RESIDUAL PAIR FORCES IN THE LIGHT NUCLEI O^{18} *AND* F^{18}

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The residual forces between two nucleons in light nuclei are determined. The energy levels and transition probabilities between them are calculated. The results are compared with the available experimental data.

INTRODUCTION

SEVERAL published papers[1] are devoted to a consistent quantum-mechanical description of the lower excited nuclear levels and to an explanation of the character of the residual pair forces between the nucleons in a nucleus. The spectra of the nuclei in the lead region were calculated and the role played by residual forces in the formation of the observed variety in the level properties has been demonstrated. In the present paper we carry out an analogous program for light nuclei. By now enough experimental data on the nuclear levels near O^{16} have been accumulated to permit such an investigation.

The basic assumption made in such a description is that the nucleons in excess of the filled shells move in a potential

$$V = V_C + V_S + V_p. \tag{1}$$

The central potential V_C is defined as the averaged local potential produced by all the nucleons of the nucleus. We can write it in rather general form as

$$V_C = V(r) - \lambda\left(\frac{\hbar}{2Mc}\right)^2 \frac{1}{r}\frac{\partial V(r)}{\partial r}\mathbf{ls};\ V(r) = -V_0/(1+e^{\alpha(r-r_0)}). \tag{2}$$

The parameters α, λ, and V_0 are determined by calculating the energies of the nuclear levels with one nucleon in excess of the filled shell[2]. In this case, indeed, the lower levels are connected with the transition of one external nucleon to a higher excited state. The levels connected with the excitation of the nucleons from the core lie high and their effect on the lower-line levels can be neglected.

The potential V_S is brought about by the quadrupole part of the interaction between the external nucleon and the core nucleons. We can assume as a convenient form for V_S the interaction between the nucleon and the surface oscillations of the core

$$V_S = -\varkappa(r)\sqrt{\frac{\hbar\omega}{2C}}\sum_{\mu}(b_\mu + (-1)^\mu b^+_{-\mu})\,Y_{2\mu}(\vartheta, \varphi). \tag{3}$$

The parameters $\hbar\omega$, C, and κ were determined previously[3] from calculations of the quadrupole moments and of the E2-transition probabilities in nuclei with one outer nucleon. Owing to the strong interaction, the introduced surface oscillations with energy $\hbar\omega$ cannot be observed independently. The form proposed by V_S by Guman[3] permits account to be taken of the nonsphericity of the potential for particles in excess of the filled shells.

When the nucleus has two outer nucleons, a potential V_p, resulting from the forces that remain after the averaging, comes into play. This involves essentially pair forces which act when two nucleons are close to each other. The residual pair forces also depend on the ordinary spin and the isospin of the particles. We choose V_p in the form

$$V_p(1,2) = -[(1-\xi\tau_1\tau_2)\,v_s\pi_s + (1-\eta\tau_1\tau_2)\,v_t\pi_t]\exp(-r_{12}^2/\rho^2). \tag{4}$$

Here v_t and v_s are the parameters of the triplet and the singlet interactions; $\pi_t = \frac{1}{4}(3+\sigma_1\cdot\sigma_2)$; $\pi_s = \frac{1}{4}(1-\sigma_1\cdot\sigma_2)$; ρ is the effective interaction radius. The parameters ξ and η determine the dependence of the forces of the isospin T.

Since the parameters entering into V_C and V_S have been selected previously[2,3], it remains to determine the parameters of the potential V_p.

SINGLE PARTICLE LEVELS. PAIR BINDING ENERGY

The nuclei O^{17} and F^{17} have three even levels. The energies of these levels were calculated previously[2,3]. After subtracting the energy of inter-

action with the surface of the core, which is not the same for the different levels, we obtain the following energies for the single-particle levels of O^{17}: -3.4, -1.8, and $+1.7$ for the levels $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ respectively (all the energies and energy parameters are given in MeV throughout). The spectrum of F^{17} is the same: the only difference is that the depth of the levels is lower by the amount of the Coulomb energy, equal to 3.5 MeV[1].

