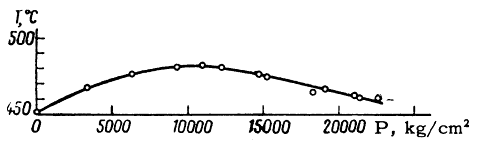
the velocity of seismic waves and the increase of the density at the boundary of the earth's core to such a transition.

Earlier one of the present authors[1] discussed the boundary of the earth's core from the point of view of the transition of the materials in the earth's mantle to the metallic state, and concluded that the melting curve of the nonmetal phase of some substances may have a maximum below the triple point which represents the equilibrium between the melt, the nonmetal phase and the metal phase. It was considered to be of interest to check this hypothesis experimentally.

Tellurium was selected as the test substance. According to Bridgman[2,3] it becomes metallic with a volume decrease of 5.5% at a pressure of about 45,000 kg/cm^2 at room temperature.

Pressure was produced in a superhigh-pressure multiplier with double mechanical support and internal heating.[4] A polysiloxane liquid was used as the pressure-transmitting medium. The pressure was measured with a manganin resistance pressure gauge accurate to ± 100 kg/cm^2. The temperature was measured with a platinorhodium thermocouple accurate to ± 1.5 deg C.

The figure gives the melting curve of tellurium up to 23,000 kg/cm^2. It shows a maximum at 10,800 kg/cm^2 and 482°C.



It is evident that at pressures above 10,800 kg/cm^2 the tellurium melted with a reduction of volume which, in our opinion, is related to the transition of the liquid to the metallic state, accompanied by a change in the short-range order.[1]

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THE EFFECT OF SPIN ON THE POSITION OF REGGE POLES

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RECENTLY Gribov and Pomeranchuk, while studying the singularities of partial wave amplitudes in orbital angular momentum, discovered an accumulation of poles in the l-plane.[1,2] First, there are accumulations at the energies corresponding to the threshold for the production of n particles. They appear at the points $l = -(3n-5)/2$.[1] This phenomenon occurs both in quantum mechanics and in field theory. Second, in field theory as a consequence of the existence of crossed channels there are accumulations at any energy at negative integer angular momenta.[2] All these accumulations impose limitations on the asymptotic behavior in momentum transfer of the amplitude, i.e., in energy in another channel.

In the present work the effect of particle spin on the position of the accumulation points described above is considered and the results are briefly discussed.

1. Consider, for simplicity, the scattering of two neutral spinless particles (pions). In the intermediate state particles with spin may appear, for example nucleon pairs. Near threshold for pair production the production amplitude is proportional to p^l, where p is the momentum of the produced particles and l is their orbital angular momentum. The value of l depends on the orbital angular momentum j of the pions and on the total spin of the produced particles. The minimum value of l is given by $j - 2\sigma$, where σ is the spin of each of the particles. It is easy to see that this value of l is never forbidden by parity selection rules.

It is easy to show by repeating the considerations of Gribov and Pomeranchuk[1] that this dependence on p results in an accumulation of poles in j at the point $j = -\frac{1}{2} + 2\sigma$ when $p \rightarrow 0$. It is clear that this accumulation is present also in the scattering amplitude of the spinless particles, as well as in all other amplitudes connected to it by the unitarity condition.

In precisely the same way the threshold for the production of n particles with the same spin σ gives rise to an accumulation of poles at the point $j = -(3n-5)/2 + n\sigma$.

All known elementary particles have spin less than unity. However nuclei could in principle, in the absence of the Coulomb interaction, have arbitrarily large spin. And even if there is a bound to the values of nuclear spin one can, by taking a sufficiently large number of nuclei of spin larger than $^3/_2$, move the position of the accumulation point at the corresponding threshold energy arbitrarily far to the right.

So far we have not taken into account the identity of the particles. As a consequence of the Pauli principle the minimum value of l for a system of many fermions is larger than $j - n\sigma$. As has been remarked by I. Ya. Pomeranchuk (private communication) this leads to the result that the accumulation points connected with the thresholds of these systems cannot move without bound to the right as $n \to \infty$. However for a system of bosons there is no such limitation.

