ROLE OF THE $\gamma + \gamma \to \gamma + \nu + \bar{\nu}$ PROCESS IN NEUTRINO EMISSION BY STARS

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The cross section for the reaction $\gamma + \gamma \to \gamma + \nu + \bar{\nu}$ is calculated and the role of this process in neutrino emission by stars is considered. The magnitude of the neutrino luminosity due to this process is found to be about 10^8 times smaller than the values based on the estimations of Chiu and Morrison.

IN the energy balance of stars that have high temperatures and densities, and particularly in the evolution of stars, processes with neutrino emission play a very important role in spite of their small cross section [1]. The reason for this is that the neutrinos carry away energy from the entire volume of the star, whereas the hottest and the densest central part of the star is excluded for the photon luminosity, owing to the short range of the photons in dense media. Therefore at high temperatures and densities ($T \sim 10^{9\circ}$ K, $\rho \sim 10^5$ g/cm^3) the luminosity connected with neutrino processes prevails over electromagnetic radiation from the star.

Various processes that cause neutrino luminosity were discussed in several papers [1-6]. We consider quantitatively one of these processes, the conversion of a photon into a neutrino pair upon collision between two photons:

$$\gamma + \gamma \to \gamma + \nu + \bar{\nu}, \qquad (1)$$

and we shall show, in contradiction to the statement of Chiu and Morrison [6], who classify process (1) as one of the basic ones at a temperature $T \sim 10^{9\circ}$K, that this process does not play an important role in the stars.

1. CROSS SECTION OF THE PROCESS (1)

In the universal theory of weak interactions [7] there exists a direct interaction between the neutrino and the electron

$$\mathscr{L} = 2^{-1/2} G (\bar{e}\gamma_\mu (1 + \gamma_5) \nu)(\bar{\nu}\gamma_\mu (1 + \gamma_5) e)$$

$$= 2^{-1/2} G (\bar{e}\gamma_\mu (1 + \gamma_5) e)(\bar{\nu}\gamma_\mu (1 + \gamma_5) \nu).$$

Owing to this, process (1) is possible in the first order of perturbation theory in the weak interaction and in third order of the electromagnetic interaction.

The process is described by diagrams which differ from the photon-photon scattering diagrams in that one of the photons is replaced by a $\nu\bar{\nu}$ pair, and in such a vertex there should be the current $\bar{e}\gamma_\mu (1 + \gamma_5) e$ in place of the electromagnetic current. However it can be demonstrated, by a method similar to that of Furry [8], that the axial part makes no contribution. Therefore the matrix element of the process (1) has the form

$$M = \frac{e^3 G (2\pi)^4 \delta^4 (k_1 + k_2 - k_3 - k_4)}{2 (4\pi)^2 (\omega_1\omega_2\omega_3)^{1/2}}$$

$$\times e_{1\mu}^{(\lambda_1)} e_{2\sigma}^{(\lambda_2)} e_{3\tau}^{(\lambda_3)} (\bar{u}_\nu \gamma_\rho (1 + \gamma_5) u_{\bar{\nu}})$$

$$\times I_{\mu\sigma\tau\rho} (k_1, k_2, -k_3, -k_4), \qquad (2)$$

where k_1, k_2 and k_3 are the 4-momenta of the photons, $k_4 = p_\nu + p_{\bar{\nu}}$ is the summary 4-momentum of the neutrino pair, $e_i^{(\lambda_i)}$ are the photon polarizations, and $I_{\mu\sigma\tau\rho}$ is the regularized photon-photon scattering tensor [9,10]. For an energy $\omega \ll m$ the tensor $I_{\mu\sigma\tau\rho}$ has a relatively simple structure. We use for this tensor the expression given by Akhiezer and Berestetskiĭ [10] (pages 588—591).

The differential cross section of the process has the form

$$d\sigma = \frac{e^6 G^2}{2^{14}\pi^9 (k_1 k_2)} I_\rho I_\eta^+ [(p_\nu)_\rho (p_{\bar{\nu}})_\eta + (p_{\bar{\nu}})_\rho (p_\nu)_\eta - (p_\nu p_{\bar{\nu}}) \delta_{\rho\eta}]$$

$$\times \frac{d^3 k_3}{\omega_3} \frac{d^3 p_\nu}{E_\nu} \frac{d^3 p_{\bar{\nu}}}{E_{\bar{\nu}}} \delta^4 (k_1 + k_2 - k_3 - p_\nu - p_{\bar{\nu}}), \qquad (3)$$

$$I_\rho = e_{1\mu}^{(\lambda_1)} e_{2\sigma}^{(\lambda_2)} e_{3\tau}^{(\lambda_3)} I_{\mu\sigma\tau\rho}. \qquad (4)$$

Integrating over the momenta of the neutrino pair and recognizing that $I_\rho (k_4)_\rho = 0$ because of gauge invariance, we obtain

$$d\sigma = \frac{e^6 G^2 k_4^2}{2^{13} 3\pi^8 (k_1 k_2)} I_\rho^2 \frac{d^3 k_3}{\omega_3}, \qquad (5)$$

where $k_4 = k_1 + k_2 - k_3$.