In O^{18} the levels are raised somewhat compared with their positions in O^{17}. This follows from formula (8) of the paper by Volchok and one of the authors[2], if it is recognized that it is O^{17} which lies on the stability curve. The approximate value obtained for the single-particle energy of the nucleon at the $d_{5/2}$ level of O^{18} is -2.8. The energy of interaction between the outer nucleon and the core was determined in [3] and found to equal 0.8. The energy of joining of a neutron to the O^{17} nucleus is $O^{17} + n = O^{18} + 8.07$. We can now find the pair binding energy due to the potential V_p: $8.07 - 2.8 - 0.8 \approx 4.5$. From the experimental data on β decay and on the neutron and proton binding energies in the F^{18} nucleus[4] it follows that in the given nucleus the average single-particle energy and the pair binding energy at the 0_1^+ level are approximately the same as in the ground state of O^{18}.

We shall use these data in the calculations that follow.

COMPUTATION METHOD

The computation method was described in detail in earlier papers[1], and only its gist will be repeated here.

The nuclei O^{18} and F^{18} are regarded as consisting of an O^{16} core plus two nucleons. The total Hamiltonian on the system is written in the form

$$H = H_S + H_p(1) + H_p(2) + V_S(1) + V_S(2) + V_p(1.2);$$
$$H_S = \hbar\omega\left(\frac{5}{2} + \sum_\mu b_\mu^+ b_\mu\right), \quad H_p = \frac{\hbar^2}{2M}\Delta_p + V_C, \qquad (5)$$

V_C, V_S, and V_p are defined by (2), (3), and (4). We seek the eigenfunctions of the Hamiltonian (5) in the form of an expansion

$$|\nu TIM\rangle = \sum_\alpha c^{TI}(\alpha)\,|\nu j_1 j_2 JTM_T, NR, IM\rangle, \qquad (6)$$

where the basis functions describe the vector addition of the total momentum J of two nucleons to the total momentum R of N phonons and the resulting momentum I with projection M.

The energy matrices were set up for different momenta I. The self-energies of the Hamiltonian (5) and the coefficients $C^{TI}(\alpha)$ were determined by diagonalizing the matrices. The known data on the levels of O^{18} and F^{18} greatly exceed the number of parameters to be determined. In addition, different levels and transition probabilities exhibit different sensitivities to individual parameters.

All this enables us to determine the sought parameters reliably from a comparison of the calculated relative energies with the experimental spectrum. In order to obtain the first two or three levels with given I accurate to ~ 0.1, we took matrices of order 20—24. Account was taken of the nucleon configurations 1d2s, and also of single-, two-, and three-phonon excitations.

We present below the basic formulas for the calculation of the energy matrix elements. The matrix elements of each interaction have the form

$$\langle j_1 j_2 JT, NR, IM\,|V_p|\,j_1' j_2' J'T', N'R', I'M'\rangle$$
$$= \delta_{JTNRIM,\,J'T'N'R'I'M'}\,NN'$$
$$\times\Big\{v_0(-1)^{j_1+j_2'-J}\Big[\sum_k (2k+1)F_k(l_1l_2l_1'l_2')\langle l_1j_1\|\mathbf{T}_k\|l_1'j_1'\rangle$$
$$\times\langle l_2j_2\|\mathbf{T}_k\|l_2'j_2'\rangle W(j_1j_2j_1'j_2';\,Jk)$$
$$+(-1)^{T+J-1}\sum_k(2k+1)G_k(l_1l_2l_1'l_2')\langle l_1j_1\|\mathbf{T}_k\|l_2'j_2'\rangle$$
$$\times\langle l_2j_2\|\mathbf{T}_k\|l_1'j_1'\rangle W(j_1j_2j_2'j_1';\,Jk)\Big] + v_1(-1)^{l_1+l_2'-J}$$
$$\times(l_1^{1}/_{2}j_1;\,l_2^{1}/_{2}j_2;\,J0J)\times(l_1'^{1}/_{2}j_1';\,l_2'^{1}/_{2}j_2';\,J0J)\Big[\sum_k(2k+1)$$
$$\times F_k(l_1l_2l_2'l_1')\langle l_1\|\mathbf{T}_k\|l_1'\rangle\langle l_2\|\mathbf{T}_k\|l_2'\rangle\times W(l_1l_2l_1'l_2';\,Jk)$$
$$+(-1)^{T+J-1}\sum_k(2k+1)G_k(l_1l_2l_2'l_1')\langle l_1\|\mathbf{T}_k\|l_2'\rangle$$
$$\times\langle l_2\|\mathbf{T}_k\|l_1'\rangle W(l_1l_2l_2'l_1';\,Jk)\Big]\Big\};$$
$$N = (1+\delta_{l_1j_1,\,l_2j_2})^{-1/2}\left(1 - \frac{1+(-1)^{T+J}}{2}\delta_{l_1j_1,\,l_2j_2}\right),$$
$$v_0 = v_t^{(T)}, \quad v_1 = v_s^{(T)} - v_t^{(T)}. \qquad (7)$$

