\Phi^{(1/2)}$ are the neutron and proton states of the Hamiltonian H_1. Repeating the arguments following Eq. (13), we find again that in the case under consideration a(0) = 1.

The absence in the β decay of effects linear in mass differences was earlier established by Behrends and Sirlin [3], but their proof is less general and is not applicable to the $\pi^+ \to e^+ + \nu + \pi^0$ decay and to hyperon decays.

2. THE $\pi^+ \to \pi^0 + e^+ + \nu$ DECAY

1. The $\pi^+ \to \pi^0 + e^+ + \nu$ decay has recently been found experimentally (cf. [4-6]). The decay was first predicted by Zel'dovich [7] in 1954. The study of this process enables us in principle to test the current vector conservation hypothesis in weak interactions, [1,2] since it follows from this hypothesis that the interaction constant of the process is equal to the constant of the vector part of the β decay interaction. It is interesting to find out how accurate is the equality of the constants, taking into consideration the approximate character of the vector current conservation.

2. Let us estimate the radiative corrections to the $\pi^+ \to \pi^0 + e^+ + \nu$ decay. In addition to the main process (see diagram 1 in the figure) we have a set of processes involving the emission of a virtual photon (diagram 2—8) and also the decay with the emission of a bremsstrahlung photon (diagram 9—11). In diagrams 9—11 we neglect the π-meson form factor. Let us first consider the case where the weak interaction form factor is independent of q^2 (the square of the difference of the meson four-momenta) and of the difference in meson mass. We are then left with diagrams 1 to 7.



[2)] We use the well-known expression for the matrix elements of rotation generators $t^{(i)}$ in canonical basis:

$$\langle m \pm 1 | t^{(1)} \pm it^{(2)} | m \rangle = \sqrt{t(t+1) - m(m \pm 1)}.$$

The resulting set of diagrams is generated in the lowest order by the following interaction Lagrangian

$$L_{int} = -e\overline{\Psi}_e \hat{A} \Psi_e + ie\left(\frac{\partial\varphi^*}{\partial x_\mu}\varphi - \varphi^*\frac{\partial\varphi}{\partial x_\mu}\right)A_\mu + e^2 A_\mu A^\mu \varphi^*\varphi$$

$$+\left[iG\left(\varphi^*\frac{\partial\varphi_0}{\partial x_\mu} - \frac{\partial\varphi^*}{\partial x_\mu}\varphi_0\right)l_\mu - eG\varphi^*\varphi_0 l_\mu A^\mu + \text{comp. conj.}\right]. \quad (17)$$

where $l_\mu = \overline{\Psi}_\nu \gamma_\mu (1+\gamma_5)\Psi_e$, $G = (1.01 \pm 0.01)\,10^{-5}/M^2$, and M is the nucleon mass. The contribution of such diagrams was calculated in [8], in which, however, the authors committed a number of errors. The diagrams were used with incorrect relative signs, and were therefore on the whole not invariant with respect to gauge transformation. An error was also made in calculating the contribution of diagram 4 and of the bremsstrahlung photon (diagrams 9—11).

We shall calculate the positron spectrum in the case where the decay with the bremsstrahlung photon emission is not singled out experimentally:

$$W(\theta)\,d\theta = G^2 m_e^5 \pi^{-3}(1 + 2m_e\mu_+^{-1}\,\mathrm{ch}\,\theta)(1+\Phi_1)$$
$$\times(\mathrm{ch}\,\theta_0 - \mathrm{ch}\,\theta)^2\,\mathrm{sh}^2\theta\,\mathrm{ch}\,\theta\,d\theta;$$

$$\Phi_1 = -\alpha\,[\pi\,\mathrm{sh}\,\theta\,\mathrm{ch}\,\theta]^{-1}\,[\theta + \theta\,\mathrm{ch}^2\theta + 2\theta^2\mathrm{ch}^2\theta - 3\,\mathrm{sh}\,\theta\,\mathrm{ch}\,\theta$$
$$-\tfrac{3}{2}\,\mathrm{sh}\,\theta\,\mathrm{ch}\,\theta\,\ln(\Lambda/m_e) + 2\,\mathrm{ch}\,\theta\,(\mathrm{sh}\,\theta - \theta\,\mathrm{ch}\,\theta)$$
$$\times\ln[2(\mathrm{ch}\,\theta_0 - \mathrm{ch}\,\theta)]\,2\,\mathrm{ch}^2\theta f(1-e^{-2\theta}) + \tfrac{1}{3}(\mathrm{ch}\,\theta_0 - \mathrm{ch}\,\theta)$$
$$\times(2\,\mathrm{sh}\,\theta - \tfrac{7}{4}\theta\,\mathrm{ch}\,\theta - \tfrac{1}{4}\theta\,\mathrm{ch}\,\theta_0)]. \quad (18)^*$$

In this expression m_e and μ_+ are the positron and π^+ meson mass, respectively, and $\cosh\theta = \epsilon/m_e$, where ϵ is the positron energy. θ varies within the limits $0 \le \theta \le \theta_0$, and θ_0 is given by the relation

$$\mathrm{ch}\,\theta_0 = \varepsilon_{max}/m_e = \Delta/m_e(1 - \Delta/2\mu_+)$$

where Δ is the mass difference of the π^+ and π^0 mesons, Λ is the ultraviolet cutoff, and $f(x) = -\int_0^x [\ln(1-t)\,dt]/t$ is the Spence function. The function Φ_1 is due to the radiative corrections and is very similar to the expressions for radiative corrections to the electron spectrum in β decay [Kinoshita and Sirlin, [9] Eq. (4.2) [3)]].