This means that the number of subtractions in the dispersion relation in momentum transfer increases without bound with increasing energy. Consequently the scattering amplitude has an essential singularity at $t \to \infty$, $s \to \infty$, and therefore the Mandelstam representation is not valid. Nevertheless the principal results obtained with the help of the Mandelstam representation remain, apparently, valid. For example, according to Greenberg and Low,[3] it is not necessary to know the behavior of the amplitude at large momentum transfers in order to obtain the result of Froissart.[4]

2. Let us consider now the accumulation points that are present at any energy.[2] For the sake of definiteness we consider a nucleon-antinucleon intermediate state. The kinematical analysis of the reaction $\pi\pi \to N\bar{N}$ was given by Frazer and Fulco.[5] The partial wave amplitudes for a given value of j are expressed in terms of $A_j(t)$, $B_{j+1}(t)$ and $B_{j-1}(t)$, where A and B are the usual invariant amplitudes and the structure of $A_l(t)$ and $B_l(t)$ is of the type

$$\frac{1}{\pi}\int_{z_0}^{\infty} Q_l(z')\, B^{(1)}(s', t)\, dz'. \quad (1)$$

The quantities $A_l(t)/(pk)^l$ and $B_l(t)/(pk)^l$ have a left and a right cut in the t-plane. The jumps across the left cut have, generally speaking, poles in l at negative integer values.[2] The residue of the pole at $l = -n-1$ is proportional to the integral

$$\int_{-z_1(t)}^{z_1(t)} P_n(z)\, b(s, u)\, dz, \quad (2)$$

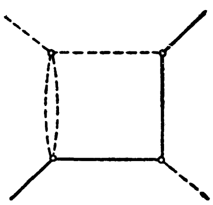
where $b(s,u)$ is the Mandelstam spectral function. The integration is at constant t over a region where $b(s,u) \neq 0$. If the Mandelstam representation is not valid then $b(s,u)$ should be interpreted as the jump $B^{(1)}(s,t)$ for $t < 0$.

As was shown in [2] the pole in the jump across the left cut gives rise to the accumulation of poles in the partial wave amplitude in the l-plane near negative integer points. This comes about as a consequence of unitarity in the t channel. Such an accumulation occurs at any energy.

For the reaction $\pi + \pi \to N + \bar{N}$, generally speaking, such an accumulation occurs not only at the negative integers but also for $j = 0$. These accumulations appear also in the amplitude for $\pi\pi$ scattering and in all the amplitudes connected with it, however the residues of the poles that accumulate at zero are proportional to j. This corresponds to the fact that the state with $l = j - 1$ is excluded for $j = 0$.

The existence of the pole accumulation at zero may be subject to experimental verification. In particular, NN scattering at large energies and not too small angles should be governed by this accumulation. Unfortunately, the experimental data[6] available at this time do not allow an unambiguous resolution of this question.

The accumulation at zero will be absent if the corresponding integral (2) vanishes for all values of t. In the region of not too large negative values of t the contribution to (2) comes essentially only from the diagram shown in the figure. At that the condition for the vanishing of the integral (2) results in an integral relation between the $\pi\pi$ scattering amplitude and the amplitudes A and B for πN scattering. For other values of t this relation would include the amplitudes for multiple processes.



If one now considers the general case of production of a pair of particles of spin σ, then accumulation points are possible at zero and at positive integers up to $j = -1 + 2\sigma$. For $\sigma > 1$ these accumulations would occur at $j = 2$ or even farther to the right. But this would contradict the unitarity condition in the s channel. For $\sigma = 1$ there is no direct contradiction. However it would then be impossible for the diffraction cone to become narrower, in contradiction with the experimental data.[7] One might therefore think that there is no accumulation of poles at $j = 1$ either. The amplitude for two-meson annihilation of particles with spin σ contains

$$N = \tfrac{1}{2}(2\sigma + 1)^2 + \tfrac{1}{4}[1 - (-1)^{2\sigma+1}]$$

invariant functions. In order that the integrals of type (2) vanish it is necessary to impose $(N-1)(N-2)/2$ integral conditions. In order to eliminate the accumulation point at zero the number of conditions must be increased to $N(N-1)/2$. Thus the number of conditions increases rapidly with increasing spin.

3. In the previous sections we have described some of the difficulties that are encountered in relativistic theory when particles of high spin are taken into account. However no elementary particles exist with spin larger than unity. Only nuclei have such spins. And nuclear amplitudes contain anomalies the precise nature of which is not sufficiently understood at the present time.

In order to derive the results of the first section we have made use only of the assumption that the partial wave amplitudes may be continued to complex j with their threshold behavior preserved. This assumption is, apparently, quite generally valid. It is not violated in the presence of the usual real anomalies.

The assumptions that were used in the second section are of a more detailed character. It could happen that when the nuclear structure is taken into account the problem is sufficiently different so that the elimination of the accumulation points requires no additional conditions.

It is interesting to note that as a consequence of gauge invariance the two-photon intermediate state does not give rise to accumulation points at either unity or zero.

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