The matrix elements of the operator of interaction with the surface were calculated from the formula

$$\langle j_1 j_2 JT, NR, IM\,|V_S|\,j_1' j_2' J'T', N'R', I'M'\rangle$$
$$= \delta_{TIM,\,T'I'M'}\delta_{N,N'+1}K(-1)^{J+R'-I-1}$$
$$\times\langle j_1j_2JT\|\mathbf{Y}_2\|j_1'j_2'J'T'\rangle\langle NR\|\mathbf{b}_2\|N'R'\rangle$$
$$\times W(JRJ'R';\,I2),$$
$$\langle j_1j_2JT\|\mathbf{Y}_2\|j_1'j_2'J'T'\rangle = NN'(-1)^{j_1'-j_2-J'}$$
$$\times\sqrt{(2J+1)(2J'+1)}\times\{\delta_{l_1j_1,\,l_1'j_1'}W(j_2Jj_2'J';\,j_12)$$
$$\times\langle l_2j_2\|\mathbf{Y}_2\|l_2'j_2'\rangle + \delta_{l_1j_1,\,l_2'j_2'}(-1)^{J'+T+1}W(j_2Jj_1'J';\,j_12)$$

[1] We neglect the small changes in the distances between levels, resulting from the decrease in their depth.

$$\times\langle l_2 j_2 \| Y_2 \| l_1' j_1' \rangle + \delta_{l_2 j_2, l_1' j_1'} (-1)^{J+T+1} W(j_1 J j_2' J'; j_2 2)$$
$$\times \langle l_1 j_1 \| Y_2 \| l_2' j_2' \rangle + \delta_{l_2 j_2, l_2' j_2'} (-1)^{J+J'} W(j_1 J j_1' J'; j_2 2)$$
$$\times \langle l_1 j_1 \| Y_2 \| l_1' j_1' \rangle. \quad (8)$$

The notation is standard[1] and $K \approx 40$ MeV.

SPECTRUM OF O^{18}

The nucleus O^{18} consists of a magic core and two neutrons. The lower levels have spins I = 0, 2, and 4. The calculations were made for the aforementioned momenta and isospin T = 1. In this case the potential V_p of the pair interaction contains three parameters: $v_t^{(1)} = (1-\eta)v_t$, $v_s^{(1)} = (1-\xi)v_s$, and ρ. An investigation has shown that experiment agrees[4,5] with the results obtained for $1.5 \le \rho \le 2.0$ (ρ is in Fermis) if a set of the parameters $v_t^{(1)}$ and $v_s^{(1)}$ is found for each value of ρ from this region. The best agreement was obtained for $\rho = 1.5$, $v_t^{(1)} = 40$, and $v_s^{(1)} = 65$; the agreement was somewhat worse for $\rho = 2.0$, $v_t^{(1)} = 25$, and $v_s^{(1)} = 50$. Both sets give approximately the same pair binding energy, amounting to ~ 4.3.

Table I lists some of the calculated energies of levels with different parameters.

