

**INVESTIGATION OF THE ENERGY SPECTRUM OF HIGH ENERGY MUONS AT A DEPTH OF  
40 METERS WATER EQUIVALENT UNDERGROUND**

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The energy spectrum of  $\mu$  mesons in the  $10^{11}$ – $3 \times 10^{12}$  eV range is derived from the spectrum of large bursts produced by high energy  $\mu$  mesons under a lead filter at a depth of 40 m water equivalent. The results for energies  $E_\mu > 10^{12}$  eV do not seem to be consistent with the usual production of  $\mu$  mesons via  $\pi^\pm$  or K decay.

**INTRODUCTION**

A study of the energy spectrum and angular distribution of high-energy muons<sup>[1-3]</sup> is of particular interest in connection with the question of the mechanism by which they are generated.

As long ago as 1952 Barret et al<sup>[4]</sup> obtained the following expression for the intensity of a flux of muons with energy  $\geq E$  (we are considering muons whose decay probability in the atmosphere is low), directed at an angle  $\vartheta$  to the vertical and resulting from  $\pi$ - $\mu$  decay:

$$I(\geq E, \vartheta) \sim E^{-\gamma} \frac{E_0}{E \cos \vartheta + \gamma E_0 / (\gamma + 1)}, \quad (1)$$

$$E_0 = B_\pi \left( \frac{\lambda_\pi}{\lambda_\pi - \lambda_p} \right) \ln \frac{\lambda_\pi}{\lambda_p}, \quad B_\pi = m_\pi c^2 r H_0 / c \tau_0, \quad (2)$$

where  $\gamma$  is the exponent of the integral energy spectrum of the generated pion<sup>1)</sup>,  $\lambda_\pi$  the range relative to the pion interaction,  $\lambda_p$  the range relative to the absorption of the nucleon component,  $H_0$  the isothermal atmosphere constant,  $\tau_0$  the pion lifetime, and  $r = \bar{E}_\mu / E_\pi$  the average fraction of the pion energy transferred to the muon in the decay.

An analogous expression for the muon intensity can be obtained by assuming that they are generated by K- $\mu$  decay. For the same number of K and  $\pi$  mesons of given energy, the number of the muons of the given energy resulting from the K- $\mu$  decay will exceed by a factor of 5 the number of muons of the same energy resulting from  $\pi$ - $\mu$

decay<sup>[5,6]</sup>. In addition, in the case of K- $\mu$  decay the constant is  $B_K = 7B_\pi$ . As can be seen from the expression for  $E_0$ , the value of  $E_0$  depends little on  $\lambda_\pi$  or  $\lambda_K$ .

Thus, in order to ascertain the origin of the high-energy muons and, in particular, the possibility of attributing their presence to  $\pi$ - $\mu$  and K- $\mu$  decay, experimental data must be obtained both on the energy distribution of the muons and on the energy spectrum of the pions and kaons generated in the atmosphere.

At the present time there are numerous and sufficiently convincing experimental data concerning the muon spectrum in the high-energy region up to  $\approx 10^{12}$  eV, obtained also by direct methods (magnetic spectrometer<sup>[1,3]</sup>). However, in the energy region  $\gtrsim 10^{12}$  eV there are so far only the poorly agreeing experimental data of Barton<sup>[3]</sup> and Bollinger<sup>[7] 2)</sup>. It is therefore of appreciable interest to supplement the measurements of the energy spectrum at  $\gtrsim 10^{12}$  eV.

Barton<sup>[3]</sup> and Barret et al<sup>[4]</sup> determined the intensity of the muons under different layers of ground in order to obtain the muon energy spectrum. As is well known, the conversion of the muon-flux absorption curve at large depths (several thousand m.w.e.) into the muon energy spectrum is fraught with a considerable uncertainty, connected with the great sensitivity of the muon energy for a specified absorption range  $x$  to the energy losses caused by "nuclear" muon interactions<sup>3)</sup>. The relative role of the latter among

<sup>1)</sup>It is assumed that: 1) the pions are produced by interaction between the air nuclei and nucleons whose absorption follows an exponential law; 2) pion regeneration does not play an appreciable role. Assumption 2) has been by now confirmed by direct measurements, both at stratospheric altitudes and at mountain altitudes<sup>[5]</sup> in the energy range  $10^{11}$ - $10^{13}$  eV.

