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SLOWING DOWN OF PROTONS AND μ^+ MESONS IN METALS

B. M. SMIRNOV

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The slowing down of a heavy positive singly-charged particle in a dense low-temperature plasma due to elastic collisions with electrons is investigated. The stopping power is calculated by means of Green's functions and the diagram method for $v \ll v_0$ (v is the velocity of the slowed-down particle and v_0 is the electron velocity on the surface of the Fermi sphere). As in the Fermi-Teller case, the change in particle energy per unit time is proportional to v^2. The possibility of applying the dense-plasma model to the calculation of the stopping power of a real metal is discussed. The calculated stopping power is compared with experimental data. The μ^+-meson slowing-down time in metals is estimated.

1. The calculation of the stopping power of condensed media presents a complicated mathematical problem because the interactions of many bodies must be taken into account. Therefore a solid must be represented by several models, each of which distinguishes the most important interaction mechanism in some particular case.

In studying the motion of positively charged particles in metals we make use of the fact that the particles are slowed down mainly by interacting with electrons and that the electron spectrum is continuous. For this reason a metal can be represented suitably by a plasma model, in which electrons are considered free with uniform density equal to the mean electron density in the metal; the positive charge is smeared out uniformly in space.

The properties of the plasma can be used to determine the stopping power on this model. However, the determination of these properties is a many-body problem which therefore encounters great mathematical difficulties in the general case. The problem is simplified when for the electron density we have $n \gg 1$ (using the atomic units $\hbar = m_{el} = e^2 = 1$). Green's functions can be used to calculate the properties of a dense plasma by summing the most highly divergent perturbation-theory diagrams.[1] It can be expected that the small parameter of the theory for calculating the stopping power will be $\sim 1/\pi p_0$, where p_0 is the electron momentum on the Fermi surface. Therefore the dense-plasma model is suitable for ordinary metals even though in such cases $p_0 = 1.5-2.5$.

The slowing down of particles in a plasma when the Born approximation ($v \gg 1$) is applicable has been studied by Larkin,[2] who derived an expression for the stopping power in terms of a two-particle Green's function. Larkin's result can be used for a dense plasma when $v \ll v_0$.

2. Let us consider a proton (or μ^+ meson) moving in a dense plasma with the a velocity $v \ll v_0$. In this case a proton can form a bound state with electrons; this strongly affects the scattering mechanism. The bound state is a of single-particle character, going over into the hydrogen atom in the limit of low electron density and disappearing in the limit of high electron density. The question as to the existence of proton-electron binding in real metals cannot be answered by comparing the Bohr radius with the Debye-Hückel length ($\sqrt{\pi/4p_0}$), since these are of the same order of magnitude. An answer is obtained from experimental studies of μ^+-meson spin precession in a transverse magnetic field,[3] which show that bound states do not exist in any of the investigated metals (Be, Mg, Al, Cu, etc). Consequently, a proton is slowed down as a result of elastic collisions with electrons.

3. The slowing down of a negatively charged meson in a plasma has been studied by Fermi and Teller.[4] These authors showed that since the energy loss of a particle colliding with an electron is proportional to the particle velocity v and the number of electrons participating in collisions (the electron layer at the Fermi surface) is $\sim v$, the stopping power is $\sim v^2$. In this calculation of the stopping power interactions between electrons were neglected, and transitions due to interactions between a meson and a Fermi-surface electron were considered. The screening effect of other electrons was taken into account in cutting off the derived divergent integral. This cutoff gives a coefficient $[(2/3\pi)\ln p_0]$ of v^2 that is valid for $p_0 \to \infty$. However, for values of p_0 corresponding to real

metals it is important to know the succeeding terms of the expansion, which assume values close to that of the logarithmic term. The Green's-function method, which automatically takes screening into account, is therefore suitable. Since the collision time of a heavy particle with an electron in small-angle scattering is $\sim 1/\sqrt{v_0}$ and the transition frequency is $\sim v/\sqrt{v_0}$, it can be assumed that for $v \ll v_0$ transitions result from two-particle (pair) interactions between the heavier particle and electrons.