We can refine the parameters $\hbar\omega$ and C further. Figure 1 shows the variation of the three lower levels as a function of C and $\hbar\omega$. The distance 0_1—4_1 differs greatly from the experimental value if C > 200 and C < 100. Approximately correct values of the relative energies of the three levels 0_1—2_1—4_1 are obtained for $\hbar\omega < 3$. However, the upper levels 0_2 and 2_2 are the most sensitive to the parameters of interaction with the surface. They shift by 1 MeV and more relative to 0_1 when $\hbar\omega$ changes from 1.8 to 3. The spectrum shown in Fig. 2 was obtained for $\hbar\omega = 1.9$ and C = 135. These values lie within the same limits as obtained in the investigation of the levels of O^{17}[3]. Table II lists the main (≥ 0.01)

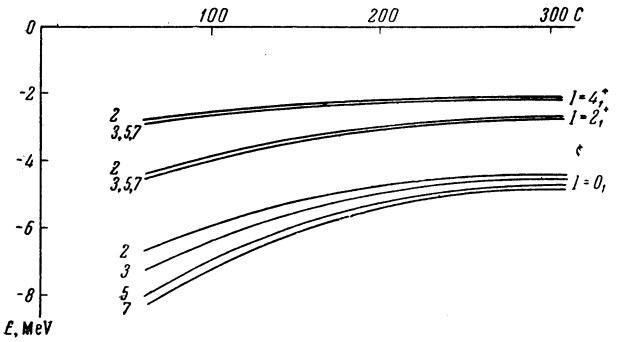

FIG. 1. Dependence of the energy of the lower levels of O^{18} on $\hbar\omega$ and C; the curves are marked with the values of $\hbar\omega$.

Table I. Energies of the lower levels of O^{18}, calculated for different parameters

	0_1	0_2	2_1	2_2	4_1
$V_0 = -60$, $V_1 = -20$, $\hbar\omega = 1.9$, $\rho = 1.5$, $C = 100$	—6,92	—3,13	—4,77	—2,6	—1,95
$V_0 = -50$, $V_1 = -30$, $\hbar\omega = 1.9$, $\rho = 1.5$, $C = 150$	—6,22	—2,63	—4,19	—2,16	—1.58
$V_0 = -50$, $V_1 = -40$, $\hbar\omega = 1.9$, $\rho = 1.5$, $C = 200$	—6.48	—2.80	—4.35	—2,41	—1,76
$V_0 = -20$, $V_1 = -23$, $\hbar\omega = 2$, $\rho = 2$, $C = 100$	—6.41	—2.41	—4,35	—2,33	—2,27
$V_0 = -25$, $V_1 = -25$, $\hbar\omega = 2$, $\rho = 2$, $C = 150$	—6,50	—2.62	—4,45	—2,41	—2,41
$V_0 = -20$, $V_1 = -20$, $\hbar\omega = 3$, $\rho = 2$, $C = 150$	—5,85	—0.169	—3.63	—1.5	—2,01
$V_0 = -20$, $V_1 = -30$, $\hbar\omega = 3$, $\rho = 2$, $C = 100$	—6.72	—1.49	—4.21	—2.00	—2,22
$V_0 = -50$, $V_1 = -30$, $\hbar\omega = 2$, $C = 100$	—6,91	—2,93	—4.66	—2,42	—1,90
$V_0 = -50$, $V_1 = -30$, $\hbar\omega = 2$, $C = 200$	—5.91	—2,23	—3,88	—2,05	—1,56
$V_0 = -50$, $V_1 = -30$, $\hbar\omega = 0$	—4,40		—2.60		—0,83

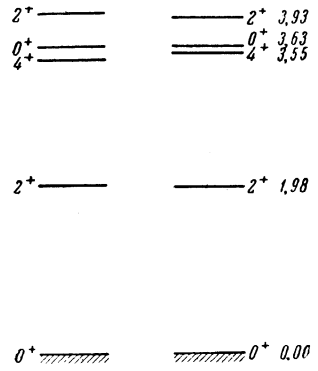

FIG. 2. Energy spectrum of O^{18}: left – calculated, right – experimental.