<sup>2)</sup>The question of experimental data on the energy spectrum of pions and kaons will be considered in the discussion of the results below.

<sup>3)</sup>We refer here to the nuclear photoeffect produced by virtual photons of the muon electromagnetic field.

the other processes (electron-positron pair production, bremsstrahlung) is still not accurately known at present. However, for values of  $Z$  and  $A$  corresponding to the ordinary ground ( $Z \sim 10$ ,  $A \sim 20$ ), the role of "nuclear" muon interactions is at any rate comparable with the role of electromagnetic interactions.

### PROCEDURE AND RESULTS

It is possible, however, to employ a different method, which has much greater transmission and at the same time is practically rid of the uncertainty due to the "nuclear" muon interactions. This method consists of investigating the large ionization bursts produced by muon interaction in a high- $Z$  absorber that screens the ionization chambers.

The array used by us was located in an underground room at 40 m.w.e. It is shown in Fig. 1.

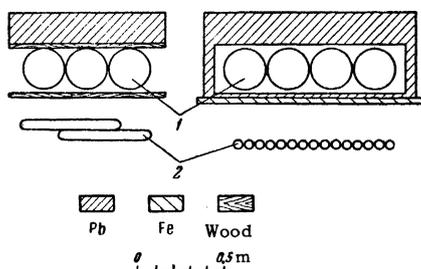
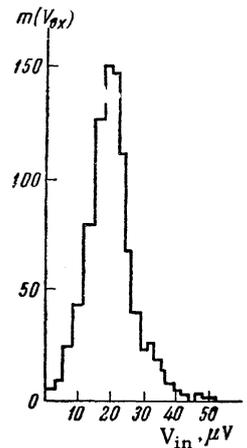


FIG. 1. Section through the array: 1 — ionization chambers, 2 — counters. The counter location is not to scale. The actual distance between the row of counters and the chambers is 75 cm.

Two groups of ionization chambers were screened on top with a layer of lead 16 cm thick. The electronic component of the pulses from the ionization chambers was registered with the aid of a pulse-height analyzer with accuracy not less than 10 per cent, over a wide pulse-height range corresponding to bursts from 30 to 100,000 relativistic particles. The high argon pressure inside the chambers enabled us to calibrate the chambers against single relativistic particles (see Fig. 2).

In view of the drooping character of the muon energy spectrum the main contribution to the ionization bursts are made by those interactions, which cause the muon to lose an energy comparable with its own energy<sup>[8]</sup>. These interactions include bremsstrahlung and the "nuclear" interaction. On the other hand, owing to the use of a filter with large  $Z$  ( $\approx 80$ ), it can be stated that the bursts observed in the ionization chambers are produced in the overwhelming majority of cases by electromagnetic muon interaction, while

FIG. 2. Distribution of pulses from ionization chamber ( $V_{in}$ ) on passage of single muons,  $m$  — number of events with given amplitude.



the "nuclear" interactions play an insignificant role. In fact, for a filter thickness expressed in  $\text{g}/\text{cm}^2$ , the probability of radiative deceleration with energy loss  $E$  increases as  $\sim Z^2$ , whereas the probability of the "nuclear" interactions with the same loss  $E$  increases in proportion to  $A$ .<sup>4)</sup> As a net result, whereas these probabilities are of the same order of magnitude for the ground, the former is eight times larger than the latter for lead.

In addition, an important role is played in the production of ionization bursts by the character of the development of the particle cascade in the filter (shape of the cascade curve). For a given muon energy released in a pure electron-photon cascade, the number of particles at the maximum will on the average be 2–3 times larger than in the case of a cascade produced by pions in "nuclear" interaction of the muons<sup>[9]</sup>. Therefore, in order for a burst to be produced by a "nuclear" interaction, the minimum required muon energy should be relatively large, which in view of the drooping character of the energy spectrum of the muons leads to an appreciable decrease in the role of the "nuclear" interactions.

After 1200 hours of operation with a total ionization-chamber area of  $1.75 \text{ m}^2$ , the spectrum of ionization bursts shown in Fig. 3 was obtained. The spectrum can be approximated over the entire observable region of the bursts ( $n \geq 200$  relativistic particles) by a single power law with exponent  $\gamma = -1.9 \pm 0.2$ . The size of the burst is expressed in terms of the number of relativistic particles  $n$  passing through the central chord of

<sup>4)</sup>We note that the expressions for the differential bremsstrahlung cross section with energy  $E$  and for the differential cross section of "nuclear" interaction with energy loss  $E$  have practically the same form when expressed as functions of  $E$ .