The probability amplitude for the transition of a particle from the state $\mathbf{P}_1$ to $\mathbf{P}_1 - \mathbf{q}$, while the medium makes a transition from state n to state m is given by

$$\Big[\sum_{\mathbf{p}} V(\mathbf{P}_1,\ \mathbf{p}-\mathbf{q} \rightarrow \mathbf{P}_1 - \mathbf{q},\ \mathbf{p})\, a^+_{\mathbf{p}} a_{\mathbf{p}-\mathbf{q}}\Big]_{nm},$$

which does not depend explicitly on time, so that the transition probability per unit time is

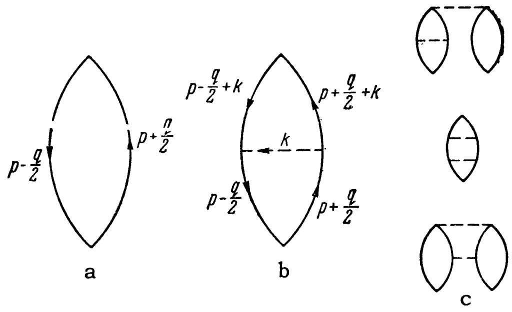
$$W_{\mathbf{q}} = 2\pi \Big| \Big(\sum_{\mathbf{p}} V a^+_{\mathbf{p}} a_{\mathbf{p}-\mathbf{q}}\Big)_{nm} \Big|^2 \delta\,(E_m - E_n + \varepsilon_{\mathbf{P}_1-\mathbf{q}} - \varepsilon_{\mathbf{P}_1}).$$

Here V is the probability amplitude for a transition from $\mathbf{P}_1$ to $\mathbf{P}_1 - \mathbf{q}$ in a collision with a free electron, which undergoes a transition from state $\mathbf{p} - \mathbf{q}$ to $\mathbf{p}$; a^+ and a are the electron creation and annihilation operators; E and ϵ are the energy of the medium and of the particle, respectively.

Since the scattering amplitude in a Coulomb field is identical in the general case and in the Born approximation ($V_{\mathbf{q}} = 4\pi/q^2$),[5] Larkin's result[2] is obtained for the transition probability per unit time. (The two-particle character of the interaction is essential for agreement of the results.) By averaging over the initial states n and summing over the final states m of the medium it becomes possible to express the stopping power of the plasma in terms of a two-particle Green's function for electrons:[2]

$$W_{\mathbf{q}} = \frac{2V_{\mathbf{q}}^2}{1-\exp(-\beta\omega)} \operatorname{Im} \frac{\Pi(\mathbf{q}, \omega)}{1 - V_{\mathbf{q}}\Pi(\mathbf{q}, \omega)}.$$

The term containing $[1-\exp(-\beta\omega)]^{-1}$, where T is the temperature and $\beta = 1/kT$, leads in the present case (where most of the stopping power comes from transitions with $\omega \gg 1/\beta$) to the forbiddenness of transitions with negative values of $\omega = \epsilon_{p_1} - \epsilon_{p_1 - q}$. This forbiddenness follows from the exclusion principle and energy conservation. $\Pi(\mathbf{q}, \omega)$ characterizes the properties of the plasma and is represented by the accompanying diagrams where the conventional solid lines correspond to free-electron Green's functions and the dashed lines correspond to interactions between electrons,



with 4-integration being performed over each vertex from which a dashed line departs. We shall confine ourselves to diagram a, corresponding to the first term in the expansion for a dense plasma. Diagram b is calculated in the Appendix in order to provide a better understanding of the expansion parameter and of the accuracy with which the given model can be used for calculations.

4. In calculating $\Pi^{(0)}(\mathbf{q}, \omega)$, corresponding to diagram a, we use the relation $v \ll v_0$, which means that an electron is scattered from an almost fixed center, resulting in a small angle between the vectors $\mathbf{p}$ and $\mathbf{q}$ (the scattered electrons are close to the Fermi surface). We obtain

$$\Pi(\mathbf{q}, \omega) = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{n(\mathbf{p}+\mathbf{q}/2) - n(\mathbf{p}-\mathbf{q}/2)}{\omega - \mathbf{p}\mathbf{q} + i\delta\,[n(\mathbf{p}+\mathbf{q}/2) - n(\mathbf{p}-\mathbf{q}/2)]}$$
$$= -\frac{p^2}{2\pi^2 p_0}\Big[2 + u \ln\Big|\frac{u-1}{u+1}\Big| + i\pi |u|\Big],$$

where $p = \sqrt{p_0^2 - q^2/4}$ and $u = \omega/pq$; the imaginary part disappears for $u \geq 1$. We note that the principal contribution to the stopping power