Note added in proof (December 6, 1962). In view of the corrections introduced in the calculated values of the 4^+ level, it should be situated between the levels 0^+ and 2^+.

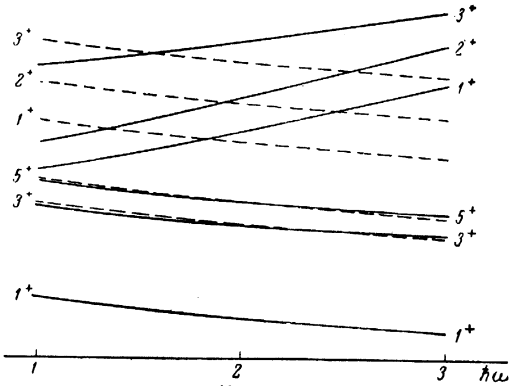

FIG. 3. Dependence of the F^{18} level energies on $\hbar\omega$. The dashed lines (numbers on the left side) show the experimental energies relative to the ground level 1_1^+, while the solid lines (numbers on the right) show the calculated values.

expansion coefficients of the functions for the first levels. Attention must be called to the large admixture of phonon states.

The probabilities of transitions between the levels of the spectrum are given in Table III. They were calculated, as usual, by means of the formula

$$T(m_\lambda) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}\left(\frac{\omega}{c}\right)^{2\lambda+1}|\langle I\|m_\lambda\|I'\rangle|^2, \quad (9)$$

where the reduced matrix element represents a sum of the form

$$\langle I\|m_\lambda\|I'\rangle = \sum_{\alpha,\alpha'} c^{TI}(\alpha)\, c^{T'I'}(\alpha')\,\langle \alpha TI\|m_\lambda\|\alpha' T'I'\rangle. \quad (10)$$

THE SPECTRUM OF F^{18}

The neutron and proton in F^{18} form levels with T = 0 and T = 1. The levels with T = 1 have the same one-particle nature as in O^{18}. Therefore the results of the calculations for O^{18}, corrected for the Coulomb energy, can be applied in their entirety to the levels of the F^{18} spectrum. The 1.043 level is known to have $I = 0^+$, T = 1. One of the levels, 3.06 or 3.13, has $I = 2^+$, T = 1; this is most likely to be the 3.06 level [6]. Further, at a height of ~ 6 MeV, there should be a level $I = 4^+$, T = 1. The spectrum of F^{18} contains, up to an excitation energy 3 MeV, at least six levels with T = 0 [6], including the ground level I = 1. In choosing the parameters it is necessary to obtain in addition to the relative level energies also the binding energy of the two nucleons, which is equal to ~ 5.5, and the relative distance between the isotopic bands (1.04).

Let us put $v_t^{(0)} = (1+3\eta)v_t$ and $v_s^{(0)} = (1+3\xi)v_s$. As in the case of O^{18}, these parameters were varied over a wide range ($-120 \le v_t^{(0)}, v_s^{(0)} \le 120$)

Table II. Coefficients of main components of the wave functions of the O^{18} and F^{18} levels