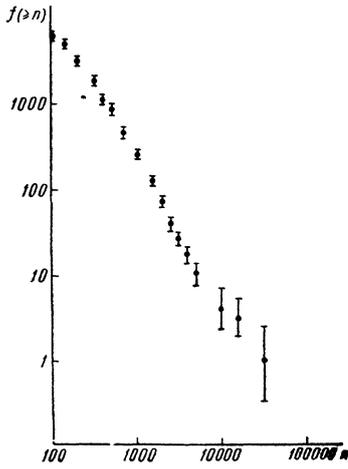


FIG. 3. Spectrum of bursts from muons at depth 40 m.w.e.,  $f$  - number of bursts.

the chamber. From the experimental spectrum of the bursts we can obtain the spectrum of the muons incident on the array, assuming that 1) the overwhelming majority of the bursts is produced by muon bremsstrahlung and 2) the muon trajectories are directed vertically<sup>5)</sup>. The spectrum of the bursts and the spectrum of the muons are in this case related by the equation<sup>[8]</sup>:

$$b(\geq n) = \int_{E_{\min}(n)}^{\infty} dE_{\mu} \int_{E_{\min}(n)}^{E_{\mu}} W_{\text{rad}}(E_{\mu}, E) l(\geq n, E) \varphi(E_{\mu}) dE, \quad (3)$$

where  $W_{\text{rad}}(E_{\mu}, E) dE$  is the effective bremsstrahlung cross section per cascade unit for a gamma quantum with energy  $E$ ;  $E_{\min}(n)$  is the minimum gamma-quantum energy necessary to produce a shower of  $n$  particles in lead (for lead we have with good accuracy  $E_{\min} = 10^8 n$ );  $\varphi(E_{\mu}) dE_{\mu}$  is the energy spectrum of the muons;  $l(\geq a, E)$  is the extent, expressed in  $t$  units, of that part of the electron-photon shower due to a gamma quantum with energy  $E$ , where the number of electrons  $\geq n$ . In the energy interval from  $\sim 10^{10}$  to  $\sim 10^{12}$  eV, the function  $l$  can be approximated by the formula  $l = 8.7 \ln(E/10^8 n)$  (we have used here Ott's curves from the review by Heisenberg<sup>[10]</sup>). In the region  $E > 10^{12}$  eV, we used the same formula. Equation (3) enables us in principle to express the muon spectrum in terms of the experimentally obtained burst spectrum. However, in the general case a solution of this integral equation is difficult.

If we assume that the energy spectrum of the muons in the region of importance to the integration has the form

$$\varphi(E_{\mu}) dE_{\mu} = A \cdot E_{\mu}^{-(\gamma+1)} dE_{\mu},$$

then it follows from relation (3) that

$$b(\geq n) = BA\gamma^{-1} (10^8 n)^{-\gamma}, \quad B \cong 9 \cdot 10^{-4} \gamma^{-2}. \quad (4)$$

Consequently, the spectrum of the bursts will have an exponent coinciding with the exponent of the muon spectrum, and will differ only in a known constant. The muon spectrum obtained from relation (4) is shown in Fig. 4.

However, relation (4) holds true for average quantities, and generally speaking it cannot be stated beforehand at what statistics the true number of muons entering the array can deviate appreciably from their average value expected from (4). We therefore employed the Monte Carlo method to obtain the energy spectrum of the muons. Three sets of plays were made<sup>6)</sup>. The results are shown in Fig. 5.

Let us consider now the question of the method-

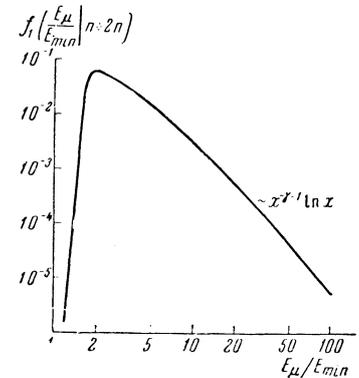


FIG. 4. Energy spectrum of muons producing bursts in the range from  $n$  to  $2n$  (for  $\gamma = 2$ ).