[3)]For a comparison, we can first simplify Eq. (4.2) in [9] taking into consideration that the complete set of Spence functions in Eq. (4.2) can be in fact reduced to one function:

$$f(\beta) - f(-\beta) + f\left(\frac{2\beta}{1+\beta}\right) + \frac{1}{2}f\left(\frac{1-\beta}{2}\right) - \frac{1}{2}f\left(\frac{1+\beta}{2}\right) = 2f\left(\frac{2\beta}{1+\beta}\right)$$
$$-\ln\frac{2}{1+\beta}\,\mathrm{arctanh}\,\beta = 2f(1-e^{-2\theta}) - \theta\ln(2e^{-\theta}\,\mathrm{sh}\,\theta)$$

and the term with $\ln(2e^{-\theta}\sinh\theta)$ cancels the last term in Eq. 4.2 in [9].

*ch = cosh; sh = sinh.

In the case where the experimental resolution of the photon energy ω_0 is small ($\omega_0 \ll m_e$), the function Φ_1 in Eq. (18) should be exchanged for Φ_2:

$$\Phi_2 = -\alpha\,[\pi\,\mathrm{sh}\,\theta\,\mathrm{ch}\,\theta]^{-1}$$
$$\times[\theta - 2\theta\mathrm{ch}^2\theta + 2\theta^2\mathrm{ch}^2\theta + 2\,\mathrm{ch}\,\theta\,(\mathrm{sh}\,\theta - \theta\,\mathrm{ch}\,\theta)$$
$$\times\ln(2\omega_0/m_e)\,2\,\mathrm{ch}^2\theta f(1-e^{-2\theta}) - \tfrac{3}{2}\,\mathrm{sh}\,\theta\,\mathrm{ch}\,\theta\,\ln\left(\frac{\Lambda}{m_e}\right)\Big]. \quad (19)$$

The integration in Eq. (18) leads to the following expression for the decay probability:

$$w = w_0(1 - \tfrac{3}{2}\Delta/\mu_+ - 5m_e^2/\Delta^2 + \delta_1), \quad (20)$$

where $w_0 = G^2\Delta^5/30\pi^3$ and $\delta_1 = (3\alpha/2\pi)[\ln(\Lambda/2\Delta) - 4\pi^2/9 + 59/20]$ arise because of the function Φ_1. When exchanging Φ_1 for Φ_2 in Eq. (20), it is necessary to exchange δ_1 for δ_2, where

$$\delta_2 = \frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{\Lambda}{m_e} + \frac{47}{60}\ln\frac{2\Delta}{m_e}\right.$$
$$\left.+2\left(1 - \ln\frac{2\Delta}{m_e} + \frac{47}{60}\right)\ln\frac{\Delta}{\omega_0} - \frac{\pi^2}{3} - \frac{2899}{900}\right).$$

δ_1 and δ_2 are of the same order of magnitude (for $\omega_0 \sim 1/10\,m_e$).

In the following we consider only δ_1. Numerically $\delta_1 = 0.012$ for $\Lambda = M$. Kinematical corrections [second and third terms in Eq. (20)] amount to -0.113. As the result, the decay probability is found to be

$$w = 3.95\cdot 10^{-1}\ \mathrm{sec}^{-1}. \quad (21)$$

The error of the known value of the mass difference of the π^+ and π^0 mesons [$\Delta = (4.59 \pm 0.01)$ MeV] causes an error of 1.5% in the decay probability. The variation of Λ by a factor of 10 leads to 0.8% change in w. An attempt to cut off the interaction with the photon in the positron-pion vertices at different Λ (similar to what was done for β decay in [10]) assuming that the electron is cut off in weak interactions at $\Lambda_e \sim 300$ BeV, and the meson in strong interactions at $\Lambda_\pi = M \sim 1$ BeV, leads to the exchange in Eq. (20) of Λ for $\Lambda_{eff} \approx 2.5\Lambda = 2.5M$, [4)] and we obtain practically the same results as earlier for $\Lambda = \Lambda_e = \Lambda_\pi = M$.

Thus, the radiative corrections and the errors in Eq. (20) are very small. There is, however, a source of an additional error, which we have not considered so far.