Nucleus		O^{18}		F^{18}		Nucleus		O^{18}		F^{18}	
IT		0,1	2,1	1,0	3,0	IT		0,1	2,1	1,0	3,0
$j_1 j_2 J$	NR	$c^{IT}(\alpha)$				$j_1 j_2 J$	NR	$c^{IT}(\alpha)$			
$5/2^2$ 0	00	0,839				$5/2^2$ 2	12	−0,366			
$5/2^2$ 1	00			0.750		$5/2^2$ 3	12			−0.262	−0,140
$5/2^2$ 2	00		−0.508			$5/2^2$ 4	12		0,234		
$5/2^2$ 3	00				0,464	$5/2^2$ 5	12				−0.368
$1/2^2$ 0	00	0,175				$1/2^2$ 0	12		0.136		
$1/2^2$ 1	00			0.193		$1/2^2$ 1	12				−0.153
$5/2\ 1/2$ 2	00		−0,271			$5/2\ 1/2$ 2	12	−0,263			
$5/2\ 1/2$ 3	00				0,488	$5/2\ 1/2$ 3	12			−0,200	−0,222
$5/2\ 3/2$ 1	00			−0.451		$5/2\ 3/2$ 1	12				0,270
$5/2\ 3/2$ 3	00				−0.112	$5/2^2$ 0	20	0.133			
$3/2^2$ 0	00	0,118				$5/2^2$ 2	22		−0.161		
$5/2^2$ 0	12		0,734			$5/2^2$ 5	22				0.134
$5/2^2$ 1	12			0.207	−0,420	$5/2\ 1/2$ 2	22		−0.109		
						$5/2^2$ 4	24	0,117			
						$5/2^2$ 5	25				0,110

Table III. Probabilities of electromagnetic transitions in the spectra of O^{18} and F^{18}

Nucleus	IT	$I'T'$	Type of transition	ΔE, MeV	Transition probability, 10^{10} sec		
					$C = 100$	$C = 150$	Experiment
O^{18}	$2_1 1$	$0_1 1$	$E2$	1.98	9.1	6.1	23 ± 8
	$0_2 1$	$2_1 1$	$E2$	1.65	1.7	1.2	32 ± 9
	$2_2 1$	$0_1 1$	$E2$	3.93	47	38	$\geqslant 0{,}23 \cdot 10^3$
		$2_1 1$	$M1$	1.95	$0{,}4 \cdot 10^3$	$2.2 \cdot 10^3$	
		$2_1 1$	$E2$	1.95	9.6	26	
		$0_2 1$	$E2$	0.30	$4{,}9 \cdot 10^{-7}$	$4{,}9 \cdot 10^{-7}$	
F^{18}	$3_1 0$	$1_1 0$	$E2$	0.94	0.27	0.16	$\geqslant 0.02$
	$0_1 1$	$1_1 0$	$M1$	1.04	$1.3 \cdot 10^4$	$1.4 \cdot 10^4$	$0.035 \cdot 10^4$
	$5_1 0$	$3_1 0$	$E2$	0.18	$0{,}91 \cdot 10^{-4}$	$0.46 \cdot 10^{-4}$	$(5{,}3 \pm 0{,}9) \cdot 10^{-4}$
	$1_2 0$	$1_1 0$	$M1$	1.70	384	276	35 ± 17
		$1_1 0$	$E2$	1.70	5.5	3.7	
		$3_1 0$	$E2$	0.76	$4{,}5 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$	
		$0_1 1$	$M1$	0,66	35	26	
	$2_1 0$	$1_1 0$	$M1$	2.10	14	5.5	87 ± 33
		$3_1 0$	$M1$	1,16	50	55	
		$0_1 1$	$E2$	1,06	$6{,}3 \cdot 10^{-5}$	$0.53 \cdot 10^{-5}$	
	$3_2 0$	$1_1 0$	$E2$	2,53	56	36	58 ± 19
		$3_1 0$	$M1$	1.59	5,9	3,8	
		$5_1 0$	$E2$	1,41	0.77	0,46	
		$1_2 0$	$E2$	0,83	0.052	0,031	
		$2_1 0$	$M1$	0.43	1.0	1.5	

for different values of ρ. $\hbar\omega$ and C were also varied.

It was established that when $\rho \approx 2$ it is impossible to obtain simultaneously the correct distances between the 1_1—3_1—5_1 levels and the binding energy. In addition, the energy of the second level 1_2 was found to be almost 1 MeV larger than the experimental one. Only by decreasing ρ was it possible to improve the agreement, the best results being obtained for $\rho \approx 1.5$. Using the fact that the distances between the 1_1—3_1—5_1 levels are practically independent of $\hbar\omega$ and C (see Fig. 3), we could use them to determine the parameters $v_t^{(0)}$ and $v_s^{(0)}$, which turned out to be $v_t^{(0)} = 110$ and $v_s^{(0)} = 80$ for $\rho = 1.5$.