$$-\frac{dE}{dt} = \int \frac{dq}{(2\pi)^3}\, \omega W_{\mathbf{q}}$$

comes from transitions with $u = vx/p \ll 1$ (x is the cosine between $\mathbf{q}$ and $\mathbf{v}$). Therefore $\operatorname{Im}\Pi \ll \operatorname{Re}\Pi$ and

$$-\frac{dE}{dt} = \int_0^{2p_0}\int_0^1 \frac{q^2 dq\, dx}{(2\pi)^2} vqx \cdot 2V_{\mathbf{q}}^2 \frac{\operatorname{Im}\Pi}{(1-V\Pi)^2}$$
$$= \frac{2}{3\pi} v^2 (1-\alpha)^{-2} \Big\{\frac{3\alpha/2 + 1}{\sqrt{\alpha+1}} \ln \frac{(1+\sqrt{1+\alpha})^2}{\alpha} - 3\Big\},$$

where $\alpha = 1/(\pi p_0 - 1)$.

Expanding the last expression in the small parameter α, we obtain

$$-\frac{dE}{dt} = \frac{2}{3\pi} v^2 \Big\{\ln\frac{4}{\alpha} - 3 + 3\alpha \ln\frac{4}{\alpha} - \frac{11}{2}\alpha\Big\},$$

so that in the limit $p_0 \rightarrow \infty$ our result agrees with that of Fermi and Teller. We note that the stopping power results mainly from transitions with small q, since a passing particle interacts with electrons at a distance equal to the screening radius.

	$_{47}Ag^{108}$	$_{79}Au^{197}$	$_{13}Al^{27}$	$_{29}Cu^{63,5}$	$_{22}Ti^{47,9}$	$_{32}Ge^{72,6}$	$_{50}Sn^{118,7}$
P_0	2.16	2.59	1.42	2.08	1.66	1.73	1.90
A_{exp}	0.30	0.28	0.23	0.11	0.39	0.26	0.21
A_{theor}	0.24	0.25	0.21	0.23	0.22	0.21	0.22

5. Our calculation based on the present model will be confined to diagram a, thus neglecting interactions between electrons. The calculations in the Appendix show that the result is not appreciably changed when such interactions are taken into account for a plasma having the density of a real metal. However, the plasma model, although convenient because of its simplicity, does not take the complex structure of metals into account. The electron density at the Fermi surface is considered constant (equal to the mean electron density). The non-constant density of states near the ground state of the system leads to violation of the law $dE/dT \sim v^2$. If electrons of many zones participate in transitions, a stopping power proportional to v^2 is obtained by averaging the effects of the different electrons.

The accompanying table compares the theoretical and experimental values [6] of the coefficients in the law $-dE/dt = Av^2$. The theoretical value of A depends only slightly on the particular metal; the agreement can be regarded as satisfactory.

We shall now examine the validity of using the dense-plasma model for a metal in the present problem. The real-metal property used in this model is the electron density. It would be more accurate to refrain from using this mean property but to sum the values of the stopping power calculated at each point of space separately. However, when the electron density is almost constant at the screening radius (where we have the electron density $n \gg 1$) our present procedure is justified. This occurs in a region where the Thomas-Fermi model is valid. Here the motion of electrons can be regarded as semiclassical and their interactions can be neglected; the paired interaction energy per electron is $\sim n^{1/3}$ and the kinetic energy of an electron is $\sim n^{2/3}$. Therefore the effective electron mass m^* ($m^* = p(\partial\epsilon/\partial p)$, p is the electron momentum, ϵ is the electron energy) does not differ much from the real mass. However, a large region exists in which these conditions are violated and the effective mass differs greatly from the real mass; the energy loss is written more correctly as $\omega = m^*qv$ when it is much greater than the separation of adjacent levels. Strong interactions introduce great mathematical difficulties, and since m^* must be the effective mass for regions contributing most of the stopping power the law $dE/dt \sim v^2$ is violated.

At velocities for which the law $dE/dt \sim v^2$ is obeyed the dense-plasma model can therefore be used, because most of the stopping power of the plasma is associated with spatial regions where it is permissible to neglect interactions between electrons. At very low particle velocities the law $-dE/dt = A_{cr}v^2$ will again be obeyed; the coefficient A_{cr}, which is smaller than A, depends on the properties of the crystal.

