"OBSERVED" PROBABILITIES OF ELASTIC SCATTERING OF NEUTRONS AND THE MÖSSBAUER EFFECT IN DEGENERATE SYSTEMS AND SOME NEW POSSIBILITIES FOR PRODUCING SUCH SYSTEMS

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It is shown that the usual definition of elastic scattering of slow neutrons and of the probability for the Mössbauer effect are incorrect for degenerate systems. Some new systems in which one can have a Mössbauer effect are considered.

1. By elastic scattering of neutrons by an atomic system one usually means scattering in which the quantum state of the scattering system does not change, i.e., to within a trivial factor the probability of elastic scattering is

$$W = \sum_m g_m |\langle m | \exp(i\mathbf{pr}) | m \rangle|^2, \qquad (1)$$

where g_m is the probability of finding the system in state $|m\rangle$, $\mathbf{p}$ is the momentum transfer, and $\mathbf{r}$ is the coordinate of the scattering nucleus. [1)] But since the quantum state of a complex atomic system cannot be determined practically either before or after the scattering, such a definition of the scattering from a system having degenerate states reduces it to the status of an unobservable quantity, since the value of (1) then depends on what linear combination of degenerate wave functions one chooses as the functions $|m\rangle$.

Let us consider, for example, the case of a two-dimensional oscillator with equal frequencies ω_0, at temperature T, when the vector $\mathbf{p}$ lies in the plane of the vector $\mathbf{r}$. Choosing the coordinate axes so that one of them makes an angle φ with $\mathbf{p}$, and taking the wave functions in the form of products of wave functions for the one-dimensional oscillators along the coordinate axes, we get

$$W = \exp\left\{-\frac{p^2}{2M\omega_0}\operatorname{cth}\frac{\omega_0}{2T}\right\} I_0\left(\frac{p^2\sin^2\varphi}{2M\omega_0 \operatorname{sh}(\omega_0/2T)}\right) \times I_0\left(\frac{p^2\cos^2\varphi}{2M\omega_0\operatorname{sh}(\omega_0/2T)}\right) \qquad (2)^*$$

(where I_0 is the Bessel function of imaginary argument). Thus W depends on the arbitrary parameter φ and this dependence disappears only when the argument of I_0 tends to zero.

On the other hand one can define elastic scattering as a process in which, in the system of the center of mass of the neutron + the scattering system, the energy of the neutron is unchanged. Using Van Hove's formalism, we can write the probability for this process in the form

$$W' \equiv \overline{\sum_m g_m \langle m | \exp[i\mathbf{p}\hat{\mathbf{r}}(t)] \exp[-i\mathbf{p}\hat{\mathbf{r}}(0)] | m\rangle}, \qquad (3)$$

where $\hat{\mathbf{r}}(t)$ is the Heisenberg operator for the coordinate of the scattering nucleus; the bar denotes an average over an infinite time interval. [2)]

The second definition is identical with the first if the scattering system has no degenerate states. If this is not the case, the second definition includes a wider class of processes than the first, i.e. $W' \geq W$.

In the example of the two-dimensional oscillator,

$$W' \equiv \exp\left\{-\frac{p^2}{2M\omega_0}\operatorname{cth}\frac{\omega_0}{2T}\right\} I_0\left(\frac{p^2}{2M\omega_0\operatorname{sh}(\omega_0/2T)}\right) \qquad (4)$$

which coincides with W only for φ equal to 0, $\pi/2$, π and $3\pi/2$.

Completely analogous arguments are also applicable to the definition of the probability for the Mössbauer effect.

2. Let us consider the elastic scattering of neutrons and the Mössbauer effect in a crystal. In this case W and W′ coincide, since the corres-

[1)]For simplicity it is assumed that the neutrons are scattered by a single nucleus. We use the system of units in which $\hbar = k = 1$.

*sh = sinh, cth = coth.

[2)]Van Hove[1] defined W′ as the limit

$$\lim_{t\to\infty}\sum_m g_m \langle m | \exp[i\mathbf{p}\hat{\mathbf{r}}(t)]\exp[-i\mathbf{p}\hat{\mathbf{r}}(0)] | m\rangle.$$

When such a limit exists, expression (3) and Van Hove's expression coincide. When the limit does not exist, as for example in the important special case of scattering by an oscillator or a finite number of oscillators, the correct expression is formula (3).