The positions of the higher levels are sensitive to the variation of $\hbar\omega$, since the corresponding states represent a strong mixture of one-particle and phonon states. It is seen from Fig. 3 that when $\hbar\omega = 3$ not a single calculated level remains in the energy interval between 2 and 3 MeV, whereas it actually contains three levels. When $\hbar\omega$ is decreased to 1.9, the calculated levels lie very near the corresponding experimental ones. The choice of $\hbar\omega \approx 2$ and $C \approx 150$ can be justified by the same independent method.

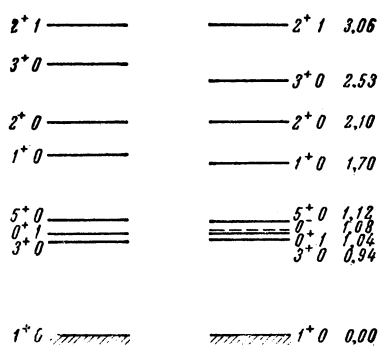

FIG. 4. Energy spectrum of F^{18}: left – calculated, right – experimental.

Figure 4 shows for comparison the calculated spectrum (with parameters $v_t^{(0)} = 110$, $v_s^{(0)} = 80$, $\hbar\omega = 1.9$, $C = 135$) and the experimental spectrum of F^{18} up to an excitation energy of 3 MeV. All the calculated levels are close to the experimental ones. The spins and parities of the calculated levels confirm the correctness of the identification made by Kuehner, Almqvist, and Bromley [6]. The spins and parities of the two levels 2.10 and 2.53 were assigned (not very reliable) values $2^+(3^+)$ and $3^+(2^+)$. Our calculations agree with the first variant, i.e., 2^+ for the level 2.10 and 3^+ for the level 2.53 MeV. The identification of higher levels is made difficult by their high density. In addition, the error in the calculated energies increases for the high levels.

To obtain the transition probabilities it was necessary to calculate the sum (10). Inasmuch as the sum alternates in sign, the probabilities can be both larger and smaller than the one-particle values. In addition, the probabilities of the E2 transitions in F^{18} are mixtures of nucleon and collective transitions. An account of the collective part sometimes intensifies the transitions by a factor 20—100. The value of the sum (10) is very sensitive to changes in the interaction parameters, particularly when the main terms in the sum have opposite signs. We did not attempt,

as in the case O^{18}, to attain exact agreement with the experimental data for the transition probabilities, since the latter are not so reliable.[5,6] The results listed in Table III have been obtained with the same parameters, for which better agreement with the experimental spectra exists. Only in the case of the M1 transition $0^+1 \rightarrow 1^+0$ doe we obtain $T_{theor}/T_{exp} \sim 40$. In the remaining cases the probabilities of the γ transitions coincide in order of magnitude or even better. The relative transition probabilities are likewise in satisfactory agreement. In particular, the transitions $1_2 \rightarrow 3_1$, $3_2 \rightarrow 5_1$, $3_2 \rightarrow 1_2$, and $3_2 \rightarrow 2_1$ are much weaker than the remaining transitions from the 1_2 and 3_2 levels, as also follows from the experimental data.

DISCUSSION OF RESULTS

The residual pair forces in light and heavy nuclei differ both in magnitude and in effective radius. Whereas in heavy nuclei the values obtained for the parameters of the potential V_p are $v_t = 15$, $v_s = 35$, and $\rho = 2$, for light nuclei with $T = 1$ we have $v_t^{(1)} = 40$, $v_s^{(1)} = 70$, and $\rho = 1.5$. After reducing to a single parameter $\rho = 2$ we get $v_t^{(1)} = 25$ and $v_s^{(1)} = 50$. Such a difference could be expected from the most general considerations. The self-consistent potential in the heavy nucleus encompasses more completely the forces acting on the nucleon, so that the residual forces in this nucleus are smaller. The effectiveness of the residual forces, which can be characterized by the magnitude of the matrix elements, increases much more in light nuclei than the forces themselves. In Table IV are listed for comparison the values of some matrix elements in light and heavy nuclei.