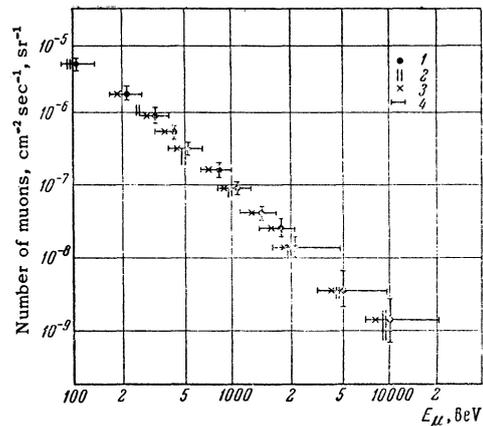


FIG. 5. Muon spectrum calculated by different methods: 1 - from relation (4), 2 - by the Monte Carlo method, 3 - with account of the corrections, 4 - uncertainty due to the inaccuracy in the exponent  $\gamma$ .

<sup>5)</sup>The corrections connected with an account of the angular distribution of the muons and the possible change in the exponent  $\gamma$  will be considered below.

<sup>6)</sup>We used in the plays the function  $f_i(E_{\mu}/n \text{ to } 2n)$ , which represents the spectrum of the muons producing bursts with magnitude between  $n$  and  $2n$  (see Fig. 4).

ological corrections to these results. On going from the burst spectrum to the muon energy spectrum two assumptions were made: 1) the muon flux is incident vertically; 2) the energy spectrum of the muons has a power-law form with constant exponent  $\gamma$  in the energy interval under consideration. In fact, as can be seen for example from (1), we should expect in our energy interval a broad angular distribution of the muons and a noticeable increase of the energy-spectrum exponent with energy.

It is obvious that formula (1) gives the maximum possible influence of both factors. We shall consider this influence, assuming (1) to be correct.

Thus, in the general case the trajectory of the muons makes an angle  $\vartheta$  with the vertical. At large values of  $\vartheta$  and with a sufficiently well directed angular distribution of the shower particles under the lead we can expect a noticeable increase in the length of shower-particle path. In the limiting case, when the muon passes through the side surfaces covered with the lead, we obtain an increase in this path by a factor 4 (see Fig. 1). However, only two of the eight side surfaces were covered with lead (for structural reasons), as can be seen from Fig. 1. If we assume an angular distribution in accordance with (1), then in the limiting case of a directional angular distribution of shower particles in the lead, neglecting the absorption of the shower particles in the walls of the ionization chamber, we obtain for  $E \gg E_0$  an average increase in the summary path  $h$  of the shower particles by a factor of not more than 2.2. The average energy  $\bar{E}_\mu$ , obtained from the distribution  $f_1(E_\mu/n)$ , and corresponding to the given value of  $n$ , should be decreased in the limiting case by the same factor.

Table I lists the calculated values of  $\overline{\cos \vartheta}$  with account of the angular distribution (1) and the geometry of the array for different values of  $E_\mu$ . As can be seen, the change in the average path does not exceed 10 per cent in the range of values of  $E_\mu$  of importance to the given experiment. Because of this change, the previously obtained value of the exponent  $\gamma$  must be increased by 0.05.

**Table I.** Change in the average path  $h_0/\overline{\cos \vartheta}$  of the muons through the chambers with increasing energy  $\bar{E}_\mu = kn$

$10^{-11} E_\mu, \text{eV}$	1	5	10	100
$\overline{\cos \vartheta}$	0.49	0.49	0.48	0.45

Further, in accordance to (1), the exponent of the energy spectrum of the muons changes, generally speaking, in the energy range under consideration. Therefore the distribution  $f_1(E_\mu/n)$  and the relation between  $\bar{E}_\mu$  and the size of the burst  $n$  (that is,  $\bar{E}_\mu = kn$ ) will depend on the energy region considered. In Table II we examine the case when the energy spectrum of the muons is given by formula (1) with  $\gamma_\pi = 1.5$ . This spectrum represents relatively well the experimental results of Ashton et al<sup>[1]</sup> and Holmes et al<sup>[2]</sup>. It is seen from the table that in the energy interval under consideration the relation between  $E_\mu$  and the size of the bursts changes little, so that at large  $E_\mu$  the ratio  $\bar{E}_\mu/n$  decreases by only 5 per cent. This means that the previously obtained value of  $\gamma$  should increase as a result of this effect by 0.05.