ponding Bessel functions I_0 are practically equal to unity. The only case of special interest for both elastic scattering and the Mössbauer effect is that of an impurity atom whose motion to a large extent is determined by a small number of vibrations localized near this atom.[2]. Thus, if as a result of the crystal symmetry the corresponding frequencies are degenerate, there may be a difference between W and W′. But in actuality, because of various inhomogeneities of the crystal such a degeneracy may be lifted. On the other hand, one should include in the elastic scattering all processes in which the energy change is less than the "resolution" of the apparatus, and include in the Mössbauer effect the probability for those processes in which the energy changes within the limits of the line width Γ. In other words, if $\Gamma \gg \Delta\nu$, the frequency splitting due to lifting of the degeneracy, the probability for the Mössbauer effect is W′; if $\Gamma \ll \Delta\nu$, the probability is W; finally, if Γ is comparable to $\Delta\nu$, broadening and distortion of the Mössbauer line can occur.

3. We mention some new possibilities, in principle, for producing degenerate systems for which the quantity W is undefined, while W′ is different from zero, and consequently elastic scattering and a Mössbauer effect can be observed. These are any systems in which there is no unique relation between the energy and momentum of an atom, so that transfer of momentum to the atom is possible without changing its energy (if by the Mössbauer effect we mean the process of absorption or emission of a γ quantum by a nucleus of the system without changing the energy of the system, or with a change of energy within the limits of "resolution" of the apparatus). For example, a Mössbauer effect can occur when γ quanta are emitted or absorbed by an ionized atom in a strong magnetic field **H**, if the momentum **P** of the quantum is perpendicular to **H**.[3)] In the case of a uniform field, if we assume that the ions have a Gibbs distribution,

$$W' = \exp\left\{-\frac{E^2}{2Mc^2\omega^2}\operatorname{cth}\frac{\omega}{2T}\right\} I_0\left\{\frac{E^2}{2Mc^2\omega\operatorname{sh}(\omega/2T)}\right\}, \quad (5)$$

where $\omega = eHZ/Mc$, and Z is the degree of ionization of the atom.

As the numerical estimates show, under actual conditions W′ is extremely small and such experiments are hardly feasible by direct methods.

A second way of recoilless emission or absorption of γ quanta is possible only for nuclei with a magnetic moment μ_0 different from zero. In a magnetic field, for fixed momentum such nuclei have various energy values. For example, in a strong magnetic field, where the coupling of the nuclear spin J to the electron shell is broken, for a given momentum there are 2J + 1 energy values, equally spaced, with the interval equal to $\mu_0 H/\sqrt{J(J+1)}$. In other words, if the momentum transfer is such that

$$E^2/2Mc^2 = m\mu_0 H/\sqrt{J(J+1)}, \quad (6)$$

where m is an integer ($|m| \le 2J$), and this equality is satisfied to an accuracy of order $2\Gamma Mc^2/E^2 \approx 10^{-4} - 10^{-2}$ (the relative spread in the energy of the radiation because of the natural width of the line), one can have emission and absorption of a γ quantum with an energy equal to the energy of the excited nucleus, and also differing from it by a multiple of $\mu_0 H/\sqrt{J(J+1)}$. The transfer of kinetic energy is compensated by the change in energy of the nucleus in the magnetic field.

However, in this simple case the experimental detection of the effect seems to be possible only if one finds a nucleus with its first excited level at $E \lesssim 100$ eV and with a sufficiently low internal conversion coefficient. More realistic is the realization of such an experiment with paramagnetic ions in a "weak" field, where the coupling of the nuclear spin to the electron shell is not broken and the emission of a γ quantum can be accompanied by a change in the projection of the spin of the atom along the direction of the magnetic field.

If the separation of the hyperfine structure levels $\Delta\nu_{hf}$ were comparable with or larger than $E^2/2Mc^2$, formula (6) would again be valid, with the one difference that in place of μ_0 one should substitute the magnetic moment of the atom, i.e., a quantity which is 10^3 as large, and in place of J use the spin of the atom. If in addition the spread in initial kinetic energy of the atom ΔE is less than Γ, the probability for such transitions is given simply in terms of Clebsch-Gordan coefficients. In the opposite case, it is a fraction of order $(\Gamma/\Delta E)(2\Delta\nu_{hf}Mc^2/E^2)^2$.

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[1] L. Van Hove, Phys. Rev. **95**, 249 (1954).

[2] Yu. M. Kagan, and Ya. I. Ioselevskiĭ, JETP **42**, 259 (1962), Soviet Phys. JETP **15**, 182 (1962).

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[3)]The deviation from $\pi/2$ of the angle between the vectors **P** and **H** must not exceed $(2Mc^2\Gamma/E^2)^{1/2} \approx 10^{-2} - 10^{-1}$, where M is the mass of the nucleus.