In the light nuclei we investigated the dependence of the residual forces on the isospin. The data obtained for the parameters $v^{(0)}$ and $v^{(1)}$ enable us to determine the quantities η and ξ which enter into (4). The final expression for the pair interaction in light nuclei has the form [2)]

$$V_p = -\{70\pi_s + 55\,(1 - 0{,}23\tau_1\tau_2)\,\pi_t\}\exp\{-r_{12}^2/(1{,}5)^2\}. \quad (11)$$

The triplet forces in the case $T = 0$ are approximately twice as large as for $T = 1$, while the singlet forces do not depend on the isospin. The inaccuracy introduced in the determination of the parameters is connected principally with the inaccuracy in the determination of the one-particle energies. However, the possible corrections to the one-particle energies cannot change the parameters V_p by more than 10—15%. Indeed, by determining the parameters from the distances between levels, produced the splitting of the main configuration (for example, $d_{5/2}^{(2)}$) we reduce the influence of the one-particle energies to a secondary effect, the action produced on these levels by the nondiagonal matrix elements.

Table IV. Comparison of matrix elements of residual forces in light and heavy nuclei

$\langle j_1 j_2 J \mid V_p \mid j_1' j_2' J \rangle$	Light nuclei	Heavy nuclei
$\langle d_{5/2}^2\, 0 \mid V_p \mid d_{5/2}^2\, 0 \rangle$	4.2	1,1
$\langle d_{5/2}^2\, 0 \mid V_p \mid s_{1/2}^2\, 0 \rangle$	1.1	0,5
$\langle s_{1/2}^2\, 0 \mid V_p \mid s_{1/2}^2\, 0 \rangle$	1.4	0,7
$\langle d_{5/2}^2\, 2 \mid V_p \mid d_{5/2}^2\, 2 \rangle$	2,1	0.5
$\langle d_{5/2}\, s_{1/2}\, 2 \mid V_p \mid d_{5/2}\, s_{1/2} 2 \rangle$	1.8	0.6

Worthy of attention is the calculation of the γ transitions, found in Table III. The probabilities of the transitions $T(\lambda)$ between different levels change from 10^{-5} to 10^4 (in the assumed units), i.e., by nine orders of magnitude. If we exclude the energy dependence and consider only the reduced probabilities $B(m_\lambda)$, they change by approximately 10^3 times. In spite of such wide a range of variation of the magnitude of $B(m_\lambda)$, theory yields results that are comparable with experiment.

The interaction between the nucleons and the core is strongest in light nuclei. The nondiagonal matrix elements of the operator V_S are 3—4 times their values in heavy nuclei. This increase is principally at the expense of reducing the effective surface tension C from 2000 to 150. Naturally, the "stiffness" of the densely compacted multi-nucleon core of Pb^{206}, for example, is greater than the stiffness of the core of O^{16}. The sensitivity of the results to the parameters $\hbar\omega$ and C also increases, and whereas the data corresponding to $\hbar\omega = 1.8$—2.0 are close to the experimental ones, a considerable discrepancy is already observed for $\hbar\omega = 3$. In essence, all the considered levels of O^{18} and F^{18} have an approximately equal mixture of particle and phonon configurations, and a noticeable contribution is made by the two- and three-phonon states.

The spectra of O^{18} and F^{18} were calculated by various authors. However, in the framework of the shell model with specified pair forces[7], and also in the framework of the deformed-nucleus

[2)]This yields the following values for the exchange-force parameters: W = 70, H = 15, M = 15, B = 0.

model[8], it was possible to obtain at best the correct order of the levels. The binding energy and the distances between levels disagreed with experiment, and the transition probabilities were not calculated at all. The results obtained in the present work, which are so complete and in such good agreement with experiment, speak in favor of the effectiveness of the method employed. It can be expected that the known levels and their properties will be obtained for more complicated nuclei, say Ne^{20}, with the same potential (1) and with the same parameters.

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