**Table II.** Change in the coefficient  $k$  which determines the average energy,  $\bar{E}_\mu = k(E)n$ , of the muons that produce bursts of size  $n$ , with increasing muon energy

$\frac{E}{\bar{E}_\mu/\cos \vartheta}$	0.2	1	5	10	100
$\frac{k(E)}{k(\infty)}$	1.07	1.06	1.03	1.02	1.01

Further, an appreciable uncertainty arises also because the exponent of the bursts spectrum in the region of large  $n$ , or in other words, the exponent of the energy spectrum of the muons in the region  $E_\mu > 10^{12}$  eV, has not been sufficiently accurately determined:  $\gamma = 1.8 \pm 0.4$ .

Finally, our spectrum obtained differs from the spectra obtained by others<sup>[1-4]</sup> in that it is the global spectrum of the muons making different angles  $\vartheta$  with the vertical. Were the angular distribution of the muons not to change with increasing energy, the global energy spectrum would be similar to the vertical spectrum. However, if the angular distribution has maximum anisotropy and changes with variation of the muon energy in accordance with (1), then the global energy spectrum of the muons incident on our array, taking into account the change in the effective registration area  $\sigma = \sigma_0 \cos \vartheta$ , has the form

$$\int_0^{\pi/2} I(\geq E, \vartheta) \cos \vartheta \sin \vartheta d\vartheta = \frac{E^{-\gamma_\pi}}{E} \left[ 1 - \frac{\gamma_\pi E_0}{(\gamma_\pi + 1)E} \ln \left( \frac{E(\gamma_\pi + 1) + \gamma_\pi E_0}{\gamma_\pi E_0} \right) \right]. \quad (5)$$

The intensity ratios of the muons with  $E \sim 10^{10}$  eV and  $E \sim 10^{13}$  eV calculated in accordance with formula (1) with  $\vartheta = 0$  and in accordance with formula (5) do not differ by more than 40 per cent. Thus, the spectra (1) and (5) are sufficiently close to each other (the difference in the exponent of the spectrum in the investigated region of muon energy does not exceed  $\Delta\gamma \lesssim 0.1$  for  $\gamma \approx 2$ ). Figure 5 shows the influence of the foregoing factors on the muon energy  $E_\mu$  and indicates the uncertainty in  $E_\mu$  due to the inaccuracy in the exponent  $\gamma$ .

We have thus, considered the maximum possible influence of all the methodological uncertainties. We note that a possible decrease in the factor for the conversion from  $n$  to  $\bar{E}_\mu$  is offset to a considerable degree by the lead-iron transition effect, which in our case amounts to 50 per cent. At the same time, it follows from the foregoing analysis that the exponent of the integral energy spectrum of the muons, with account for all the corrections, cannot exceed  $\gamma = 2.1 \pm 0.2$  in the region  $10^{11}$  eV  $< E_\mu < 3 \times 10^{12}$  eV.

Figure 6 shows the muon spectrum with allowance for all the foregoing corrections. It also

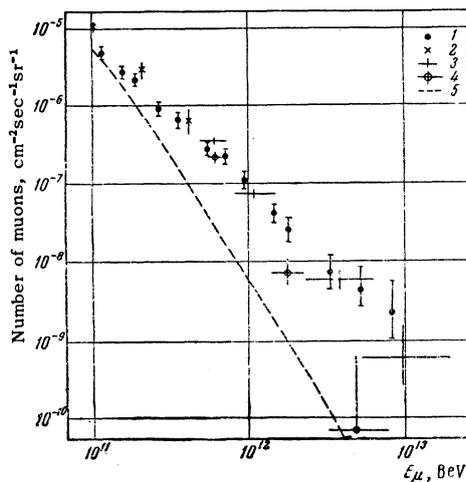


FIG. 6. Energy spectrum of muons from various data: 1 – present work, 2 – Ashton et al.<sup>[1]</sup> 3 – Bollinger,<sup>[7]</sup> 4 – Barton,<sup>[3]</sup> 5 – curve calculated by Rodgers.<sup>[11]</sup>

shows the data of Ashton et al.<sup>[1]</sup> and Holmes et al.<sup>[2]</sup>, obtained with the aid of a magnetic spectrometer, and also the data of Bollinger<sup>[7]</sup> and Barton<sup>[8]</sup>. We note that the data of Bollinger and Barton, obtained under identical assumptions concerning the role of the “nuclear” muon interactions, are represented by respectively equal sides of the horizontal segments, so that their discrepancy is appreciable. The data of the present work do not contradict those of Bollinger.

