EXCITATION OF COLLECTIVE STATES OF NUCLEI IN THE SCATTERING OF CHARGED PARTICLES

S. I. DROZDOV

Submitted to JETP editor August 15, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) 44, 335-340 (January, 1963)

Inelastic scattering of fast charged particles (protons) by translucent nuclei with concomitant excitation of the first collective levels is considered in the diffraction approximation. Comparison of the results of the calculation with the experimental data permits one to estimate the radius and nonsphericity parameter of the nucleus and also the parameter of the imaginary part of the optical potential.

1. INTRODUCTION

SCATTERING of nucleons by nuclei is accompanied by excitation of collective nuclear states, owing to direct interaction. In this connection, we consider the scattering of charged particles, say protons, by a translucent nucleus, when the nuclear interaction is described by the optical potential

$$V(\mathbf{r}) = -V_0(1 + i\zeta)\, n(\mathbf{r}). \tag{1}$$

To calculate the scattering cross section we use, as is customary, the adiabatic approximation, which is applicable if $kR\Delta E/E \ll 1$, where ΔE is the energy of the excited level, k the wave number of the incident particle, and R is radius of the nucleus. The solution of the scattering problem reduces then to the determination of the amplitude $f(\Omega, \omega)$ of particle scattering by the stationary nucleus, with $\Omega = (\theta, \phi)$ determining the scattering direction and $\omega = (\vartheta, \varphi)$ determining the orientation of the symmetry axis of the nucleus. Then, for example, the differential cross section for scattering with excitation of the rotational levels of an even nucleus with momentum λ is determined by the expression

$$\sigma_\lambda(\theta) = \sum_\mu |\langle Y^*_{\lambda\mu}(\omega) f(\Omega, \omega) Y_{00}\rangle|^2. \tag{2}$$

The amplitude $f(\Omega, \omega)$ of scattering by a stationary nucleus can be calculated by diffraction theory[1,2], where the interaction energy is considered everywhere to be small compared with the energy: $|V(\mathbf{r})| \ll E$. As applied to nuclei, with sufficiently strong absorption, the condition under which the diffraction approximation is applicable turns out to be weaker, since it pertains only to the Coulomb field of the nucleus. In this case it has the form $kR \gg \eta$, where $\eta = ZZ'e^2/\hbar v$ is the Coulomb parameter.

Using these approximations, let us calculate the angular distributions of charged particles (protons) scattered by translucent nuclei with excitation of the first collective levels 2^+ and 3^- of the nucleus (single-phonon excitations).

2. SCATTERING AMPLITUDE

In the diffraction approximation the scattering amplitude in the region of small angles $\theta < 1$ has the form

$$f(\Omega, \omega) = \frac{ik}{2\pi}\int d\boldsymbol{\rho}\, e^{-i\mathbf{k}'\boldsymbol{\rho}} \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{\infty}[V(\mathbf{r}) + U(\mathbf{r})]\,dz\right\}. \tag{3}$$

$U(\mathbf{r})$ is the energy of the electric interaction and $\mathbf{k}'$ is the wave vector of the scattered particle. For simplicity we assume that the nuclear charge distribution coincides with the density distribution $n(\mathbf{r})$ (1). The electrical interaction energy is then determined by the expression

$$U(\mathbf{r}) = \frac{3ZZ'e^2}{4\pi R^3}\int\frac{n(\mathbf{r}')\,d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} = ZZ'e^2 m(\mathbf{r}). \tag{4}$$

Here the function $n(\mathbf{r})$ depends on the nonsphericity parameters α_λ, which enter in the equation for the nuclear surface (in the proper reference frame):

$$R(\alpha) = R\left(1 + \sum_\lambda \alpha_\lambda P_\lambda(\mu')\right). \tag{5}$$

Taking R to be unity and going over in integrals (3) and (4) to the dimensionless variables ρ and z, we obtain

$$f(\Omega, \omega) = \frac{ikR^2}{2\pi}\int_0^\infty \rho\, d\rho \int_0^{2\pi} d\varphi' e^{-ia\rho\cos\varphi'}\mathscr{E}(\rho, \alpha). \tag{6}$$