Averaging over the initial-photon polarizations and summing over the final-photon polarizations, we obtain the following expression for the c.m.s. differential cross section:

$$\frac{d\sigma}{d\Omega\, d\omega_3} = \frac{e^6 G^2 \omega_0^3 (\omega_0 - \omega_3)\, \omega_3^3}{(2\pi)^8\, 3\, (90)^2\, m^8} \{\omega_0^2 [1760 \sin^2\theta + 2224] - 2872\, \omega_0\omega_3 \sin^2\theta + 139\, \omega_3^2 \sin^4\theta\}. \quad (6)$$

Here ω_0 is the energy of the colliding photon in the c.m.s. and θ is the angle between the directions of the momenta k_1 and k_3 in the same system.

Integrating over the angles we get

$$\frac{d\sigma}{d\omega_3} = \frac{2\alpha^3 G^2 \omega_0^3 \omega_3^3}{(45)^3 \pi^4 m^8} \{6370\omega_0^3 - 9960\omega_0^2\omega_3 + 3729\omega_0\omega_3^2 - 139\omega_3^3\}, \quad (7)$$

hence

$$\sigma = \frac{2G^2\alpha^3\omega_0^{10}}{(45)^3\pi^4 m^8}\left(202 + \frac{1}{7}\right). \quad (8)$$

This result is valid when $\omega_0 \ll m$. From dimensionality considerations it follows that in the opposite case ($\omega_0 \gg m$) the cross section has the form

$$\sigma\,(\omega \gg m) = A G^2 \alpha^3 \omega^2/\pi^4. \quad (9)$$

The coefficient A can be estimated by comparing the photon-photon scattering cross sections at low energy [10] and at high energy [11], from which we get $A \sim 1$.

2. ENERGY CARRIED AWAY BY THE NEUTRINO

We are interested in the energy carried away by the neutrino from a unit volume of stellar matter. This quantity is equal to

$$Q_\nu = \frac{1}{(2\pi)^6} \int \frac{2d^3k_1}{e^{\omega_1/T} - 1} \int \frac{2d^3k_2}{e^{\omega_2/T} - 1} \frac{(k_1k_2)}{\omega_1\omega_2}\varkappa, \quad (10)$$

where κ is determined in the following manner:

$$\varkappa = \int (\omega_1 + \omega_2 - \omega_3)\, d\sigma, \quad (11)$$

with all the quantities in the right half pertaining to some element of star volume, and not to the c.m.s. of the photons.

But κ is the fourth component of the 4-vector

$$\int (k_1 + k_2 - k_3)_\alpha d\sigma. \quad (12)$$

We can therefore express κ with the aid of the Lorentz transformation in the form of an integral pertaining to the photon c.m.s.:

$$\varkappa = (1 - v^2)^{-1/2} \int_{\text{c.m.s.}} [(2\omega_0 - \omega_3) - \mathbf{k}_3\mathbf{v}]\, d\sigma;$$

$$\mathbf{v} = (\mathbf{k}_1 + \mathbf{k}_2)\,/\,(\omega_1 + \omega_2), \qquad \omega_0 = {}^1/_2\,(\omega_1 + \omega_2)\,(1 - v^2)^{1/2}. \quad (13)$$

It is easy to see that the second term in the right half of (13) vanishes, since the cross section $d\sigma$ [see formula (6)] depends only on the even powers of $\cos\theta$.

Using (6), we obtain for the case $\omega \ll m$:

$$\varkappa = \frac{2\alpha^3 G^2 \omega_0^{11} (275 - {}^3/_{56})}{(45)^3 \pi^4 m^8 (1 - v^2)^{1/2}}. \quad (14)$$

Substituting (14) in (10) we get

$$Q_\nu = \frac{4\,(275 - {}^3/_{56})\, 8!\, 7!\, \zeta(9)\, \zeta(8)\, T^{17}\alpha^3 G^2}{7\,(45)^3\, \pi^8 m^8}. \quad (15)$$

If we measure the temperature in kiloelectron volts, then

$$Q_\nu = 1.7\cdot 10^{-28} T^{17}\ [\text{erg}\cdot\text{cm}^{-3}\cdot\text{sec}^{-1}]. \quad (16)$$

Thus, when $\omega \ll m$ we have $Q_\nu \sim T^{17}$. In the opposite case (when $\omega \gg m$), we have $Q \sim T^9$ from (9), (10) and (13). The latter case, however, is of no interest since it is connected with a temperature much higher than that of the hottest stars (white dwarfs).