## DISCUSSION

We have already noted above that to solve the question of the origin of the high-energy muons it is necessary to have independent data on the energy spectrum of the  $\pi^\pm$  mesons in the entire interval of investigated muon energies, that is, from  $10^9$  to  $\sim 10^{13}$  eV. These data were obtained already by Rossi at  $\sim 1$  BeV (see<sup>[11]</sup>). Recently Duthie, Perkins, et al.<sup>[6]</sup> obtained data on the spectrum of  $\pi^0$  mesons generated in the stratospheres in the energy range from 300 to 5000 BeV. All these data are listed in Table III. We can thus state that the exponent of the pion energy spectrum apparently increases noticeably with increasing energy.

Table III

Exponent of integral energy spectrum of the pions, $\gamma_\pi$	Energy interval, BeV	Height, g/cm <sup>2</sup>	Literature reference
1.6	1		Rossi <sup>[11]</sup>
$2.0 \pm 0.5$	300–1500	8–25	Duthie et al. <sup>[6]</sup>
$2.5 \pm 0.3$	400–1500	100–200	The same
$2.8 \pm 0.3$	1500–5000	100–200	”

Rodgers<sup>[12]</sup> calculated the energy spectrum of the muons under the assumption that the spectrum of the generated  $\pi^\pm$  mesons has a power-function law with slowly increasing exponent:

$$\gamma_\pi = 1.6 + 0.07 \ln(E_\pi / 10^{10}).$$

The remaining assumptions are the same as in<sup>[4]</sup>. The change in the exponent  $\gamma_\pi$  with  $E_\pi$  corresponds fully to the data of Table III.

As can be seen from Fig. 6, the spectrum calculated by Rodgers, which is in good agreement with experiment for  $E_\mu \lesssim 100$  BeV, greatly deviates from the experimental data when  $E_\mu > 100$  BeV.

Apparently it is possible to reconcile Rodgers' results<sup>[12]</sup> with experiment in the energy region  $E_\mu > 100$  BeV only by assuming that additional sources of muon production (other than the  $\pi^\pm$  mesons) exist. If Barton's data<sup>[3]</sup> are correct, then it is sufficient to assume a contribution from the K- $\mu$  decay, which is quantitatively fully compatible with the experimental estimates ( $\lesssim 20$  per cent)<sup>[5]</sup> of the share of the kaons generated in nuclear interactions at the energies considered. On the other hand, Bollinger's<sup>[7]</sup> and our own data can hardly be explained without making new

assumptions that the muons are generated more rapidly than in the K- $\mu$  decay.

We note, however, that Barton's data<sup>[3]</sup> can be reconciled with our present data if we assume that the role of the "nuclear" interactions of the muons in ground is 1.7 times larger than was assumed in Barton's paper.

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<sup>1</sup> Ashton, Brooke et al, Nature 185, 364 (1960).

<sup>2</sup> Holmes, Owen, and Rodgers, Proc. Phys. Soc. 78, 496 (1961).

<sup>3</sup> J. C. Barton, Phil. Mag. 6, 1271 (1961).

<sup>4</sup> Barret, Bollinger, Cocconi, Eisenberg, and Greisen, Revs. Modern Phys. 24, 133 (1952).

<sup>5</sup> J. Perkins, CERN 61-22 (1961).

<sup>6</sup> Duthie, Fisher, Fowler, Kaddoura, Perkins, and Pinkau, Phil. Mag. 6, 89 (1961).

<sup>7</sup> L. Bollinger, Phys. Rev. 79, 207 (1950).

<sup>8</sup> S. Z. Belen'kiĭ, Lavinnye protsessy v kosmicheskikh luchakh (Cascade Processes in Cosmic Rays), Gostekhizdat, 1948.

<sup>9</sup> Grigorov, Murzin, and Rapoport, JETP 36, 1068 (1959), Soviet Phys. JETP 9, 759 (1959).

<sup>10</sup> H. Heisenberg, Vorträge über die Kosmische Strahlung, Berlin, 1953.

<sup>11</sup> G. H. Fowler and A. W. Wolfendale, Handbuch d. Physik 46/1, 231 (1961).

<sup>12</sup> A. L. Rodgers, Proc. Phys. Soc. 78, 918 (1961).

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