Here $a = kR\theta$ and the function $\mathscr{E}(\rho, \alpha)$, which depends on the parameters α_λ, is defined as

$$\mathscr{E}(\rho, \alpha) = \exp\left[\frac{i\xi}{2}\int_{-\infty}^{\infty} n(\mathbf{r})\,dz - i\eta\int_{-\infty}^{\infty} m(\mathbf{r})\,dz\right], \qquad (7)$$

where the complex parameter is

$$\xi = \frac{V_0}{E}(1 + i\zeta)\,kR = \xi_1 + i\xi_2. \qquad (8)$$

Let us expand the amplitude $f(\Omega, \omega)$ in powers of the small nonsphericity parameters α_λ, and confine ourselves to the linear approximation, which corresponds to the consideration of single-phonon excitations. The smallness parameters of this expansion are $\alpha_\lambda kR\theta \ll 1$ [3]. In our case we can disregard the diffuseness of the nuclear surface, assuming that the density distribution in the nucleus is expressed by a stepwise Θ function

$$n(\mathbf{r}) = \Theta(R(\alpha)/R - r). \qquad (9)$$

Then after several calculations analogous to those in [3] we obtain the following expression for the amplitude of inelastic scattering with excitation of one phonon ($\lambda\mu$):

$$\langle Y^{*}_{\lambda\mu}(\omega)\, f(\Omega, \omega)\, Y_{00}\rangle = \frac{\alpha_\lambda kR^2}{2\lambda+1} F_{\lambda\mu}(a);$$

$$F_{\lambda\mu}(a) = \sqrt{4\pi}\int_0^1 \rho d\rho\, J_\mu(a\rho)\left[\xi\,\frac{Y_{\lambda\mu}(\sqrt{1-\rho^2}, 0)}{\sqrt{1-\rho^2}} - 2\eta\, Q_{\lambda\mu}(\rho)\right](1+\sqrt{1-\rho^2})^{i2\eta}$$

$$\times \exp\left[i\left(\xi - 2\eta\,\frac{4-\rho^2}{3}\right)\sqrt{1-\rho^2}\right]$$

$$- \sqrt{4\pi}\,2\eta\int_1^\infty d\rho J_\mu(a\rho)\, Q_{\lambda\mu}(\rho)\,\rho^{1+i2\eta}. \qquad (10)$$

Here $Q_{\lambda\mu}(\rho)$ is an irrational function, which is expressed in terms of the normalized spherical function $Y_{\lambda\mu}(x, 0)$:

$$Q_{\lambda\mu}(\rho) = \int_0^\infty dz\, Y_{\lambda\mu}\left(\frac{z}{r}, 0\right) m_\lambda(r);$$

$$m_\lambda(r) = \frac{3}{2\lambda+1}\begin{cases} r^\lambda, & r<1 \\ r^{-\lambda-1}, & r>1 \end{cases}. \qquad (11)$$

The amplitude (10) vanishes when the sum $\lambda+\mu$ is odd.

In the expression (10) for the inelastic scattering amplitude it is convenient to separate the part $F_{E\lambda\mu}$ corresponding to the amplitude of the electrical Coulomb excitation [4]:

$$F_{\lambda\mu} = F_{E\lambda\mu} + F_{n\lambda\mu}. \qquad (12)$$

As shown in [3], the amplitude $F_{E\lambda\mu}$ is expressed by the last integral in (10), provided it is complemented to the integral with respect to $d\rho$ from zero to infinity, which can be evaluated. Thus we obtain

$$F_{E\lambda\mu}(a) = -\delta_{\mu\pm\lambda} c_{\lambda\mu}\eta\,\frac{\Gamma(1+i\eta)}{\Gamma(\lambda-i\eta)}\left(\frac{a}{2}\right)^{\lambda-2-i2\eta}, \qquad (13)$$

where

$$c_{\lambda\mu} = \frac{3\sqrt{4\pi}}{2\lambda+1}\int_0^1 Y_{\lambda\mu}(x, 0)\,(1-x^2)^{\lambda/2-1}dx.$$

Going over in the first integral of (10) to a new integration variable $x = \sqrt{1-\rho^2}$, we obtain the following expression for the so-called nuclear part of the amplitude (12):

$$F_{n\lambda\mu}(a) = \sqrt{4\pi}\int_0^1 dx\, J_\mu(a\sqrt{1-x^2})\,[\xi Y_{\lambda\mu}(x, 0) - 2\eta x\, Q_{\lambda\mu}(\sqrt{1-x^2})]\,(1+x)^{i2\eta}\exp\left[ix\left(\xi - 2\eta\,\frac{3+x^2}{3}\right)\right]$$

$$+ \delta_{\mu\pm\lambda} 2\eta\Phi_{\lambda\mu}(a, \eta)\, c_{\lambda\mu}. \qquad (14)$$

Here

$$\Phi_{\lambda\mu}(a, \eta) = a^{\lambda-2-i2\eta}\int_0^a dx\, J_\mu(x)\, x^{1-\lambda+i2\eta},$$

and it is convenient in the computations to represent the function $\Phi_{\lambda\lambda}(a, \eta)$ in the form [3]

$$\Phi_{\lambda\lambda}(a, \eta) = \sum_{m=0}^\infty \frac{a^m J_\lambda(a)}{2^{m+1}} \prod_{p=0}^m \frac{p+1-i\eta}{(p+1)^2+\eta^2}. \qquad (15)$$