In order to find the total energy carried away by the neutrino from a star, it is necessary to integrate Q_ν over the entire volume of the star:

$$L_\nu = 4\pi \int_0^R Q_\nu r^2 dr. \quad (17)$$

Using the model with a central source and assuming that the convective core of the star has a constant temperature equal to T_c (temperature at the center), and that in the rest of the volume the temperature has a distribution [12]

$$T = T_c\,(R/r - 1)\,/\,(1/0.169 - 1),$$

we obtain

$$L_\nu \approx 4.5\cdot 10^{-30} T_c^{17} R^3\ [\text{erg}\cdot\text{sec}^{-1}]. \quad (18)$$

The radius of the star is connected with its temperature and density at the center by the relation [13]

$$R = 1{,}18\cdot 10^{11} T_c^{0,5} \rho_c^{-0,5}\, \mu^{-0,5}, \qquad \mu^{-1} = \sum_i C_i\,(Z_i + 1)/A_i.$$

Substituting this value of R in (18) and expressing the result in solar units ($L_\odot = 3.78 \times 10^{33}$ erg/sec), we obtain

$$L_\nu = 1{,}97\cdot 10^{-30} T_c^{18,5} \rho_c^{-1,5} \mu^{-1,5}.$$

Thus, the luminosity due to process (1) amounts to 10^{-8} of the luminosity due, say, to the process $e + \gamma \rightarrow e + \nu + \bar{\nu}$ (at T = 100 keV and $\rho = 10^5$ g/cm^3).

3. COMPARISON WITH NEUTRINO LUMINOSITY DUE TO OTHER PROCESSES

It is useful to compare the value which we obtained for the energy carried away by the neutrino from a gram of stellar matter, with the values of

Energy of neutrino emission for different processes

№	Process	Q_ν/ρ (T in keV)	L_ν (erg-g^{-1}sec^{-1}) at T=100 keV, $\rho=10^5$ g/cm^3
I	$Z + e^- \rightarrow (Z-1) + \nu$, $(Z-1) \rightarrow Z + e^- + \bar{\nu}$ [1]		10^4—10^6
II	$e + Z \rightarrow e + Z + \nu + \bar{\nu}$ [2]	$2.75 \cdot 10^{-10} \rho \nu^{-1} \mu_e^{-1} T^{4,5}$	$8.0 \cdot 10^4$
III	$\gamma + e \rightarrow e + \nu + \bar{\nu}$ [3]	$3.32 \cdot 10^{-8} \mu_e^{-1} T^8$	$1.66 \cdot 10^8$
IV	$e^- + e^+ \rightarrow \nu + \bar{\nu}$ [4]	$7.45 \cdot 10^{12} \rho^{-1} T^3 e^{-2m/T}$	$2.8 \cdot 10^9$
V	$\gamma + Z \rightarrow Z + \nu + \bar{\nu}$ [5]	$3.6 \cdot 10^{-11} \nu^{-1} T^6$	$2.1 \cdot 10^2$
VI	$\gamma + \gamma \rightarrow \nu + \bar{\nu}$ [5]	$1.8 \cdot 10^{-8} \rho^{-1} T^9$	$1.8 \cdot 10^5$
VII	$\gamma + \gamma \rightarrow \gamma + \nu + \bar{\nu}$	$1.7 \cdot 10^{-28} \rho^{-1} T^{17}$	$1.7 \cdot 10^1$

Note: The quantities μ_e and ν are defined in the following manner:

$$\mu_e^{-1} = \sum_i C_i Z_i / A_i, \qquad \nu^{-1} = \sum_i C_i Z_i^2 / A_i,$$

where A_i and Z_i are the mass number and charge of the nucleus, and C_i is its concentration.

Q_ν/ρ in other processes. The results are gathered in the table. The temperature and density regions of interest to us are $T \sim 10^9$°K and $\rho \sim 10^5$ g/cm^3. Under these conditions the electron gas in processes I—IV can be regarded as nondegenerate and nonrelativistic.

The energy relase in the Gamow-Shoenberg process [1] depends in essential fashion on the concentration of certain definite isotopes in stellar matter. The quantity listed in the table is taken from the review of Chiu [14], where it was obtained under the assumption that the content of a certain combination of isotopes with mass number $A < 60$ amounts to 1 per cent of stellar matter. Processes V and VI were considered in the paper by Matinyan and Tsilosani [5]. It must be pointed out, however, that the last paper contains an error, and formula (6) of that paper must be multiplied by 0.14/137. The formula in our table incorporates this correction. We recall that process VI is possible only in the case of nonlocal interaction [15].

The result which we obtained for the process $\gamma + \gamma \rightarrow \gamma + \nu + \bar{\nu}$, differs in essential fashion from the result of Chiu and Morrison [6]. According to their estimates $Q_\nu/\rho = 10^7$ erg-g^{-1} sec^{-1} at $T = 10^9$°K and $\rho = 10^6$ g/cm^3, whereas our formula yields $Q_\nu/\rho = 1.4 \times 10^{-1}$ erg-g^{-1} sec^{-1}. Thus, apparently, the excessively crude estimate of Chiu and Morrison has led to an exaggeration of the result by eight orders of magnitude.

From a comparison of the value we obtained with the neutrino energy release in other processes it follows that the process considered cannot play an important role in the neutrino radiation from stars.

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