3. If we take the weak interaction form factor

[4)]Such a result is obtained if we choose the cut-off factor in the positron vertex in the form $\Lambda_e/[\Lambda_e^2 + k^2]^{1/2}$ and in the π meson vertex in the form $\Lambda_\pi/[\Lambda_\pi^2 + k^2]^{1/2}$. It is then necessary to make use of the fact that $\Lambda_e \gg \Lambda_\pi$.

into account, then the matrix element corresponding to diagram 1 in the figure is proportional to the expression [compare Eqs. (4) and (5)]

$$GF_\mu \bar{u}_\nu \gamma^\mu (1 + \gamma_5) v_e, \qquad (22)$$

$$F_\mu = f(q^2)(p^+ + p^0)_\mu + h(q^2) q_\mu. \qquad (23)$$

We can neglect the second term in Eq. (23) (see footnote [1]). In the theory with conserved vector current the quantity $f(q^2)$ coincides, with an accuracy to the meson mass difference, with the π-meson electromagnetic form factor. We can assume that $f(q^2) \equiv f(q^2/\mu^2) \approx f(0)$ since $q^2/\mu^2 \sim \Delta^2/\mu^2 \sim 10^{-3}$. The theorem about the absence of effects linear in the mass difference (see section 3.1) enables us to put $f(0) = 1 + O(\Delta^2/\mu^2)$. Thus, taking the form factor in diagram 1 into account does not lead to a considerable uncertainty in the decay probability.

4. The source of an additional error lies in an effect analogous to that discussed in [10] for the radiative corrections to the β decay. In the presence of form factors, the diagrams 2—8 are gauge-invariant and processes 6—8 occur therefore in addition to those discussed by us in Sec. 2.2. (It should be noted that the cases of a local interaction of five fields, contained in diagrams 6 and 7, were also taken into account in Sec. 2.) Let us denote by $F_\mu^\lambda(p^+, p^0, q, k)$ and $F_\mu^{\lambda\lambda'}(p^+, p^0, q, k, k')$ the form factors involving the emission, directly from F_μ in diagram 1, of one and two photons respectively, with momenta k and k′ and polarizations λ and λ'. We have also the following identities of the Ward type for F_μ^λ and $F_\mu^{\lambda\lambda'}$:

$$k_\lambda F_\mu^\lambda (p^+, p^0, q, k) = e\,[F_\mu(p^+ - k, p^0, q - k) - F_\mu(p^+, p^0, q)], \qquad (24)$$

$$k_\lambda k_{\lambda'} F_\mu^{\lambda\lambda'}(p^+, p^0, q, k, -k) = e^2\,[F_\mu(p^+ + k, p^0, q + k) - F_\mu(p^+ - k, p^0, q - k) - 2F_\mu(p^+, p^0, q)]. \qquad (25)$$

The proof of these relations obtained by summing the perturbation theory diagrams, was given in [10].[5] The contribution of diagrams 6—8 can be estimated as follows: Eqs. (24) and (25) enable us to express the quantities F_μ^λ and $F_\mu^{\lambda\lambda'}$, for k and k′ equal to zero, through the derivatives of the form factor F_μ. Thus, e.g.,

$$F_\mu^\lambda|_{k=0} = -eg_{\lambda\mu}, \qquad (26)$$

$$F_\mu^{\lambda\lambda'}|_{k=k'=0} = -2e^2(p^+ + p^0)_\mu g_{\lambda\lambda'}/\mu_+^2. \qquad (27)$$

In calculating the contribution from diagrams 6—8 we used Eqs. (26) and (27) and cut off the integration over the photon momentum at the nucleon mass. The resulting correction to the decay probability equals ~ 0.3%. Strictly speaking this quantity is actually not a correction but characterizes only the magnitude of additional errors, since the above discussion does not pretend to great accuracy.

5. Summarizing the results of the preceding sections we come to the conclusion that the probability of the $\pi^+ \to \pi^0 + e^+ + \nu$ decay is given by Eq. (21) with an error of 2.5—3%. Moreover, a greater part of this error (1.5%) is due to the errors in the experimental value of the π^+ and π^0 meson mass difference [$\Delta = (4.59 \pm 0.01)$ MeV].

The probability considered usually is that of the $\pi^+ \to \mu^+ + \nu$ decay, which essentially determines the lifetime of the π^+ meson. The ratio R is found to be

$$R = (1.01 \pm 0.03) \cdot 10^{-8}. \qquad (28)$$

For comparison let us quote the values of R obtained recently by three different experimental groups:

R	Experimental group
$(1.1^{+1.0}_{-0.5}) \cdot 10^{-8}$	Dubna [4]
$(1.7 \pm 0.5) \cdot 10^{-8}$	Geneva [5]
$(2.0 \pm 0.6) \cdot 10^{-8}$	Berkeley [6]

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[5]In this proof, the interactions in which the photon is emitted from the same point at which the interaction with the lepton current takes place were erroneously omitted. Further analysis shows that the form factors in the presence of such interactions satisfy, as before, Eqs. (24) and (25) and the complete set of diagrams including the emission of photons and of lepton currents from the same point is separately gauge-invariant.

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