When $\eta = 0$ formulas (10) and (14) describe the scattering of neutrons [5,6]:

$$F_{\lambda\mu}(a) = \xi\sqrt{4\pi}\int_0^1 dx\, J_\mu(a\sqrt{1-x^2})\, Y_{\lambda\mu}(x, 0)\, e^{i\xi x}. \qquad (16)$$

It is also easy to see that if

$$\xi_2 = \operatorname{Im}\xi = V_0\zeta kR/E \gg 1 \qquad (17)$$

the expressions (10), (14), and (16) go over into formulas [3,7,8] that describe inelastic scattering by a black nucleus. Indeed, evaluating the first integral of (14) by parts and assuming at the same time $dv = \exp(i\xi x)dx$, we obtain, taking (17) into account,

$$F_{n\lambda\mu}(a) = i\sqrt{4\pi}\, Y_{\lambda\mu}(0, 0)\, J_\mu(a) + \delta_{\mu\pm\lambda} c_{\lambda\mu} 2\eta\Phi_{\lambda\mu}(a, \eta), \qquad (18)$$

which corresponds to formula (19) of [3].

3. ANGULAR DISTRIBUTIONS AND COMPARISON WITH EXPERIMENTAL DATA

According to (2) and (10), the differential cross sections λ for scattering with excitation of one photon can be represented in the form

$$\sigma_\lambda(\theta) = \left(\frac{\alpha_\lambda k R^2}{2\lambda+1}\right)^2 s_\lambda(a), \tag{19}$$

where the function

$$s_\lambda(a) = \sum_\mu |F_{\lambda\mu}(a)|^2 \tag{20}$$

does not depend on the deformation parameters α_λ. In accordance with (12), $s_\lambda(a)$ can be represented in the form of a sum of a Coulomb part, a nuclear part, and an interference term:

$$s_\lambda(a) = s_{E\lambda}(a) + s_{n\lambda}(a) + s_{nE\lambda}(a), \tag{21}$$

where, for example

$$s_{E3}(a) = \sum_\mu |F_{E3\mu}(a)|^2 = \frac{2\eta^2 a^2}{35(4+\eta^2)(1+\eta^2)}$$

etc [3]. A comparison of the results of the calculation with the experimental data by means of formula (19) enables us to estimate the parameters α_2 and α_3, which can correspond to stable deformation of the nucleus or to the amplitude of the zero-point oscillations near the equilibrium shape [9].

Using (12)—(16), (18), and (20) we calculated the angular distributions of the protons and neutrons with energy E = 40 MeV, scattered with excitation of first levels 2^+, 3^- of the nuclei Mg^{24}, Ni^{58}, and Gd^{156} (Figs. 1—3). The absolute value of the function $s_\lambda(a)$, and particularly its form, depend quite strongly on the parameter ζ of the imaginary part of the optical potential. However, in accordance with the criterion (17), even when Im $\xi \geq 2$ the angular distributions $s_\lambda(a)$ are close to the distributions corresponding to scattering by a black nucleus. The calculations merely confirm the important role of the Coulomb effects: the angular distributions $s_\lambda(a)$ of the

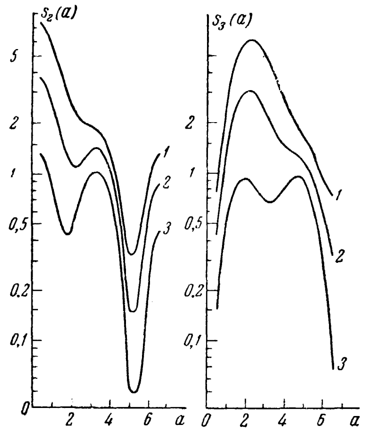

FIG. 1. The functions $s_2(a)$ and $s_3(a)$, with $a = kR\theta$ describing the angular distributions of the protons inelastically scattered by the nucleus Ni^{58}(E_p = 40 MeV, η = 0.70, ξ_1 = 6.5). These calculations were made for V_0 = E, R = 4.6 × 10^{-13} cm, and various values of the parameter ζ of the imaginary part of the potential: curve 1 – ζ = 0.10; 2 – ζ = 0.25; 3 – $\xi_2 \to \infty$. The lower curves correspond to scattering by a black nucleus.

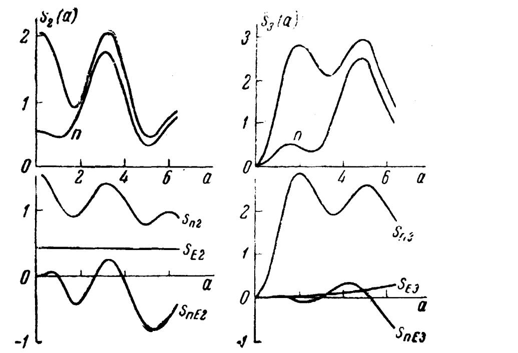

FIG. 2. The functions $s_\lambda(a)$, $s_{n\lambda}(a)$, $s_{E\lambda}(a)$, λ = 2 and 3, describing the angular distribution of protons inelastically scattered by Gd^{156}(E_p = 40 MeV, ξ_1 = 9.8, ζ = 0.10; η = 1.60). The calculation was made for V_0 = E, R = 7.0 × 10^{-13} cm. Angular distributions $s_\lambda(a)$ of inelastically scattered neutrons (n) are plotted for comparison.