6. The foregoing results can be used to study the behavior of μ^+ mesons in metals. As long as the relation $dE/dx \gg E\mu/\lambda M$ is fulfilled a μ^+ meson is slowed down by elastic collisions with electrons, and interactions with nuclei accompanied by phonon excitation are insignificant. Here μ and M are the μ^+-meson and nuclear mass, $\lambda \sim 1/\sigma n$ is the μ^+-meson mean free path (so that n is the density of nuclei), and σ is the cross section for the elastic scattering of a meson by an atom as determined from the Thomas-Fermi atomic model. For real metals the foregoing relation is found to be well fulfilled in the energy region $E \gtrsim kT$ ($T \sim 300°$). The time required for μ^+-meson slowing down to thermal energy as calculated from the dense-plasma model is of the order $\sim 10^{-13}$ sec and varies little from metal to metal. The fastest mechanism of μ^+-meson spin depolarization resulting from interactions with electron spin has $\tau \sim 3 \times 10^{-11}$ sec. Thus when a μ^+ meson is slowed down its spin is not depolarized.

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APPENDIX

To investigate the expansion parameter used in calculating the stopping power it is interesting to calculate $\Pi^{(1)}(q, \omega)$ for the case $\omega/p_0q \ll 1$. $\Pi^{(1)}$ is given by the diagram b in the figure:

$$\Pi^{(1)} = \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} V_k G_0\left(p - \frac{q}{2}\right) \times G_0\left(p + \frac{q}{2}\right) G_0\left(p - \frac{q}{2} + k\right) G_0\left(p + \frac{q}{2} + k\right),$$

where $V_k = 4\pi/(k^2 + \kappa^2)$, $\kappa^2 = -\Pi^{(0)}(0,0)$, and the vectors p, q, k have the components $p = (\mathbf{p}, \epsilon)$, $q = (\mathbf{q}, \omega)$, $k = (\mathbf{k}, k_4)$. Integrating with respect to $d\epsilon/2\pi$ and $dk_4/2\pi$, we obtain

$$\Pi^{(1)} = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{d\mathbf{k}}{(2\pi)^3} \frac{n^+ - n^-}{\omega - \mathbf{pq} + i\delta(n^+ - n^-)}$$

$$\times V_{\mathbf{k}} \frac{n_+ - n_-}{\omega - (\mathbf{p}+\mathbf{k})\mathbf{q} + i\delta(n_+ - n_-)},$$

$$n^\pm = n(\mathbf{p} \pm \mathbf{q}/2), \quad n_\pm = n(\mathbf{p} + \mathbf{k} \pm \mathbf{q}/2).$$

We make use of the fact that one of the extreme vertices corresponds to the scattering of an electron by a heavy particle, so that the cosine of the angle between the vectors $\mathbf{p} + \mathbf{k}$ and $\mathbf{q}$ is small; this is associated with the fact that electrons close to the surface of the Fermi sphere are scattered. We note that $\omega \ll (p+k)q$, and obtain

$$\int \frac{d\mathbf{k}}{(2\pi)^3} V_{\mathbf{k}} \frac{n_+ - n_-}{\omega - (\mathbf{p}+\mathbf{k})\mathbf{q} + i\delta}$$

$$= \int \frac{d\mathbf{k}}{(2\pi)^3} V_{\mathbf{k}} \frac{p_0^{-1}(\mathbf{p}+\mathbf{k})\mathbf{q}\delta\left(\sqrt{(\mathbf{p}+\mathbf{k})^2 + q^2/4} - p_0\right)}{\omega - (\mathbf{p}+\mathbf{k})\mathbf{q} + i\delta}$$

$$\approx \frac{1}{8\pi p_0^2 p}\left\{\left(p_0^2 - p^2 - \frac{q^2}{4} + \varkappa^2\right)\right.$$

$$\left.\times \ln\left[1 + \frac{\left(\sqrt{p_0^2 - q^2/4} + p\right)^2}{\varkappa^2}\right] - \left(\sqrt{p_0^2 - q^2/4} + p\right)^2\right\}.$$

A calculation yields $\Pi^{(1)} \lesssim \Pi^{(0)}/2\pi p_0$. A correction of this order takes account of self-energy in $\Pi^{(0)}$. We can therefore assume that the stopping power for the dense-plasma model is computed with a practically sufficient degree of accuracy ($\sim 1/\pi p_0$).

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35