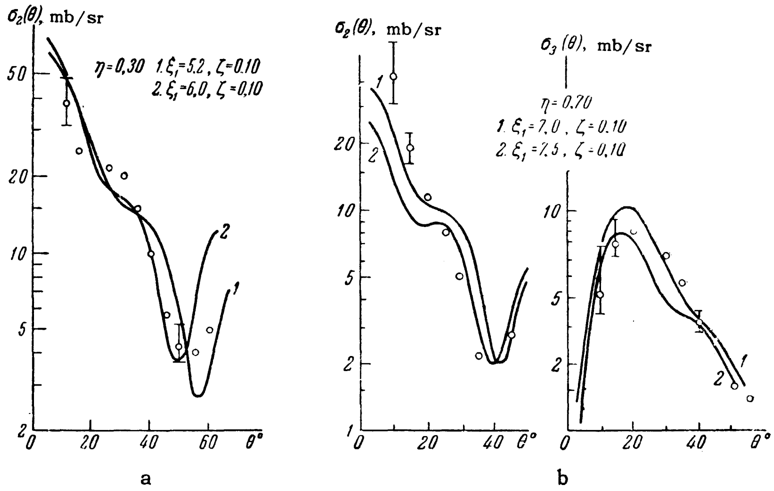

FIG. 3. Differential cross section $\sigma_2(\theta)$ and $\sigma_3(\theta)$ of inelastic scattering of protons (E = 40 MeV) on Mg^{24}(a) and Ni^{58}(b) with excitation of the first collective levels. The circles denote Hintz's experimental data. The calculations (solid curves) have been made for V_0 = E, ζ = 0.10; R(Mg^{24}) = 3.7 × 10^{-13} (curve 1) and R(Mg^{24}) = 4.3 × 10^{-13} cm (curve 2); R(Ni^{58}) = 5.0 × 10^{-13} (curve 1) and R(Ni^{58}) = 5.3 × 10^{-13} cm (curve 2). Comparison of the theoretical and experimental data yields the nonsphericity parameters α_2 = 0.19-0.24 for Mg^{24} and α_2 = 0.09-0.11, α_3 = 0.07-0.09 for Ni^{58}.

protons and neutrons differ quite strongly in the region of the first diffraction maximum.

A comparison of the calculations with Hintz's experimental data (private communication) on the inelastic scattering of protons E = 40 MeV on Mg^{24} and Ni^{58} indicates that the nuclei are quite transparent to protons at this energy (Fig. 3). The best agreement between the calculated data and the experimental data is obtained for $\zeta \approx 0.1$. Comparison by means of formula (19) gives an estimate of the deformation parameter of Mg^{24}, namely $\alpha_2 = 0.19-0.24$, which coincides with the results obtained in the analysis[3] of the experimental data by Watters[10] on the scattering of α particles by Mg^{24}. An analogous analysis for Ni^{58} yields $\alpha_2 = 0.09-0.11$ and $\alpha_3 = 0.07-0.09$.

The author is grateful to V. K. Saul'ev and M. M. Egorova, and also to P. Naur for programming and operating the computer. The author is grateful to B. T. Geĭlikman, O. Winther, and G. E. Brown for a discussion of the work.

[1] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Gostekhizdat, 1948, p. 184 (Engl. Transl., Pergamon, 1958).

[2] A. I. Akhiezer and A. G. Sitenko, JETP **32**, 794 (1957), Soviet Phys. JETP **5**, 652 (1957).

[3] S. I. Drozdov, JETP **36**, 1875 (1959), Soviet Phys. JETP **9**, 1335 (1959).

[4] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

[5] E. V. Inopin, JETP **30**, 210 (1956), Soviet Phys. JETP **3**, 134 (1956).

[6] G. Corman, Phys. Rev. **125**, 359 (1962).

[7] E. V. Inopin, JETP **31**, 901 (1956), Soviet Phys. JETP **4**, 764 (1957).

[8] J. S. Blair, Phys. Rev. **115**, 928 (1959).

[9] S. I. Drozdov, JETP **34**, 1288 (1958), Soviet Phys. JETP **7**, 889 (1959).

[10] H. J. Watters, Phys. Rev. **103**, 1763 (1956).

Translated by J. G. Adashko
57